**Homework #4: Due 11-22–2024**

**PHYSICS 561 - Fall 2024**

1. Problem 11-1

2. Problem 11-2

3. Problem 12-4

4. Problem 12-7

5. Problem 13-1

6. Using the material in Sec. 12.5, prove that the Kondo interaction, $H'$ in Eq. 8.15, is indeed marginal relative to the kinetic term in Eq. 8.4.

7. The Hamiltonian for a two-site system is

$$H = t(a_1^\dagger a_2 + h.c.) + U a_1^\dagger a_1 a_2^\dagger a_2 \tag{1}$$

where the creation and annihilation operators create and annihilate electrons on sites 1 and 2. Both $t$ and $U$ have units of energy. Respectively, they represent the hopping energy between sites 1 and 2 and $U$ is the repulsive energy when electrons are placed on sites 1 and 2. Each site can hold at most one electron. Spin is not included here. So these are spinless fermions. a) Calculate the 0, 1 and 2 particle energies for this system. That is, diagonalize the Hamiltonian in the subspace of zero, single and double occupancy. b) Construct the grand canonical partition function for this system:

$$Z = \sum_i e^{\beta\mu n_i} Q_i \tag{2}$$

where $Q_i$ is the canonical partition function for $i$ particles. c) You should obtain a second order polynomial in the fugacity, $z = e^{\beta\mu}$. Any second order polynomial can be

factored. Factor this polynomial by writing it in the form

$$Z = \prod_i (1 + ze^{-\beta E_i}) \tag{3}$$

Solve for the $E_i's$ explicitly. What is the significance of these "energies"? Hint, think about the form of the factored partition function.

d) Construct an effective "Hamiltonian" in terms of the $E_i's$. Is your Hamiltonian Hermitian? If not elaborate on what the lack of Hermiticity means

e.) Your effective Hamiltonian is expressed in terms of new creation and annihilation operators. Find a mapping between the original creation and annihilation operators and the ones appearing in the efffective Hamiltonian. Don't get frustrated. This is an extremely difficult problem.