561 F 2005 Lecture 20

The Superconducting Condensed State: Gauge Invariance, Flux Quantization, Persistent Currents, Landau-Ginsburg Theory, Josephson Effect, SQUID devices

References: de Gennes, "Superconductivity of Metals and Alloys, Ch Ch 5-1,5-2; 6-1 - 6-5. (also p 118-121, p 234 - 246); Tinkham, "Intro. to Superconductivity", Ch 4,6; Brief summaries in Aschroft and Mermin and Kittel, "Intro. to SSP"

1. General relations for currents

- The current operator for particles with charge q and mass m has the form
  $\mathbf{j} = \frac{q}{m}(\mathbf{p} - \frac{q}{e}\mathbf{A})$; $\mathbf{p} = -i\hbar\nabla$
- If a many-body state ϕ_0 has current $\mathbf{j} = 0$, then the state $\phi = \phi_0 exp(i[S(r_1) + S(r_2) + ...])$ has current
  $\mathbf{j} = \frac{q}{m}\langle\phi|(\mathbf{p} - \frac{q}{c}\mathbf{A})|\phi\rangle = \frac{nq}{m}(\hbar\nabla S - \frac{q}{c}\mathbf{A})$.
- Note: Only the combination $(\hbar\nabla S - \frac{e}{c}\mathbf{A})$ enters the expressions. S and $\mathbf{A}$ are each subject to gauge transformations. We will use this in general property in several ways.
- BCS theory and experiment lead to the "particles" being pairs with charge $2e$ and mass $2m_e$

2. Bogoliubov Equations and Gauge Invariance:

- Generalizing the Bogoliubov transformation to spatially varying u(r) and v(r). If $Delta(r) = V\sum_n v_n^*(r)u_n(r)(1 - 2f_n)$ is the pair potential operator as before, but generalized to be a (slowly varying) function of position, then the Bogoliubov equations become:

  $\epsilon u(r) = H_0 u(r) + Delta(r)v(r)$
  $\epsilon v(r) = -H_0 v(r) + Delta^*(r)u(r)$
- Simple example of constant current with $Delta(r) = |Delta|exp(i2i\mathbf{r}\cdot\mathbf{r})$
  Example of "critical current" in simple case
- Gauge invariance: Physical quantities are invariant to
  $\mathbf{A}(r) \rightarrow \mathbf{A}(r) + \nabla\chi(r)$ and $Delta(r) \rightarrow Delta(r)exp(i\frac{2e}{\hbar c}\chi(r))$
- Restriction: $Delta(r)$ must be single valued

3. Flux Quantization and Persistent Currents:

- For a multiply connected superconductor of macroscopic dimensions. In the interior there are no currents, but supercurrents can flow of the boundary.
- The condition that $Delta(r)$ is single valued leads to quantization of flux through any hole completely surrounded by the superconductor.
  Flux $\Phi = N\Phi_0$, $\Phi_0 = \frac{ch}{2e}$

- Persistent currents must flow in the ring if the flux remains the same in the hole. But the flux can escape only if it passes through the superconductor. For a thick type I material the probability of a region becoming normal to let the flux escape exceeds the age of the universe (estimate in Kittel). Thus the "rigidity" of the SC state against an H field, together with the quantization condition, leads to persistent currents.
- Flux quantized through vortices of Type II superconductor
- Similar ideas will appear later in the quantum Hall effect, the Bohm-Aharonov effect, and general aspects of boundary conditions on wavefunctions.

4. Landau-Ginsburg Theory for the Free Energy

- Well before the BCS theory, Landau and Ginsburg recognized that the superconductor must be described by a complex order parameter Ψ, and that near T_c they could describe the free energy using the theory of phase transitions. (We will use the notation $\Psi \rightarrow \Delta$ since the BCS order parameter turns out to be the correct description, and we will use mass $= 2m_e$ and charge $= 2e$, which was follows from the BCS theory.)

  Free energy valid near $T = T_c$ where all length scales for the SC state are long:

  $F_{LG} = F_N + \frac{1}{8\pi}h^2 + A(T)|\Delta|^2 + \frac{1}{2}B(T)|\Delta|^4 + C|(-i\nabla - \frac{2e}{c}A)\Delta|^2$

  Key points: Exactly like a non-linear generalization of Schr. Eq. for a single particle if we set $C \rightarrow \frac{1}{2m}$, where m is the appropriate mass (which is $m = 2m_e$ as shown by the NCS theory). The magnetic field is included in the vector potential $\mathbf{A}$ that enters only in KE term. This describes the spatially varying superconducting state with a parameterized form that is valid near T_c.
- Two lengths:
  - Coherence length: $\xi(T) \propto (T_c - T)^{-0.5}$
  - Penetration Depth: $\lambda(T) \propto (T_c - T)^{-0.5}$
  - Landau-Ginsburg Parameter: $\kappa = \frac{\xi(T)}{\lambda(T)}$ independent of T
  - $\kappa >> 1 \rightarrow$ Type I; $\kappa << 1 \rightarrow$ Type II
- In simplest case (no scattering) of BCS,

  $\xi(T) = 0.74\xi_0(\frac{T_c}{T_c - T})^{\frac{1}{2}}$; $\lambda(T) = 0.707\lambda_L(\frac{T_c}{T_c - T})^{\frac{1}{2}}$

5. Josephson Effect

Discussed by deGennes on p. 118 and following.

- Tunneling of pairs - isomorphic to a quantum particle in a 1-d chain
- Supercurrent through barrier $I = -4\frac{e}{h}J_0 sink$.
- AC current for a constant voltage $I \propto sin(\frac{2eV}{h}t)$.
- General result for weak links $I \propto sin(\Delta\phi)$

6. SQUID devices
Interference Effects between two weak links
Sensitive to magnetic flux enclosed

7. Is a Gap required for superconductivity? No!
Examples where a gap clearly is not present:

- In any superconductor at T¿0, there are low energy electronic excitations (as found in specific heat) yet there are persistent currents
- In the mixed state of a type I or type II superconductor, the pairs are in intimate contact with normal regions which have no gap
- Josephson Effect - supercurrents flow through normal regions
- "Gapless" superconductivity in samples with magnetic impurities

The stability of the supercurrent is due to the collective nature of the state; macroscopic numbers of pairs together form the SC state, which can decay only by breaking up the entire state. See the clear statement in Aschroft and Mermin: the gauge arguments that "show" there must be persistent current even without a gap are a result of this cooperative effect