561 Fall 2005 Lecture 10

Fermi Liquid Theory: Quantum Liquids

Short summary in Phillips, Sec. 11.5. Many good standard references: Pines and Nozieres, Quantum Liquids, Vol 1, Ch 1; Jones and March, Theoretical Solid State Physics, Sect. 2.9; Abrikosov, A., et.al., Quant. Field Th. Methods in Statistical Phys., Sect. 1.2; Nozieres, Theory of Interacting Fermi Systems. Review by Imada, et al., Sect. 2D.

1. What is Fermi Liquid Theory (FLT) ?

- Historically introduced by Landau in 1956 to describe liquid ^3He, a quantum liquid composed of strongly interacting fermions. (Here we deal with a system of electrons, which introduces the additional features due to charge of the particles and long range interactions.)
- FLT is a phenomenological approach which describes a liquid of strongly-interacting fermions as a weakly interacting systems of *quasiparticles* with interaction parameters that can be deduced from experiment
- FLT is NOT a description of the ground state. It is a theory of low energy excitations near the Fermi surface ($E << E_{Fermi}$) and low temperature properties ($T << T_{Fermi}$) of the interacting system of Fermions.
- Key Assumptions:
  - 1. Excitations of the interacting system are *quasiparticles* in one-to-one correspondence with states of a non-interacting system.
  - 2. Interactions included as effective interactions between the quasiparticles.
  - 3. In addition, interactions lead to *collective excitations.*

2. Excitations in a non-interacting system of Fermions

- Example of homogeneous gas (Jellium):
  Eigenvalues of non-interacting system: $\epsilon(k,\sigma) = k^2/2m$
  Ground state occupation (T=0): $n(k,\sigma) = 1,\ k < k_F,\ n(k,\sigma) = 0,\ k > k_F$
  Excited state energies: $E = E_0 + \sum_{k,\sigma} \epsilon(k,\sigma)\delta n(k,\sigma)$
  Specific Heat: $C_V = \gamma T \propto mT,\ \gamma \propto \rho(E_F)T$
  Pauli spin susceptibility: $\chi_{spin} \propto \gamma$
  Wilson ratio - χ_{spin}/γ = universal constant
- Metallic crystal
  Eigenvalues $\epsilon(k,\sigma)$ determined by crystal potential
  Ground state occupation: $n(k,\sigma) = 1$ for $\epsilon(k,\sigma) < \mu$; $n(k,\sigma) = 0$ for $\epsilon(k,\sigma) > \mu$.
  Same expression for excited state energies: $E = E_0 + \sum_{k,\sigma} \epsilon(k,\sigma)\delta n(k,\sigma)$
  Specific Heat $C_V = \gamma T \propto \rho(E_F)T$
  Pauli spin susceptibility: $\chi_{spin} \propto \gamma$
  Wilson ratio - χ_{spin}/γ = universal constant in all non-interacting electron systems
  Resistivity due to phonons: $\rho \propto T^{-5}$ at low T due to acoustic phonons; $\rho \propto T$ at high T $k_BT >> \hbar\omega_{phonon}$

- Insulating crystal
  Filled bands $\epsilon(k,\sigma)$ with a gap to excited states.
  No low energy electronic excitations.

3. Fermi Liquid Theory for interacting metallic system

- Example of homogeneous gas:
  Ground state energy E_0 considered a separate issue.
  Excitation energies for single quasiparticles at T=0 for $\epsilon \approx E_F$, $k \approx k_F$:
  $\epsilon(k,\sigma) = k^2/2m^*$, m^* = effective mass
  Ground state occupation $n(k,\sigma) = 1$, $k < k_F$, $n(k,\sigma) = 0$, $k > k_F$
  Excited state energies affected by effective interactions between excitations:
  $E - E_0 = \sum_{k,\sigma} \epsilon(k,\sigma)\delta n(k,\sigma) + \frac{1}{2}\sum_{k,\sigma,k',\sigma'} f(k,\sigma,k',\sigma')\delta n(k,\sigma)\delta n(k',\sigma') + \ldots$
  Specific Heat $C_V = \gamma T \propto m^* T$, i.e., the linear specific heat coefficient involves only the lowest order terms

- In an isotropic system the interactions are functions only of the relative orientation of $\mathbf{k}$ and $\mathbf{k}'$, and it asumed that only states with magnitude of $k = k_F$ are needed for $E \approx E_F$
  $f(\mathbf{k},\sigma,\mathbf{k}',\sigma') = f_s(\mathbf{k}\cdot\mathbf{k}') + f_a(\mathbf{k}\cdot\mathbf{k}')\sigma\cdot\sigma'$

- If the variation with θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$ is smooth, then the interaction can be described by a small number of parameters.
  Also we can define dimensionless parameters F_s and F_a to give:
  $F_s(\mathbf{k}\cdot\mathbf{k}') = \rho(E_F)f_s(\mathbf{k}\cdot\mathbf{k}') = \sum_l F_{s,l}P_l(cos(\theta)$,
  F_a = similar expression

- Parameters m^*, $F_{s,l}$, and $F_{a,l}$ are the Fermi Liquid parameters. It is reasonable that the parameters are non-zero for only a few low angular momenta l.

- Pauli spin susceptibility and Wilson ratio modified by $F_{a,0}$

4. Rationalization of Fermi Liquid Theory from our previous derivations of quasiparticles

- We have derived the Green's function for electrons from the Dyson's equation, *assuming* the excitations have the same quantum numbers as non-interacting particles:

  $G_\lambda(E) = G^0_\lambda(E) + G^0_\lambda(E)\Sigma^\star_\lambda(E)G_\lambda(E)$,

  or

  $G_\lambda(E) = \frac{1}{E-\epsilon^0_\lambda-\Sigma^\star_\lambda(E)}$

  where $\Sigma^\star_\lambda(E)$ is the proper self-energy.

- For energies near the Fermi energy, the self energy describes a well-defined quasiparticle of renormalized energy and imaginary energy (inverse lifetime) small $\propto (E - E_F)^2$.

- The Fermi surface is well-defined surface in k-space defined by analytic singularity in $n(\mathbf{k},\sigma)$.

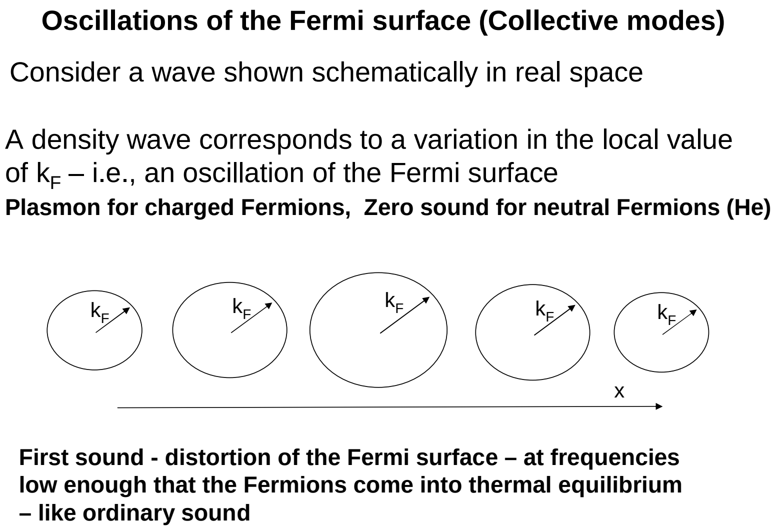

Figure 1: Collective oscillations of Fermi systems

- Follows from assumptions that interacting system is analytically related to non-interacting system and that perturbation expansion converges
- Luttinger theorem: Volume enclosed by Fermi surface same as for non-interacting system, determied simply by counting electrons
- Related to Friedel sum rule for impurities

- These aspects are consistent with the phenomenological FLT. For example, the interactions F_s and F_a, together with the phase space arguments for scattering of particles near the Fermi surface, lead to the scattering rate for quasiparticles $\propto |F|^2(E - E_F)^2$

5. Properties of a Fermi Liquid

- The quasiparticles have renormalized energies, which are well-defined at $T = 0$ at the Fermi energy, and have a scattering rate which varies as $\propto (E - E_F)^2$ at $T = 0$ and as $\propto T^2$ for $T \neq 0$

- Collective modes (See Figure)

  For a neutral Fermi Liquid (like helium) with short range potentials, sound waves are collective modes:

  - First Sound: Ordinary sound - density wave with the Fermi energy varying in space as the density varies
    Here one assumes the oscillations are slow enough that the Fermi energy is always in equilibrium with the local density
    First sound velocity slower than Fermi velocity
  - Zero Sound: Oscillation of the Fermi surface itself
    The Fermi surface is out of equilibrium for oscillations faster than relaxation

time of excitations
Zero sound velocity higher than Fermi velocity

For a charged Fermi Liquid collective excitations are plasmons with finite frequency ω as $q \to 0$

6. Response functions and instabilities of Fermi Liquid

- Interactions modify the linear response of the system to perturbations, i.e., the susceptiblities, such as magnetic susceptiblity, etc.
  - RPA-type expressions
- Interactions can lead to instabilities where excitations lower the energy $\to$ new ground state
  - Indicated by divergent susceptiblity
  - May lead to Broken symmetry solution with a qualitatively different ground state
  - Examples: Magnetic, charge density wave, superconductivity, ...
  - May lead to complete breakdown of FLT, e.g., a gap in the density of states
  - Continued in section on strong interactions: Broken symmetry and order parameters

7. Examples from experiment - strong support for FLT in many cases - also counter examples where FLT appears NOT to be sufficient
Examples given in class.

- Specific heat
- Susceptibility
- Resistivity
- DeHass-Van Alfen
- Photoemission