**561 Fall 2005 Lecture 17**

**Bosonization and Luttinger Liquids**

Phillips, Ch. 9

Original papers: J. M. Luttinger, J. Math. Phys 4, 1154 (1963) and D. C. Mattis and E. H. Lieb J. Math. Phys 6, 304 (1965).

Paper by Haldane that coined the phrase "Luttinger Liquid", F.D.M. Haldane, J. Phys. C 14, 2585 (1981)

Nice readable set of lectures by C. Kane at the Boulder Summer School, 2005. See http://research.yale.edu/boulder/Boulder-2005/Lectures/Kane/lectures.pdf

**1. Why is one dimension special? Qualitative description**

- Key point: In one dimension identical fermions cannot exchange position. They are in effect distinguishable because they can be numbered in order and the order never change order.
- Important developments for 1D
  Jordan-Wigner transformation - spinless fermions $\leftrightarrow$ $\sigma = 1/2$ spins
  Exact solutions using Bethe Ansatz - not covered here.
- Since the fermions cannot change order, all excitations are the same as for distinguishable particles in a line – the same as vibrations of atoms on a line as is illustrated in Fig. 1a.
- It may be difficult to find the exact mapping but we expect that there is always a mapping of the excitations of any fermion system in one dimension to a boson representation.
- Example: non-interacting electrons. Consider the spectrum of excitations of the form $c^\dagger_{k+q} c_k |0\rangle$. The spectrum is shown in Fig. 1b. For small q, the spectrum resembles a sound mode with dispersion $\epsilon_q = v_F q$. Thus the low energy excitations of the electron gas are similar to that of a 1D elastic medium. (There are also other excitations since the fermions are not restricted to the harmonic approximation.)
  Note the difference from a 2D or 3D spectrum.
- Interpretation as an oscillation of the Fermi surface.
- In the presence of interactions, there are still such modes. To see this, consider the opposite extreme of strongly interacting electrons, which form a Wigner crystal. This problem maps directly to a phonon problem!
- Correlation functions are different in 1-D compared to 3-D. Thermal vibrations at any temperature destroy long range order - well known that thermal vibrations destroy Bragg peaks, which have power law form instead of delta functions. (2-D is the critical dimension.) Thus we expect power law singularities in correlation functions in the electron problem.

**2. Tomanaga-Luttinger Model**

See Phillips, Ch. 9 and blackboard notes in class.

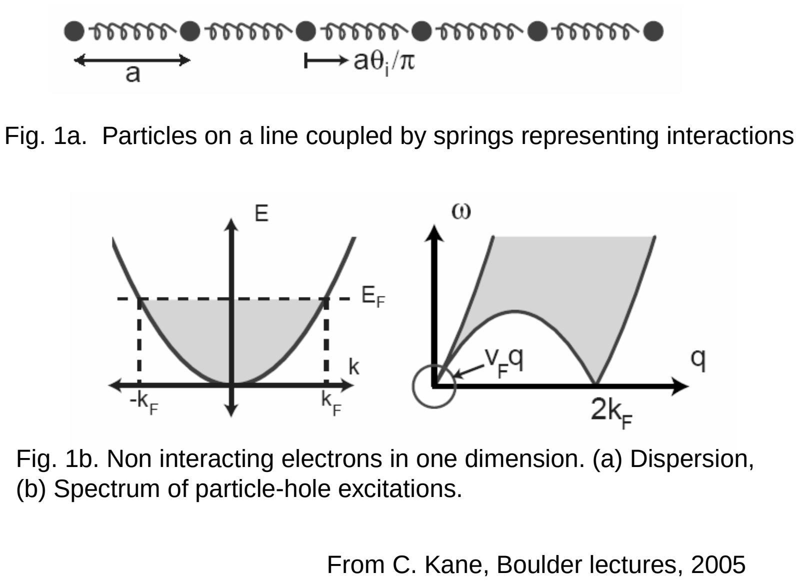


Figure 1: 1a. Schematic illustration the 1D phonon model that can represent low energy excitations in 1D Fermion systems. 1b. The spectrum for non-interacting fermions in 1D. Note the sound wave like mode at low energy. From C. Kane, Boulder School lectures, 2005