Physics 570 Final Exam, Fall 2017

Due 9:00am, Thursday, December 21, 2017

This is a Take-Home exam. For this exam, you may consult any textbooks and any 570 course material. You may NOT consult with any other person or use other person's note.

This exam is due on Thursday, December 21, 2017 at 9am. Please put your exam solutions, together with the question sheets, in the P570 Homework box. You are also very welcome to hand them to the instructor at Loomis 409.

Please fill out your online course evaluation form when you have a chance (Thanks!).

Have a great winter break, and happy holidays. See you next semester!

Academic Integrity

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including repulsion from the University.

Problem 1 (15 points)

The spin-averaged invariant amplitude squared for the $e^-\mu^- \to e^-\mu^-$ reaction is expressed in terms of the Mandelstam variables s, t, u as

$$\overline{|M|^2} = 2e^4 \frac{s^2 + u^2}{t^2}.$$

What are the $\overline{|M|^2}$ for the following reactions?

a) $e^-\mu^+ \to e^-\mu^+$

b) $e^-e^+ \to \mu^-\mu^+$

c) $e^+\mu^+ \to \mu^+e^+$

Problem 2 (20 points)

Consider the decays $\pi^- \to \mu^-\bar{\nu}_\mu$ and $\pi^- \to e^-\bar{\nu}_e$. Their decay rates have been derived in Section 11.6.1 of Thomson, and the ratio R, defined as

$$R = \frac{\Gamma(\pi^- \to e^-\bar{\nu}_e)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu)},$$

was also derived. These decays could have proceeded via a scalar coupling for the hadronic current, if charged-current interaction is a scalar interaction rather than a V-A interaction. In this case, we have the following form for the matrix (compare this with the expressions in Section 11.6.1 of Thomson)

$$M_{fi} = G_X f_\pi m_\pi \bar{u}(p_3) v(p_4),$$

where f_π^p is the pseudoscalar pion decay strength and G_X is analogous to the Fermi constant. This coupling now favors the same helicity for lepton and antilepton. What is the value of R in this case? Comment on this result from the helicity consideration. The experimental value for R can be used to set a limit on the strength of the scalar coupling.

Problem 3 (15 points)

Without working out the Feynman diagrams (unless you insist), what would be the polar angle dependence of the following reactions in the center-of-mass frame? Use the approach similar to that described on page 33 of the lecture note for Chapter 5.

a) $e^+ + e^- \rightarrow \gamma^* \rightarrow s^+ + s^-$

b) $s^+ + s^- \rightarrow \gamma^* \rightarrow s^+ + s^-$

c) $\bar{\nu}_e + \nu_e \rightarrow Z^0 \rightarrow \bar{\nu}_\mu + \nu_\mu$

s^- and s^+ are spin-0 point particles. Assume that the reactions occur at relativistic energies, and ignore the rest masses of the particles.

Note that the relevant rotation matrices are given below:

$$d^1_{01}(\theta) = -d^1_{10}(\theta) = -d^1_{0-1}(\theta) = d^1_{-10}(\theta) = \sqrt{\tfrac{1}{2}}\sin\theta$$

$$d^1_{11}(\theta) = d^1_{-1-1}(\theta) = \tfrac{1}{2}(1+\cos\theta)$$

$$d^1_{-11}(\theta) = d^1_{1-1}(\theta) = \tfrac{1}{2}(1-\cos\theta)$$

$$d^1_{00}(\theta) = \cos\theta$$

Problem 4 (15 points)

In proton-proton or antiproton-proton collider, W^--boson production is due to the process $q\bar{q}' \rightarrow W^-$.

a) For what quark pairs (u, d, s, c, b) is this possible (Ignore the t quark)?

b) Show by diagrams (no equations) that in the $q\bar{q}'$ center of mass frame the direction of spin of the W^- is determined.

c) In the reaction $q\bar{q}' \rightarrow W^- \rightarrow e^-\bar{\nu}$, will the electrons be emitted preferentially along the direction of the initial antiquark or quark?

Problem 5 (15 points)

Neutral current was first discovered in a neutrino scattering experiment. For neutrino scattering off a nucleus having equal number of protons and neutrons (isoscalar nucleus), show that the ratios of neutral to charged-current cross sections are

$$R = \frac{\sigma^{\nu N}(NC)}{\sigma^{\nu N}(CC)} = \frac{1}{2} - sin^2\theta_W + \frac{20}{27} sin^4\theta_W$$

and

$$\bar{R} = \frac{\sigma^{\bar{\nu} N}(NC)}{\sigma^{\bar{\nu} N}(CC)} = \frac{1}{2} - sin^2\theta_W + \frac{20}{9} sin^4\theta_W$$

Neglect the sea quarks.

Problem 6 (20 points)

State whether the following processes are possible or impossible, according to the Standard Model. If it is possible, state which interaction is primarily responsible (strong, electromagnetic, or weak). If it is only allowed by higher-order diagrams, state so. If it is not possible, explain why.

a) $K^- + p \rightarrow \Lambda + \pi^0$

b) $\eta \rightarrow \pi^+ + \pi^-$

c) $K^+ \rightarrow \pi^+ + \pi^0$

d) $\Sigma^+ \rightarrow n + e^+ + \nu_e$

e) $K^+ + n \rightarrow \pi^+ + \Lambda$

f) $K^+ \rightarrow \pi^+ + e^+ + e^-$

g) $e^+ + e^- \rightarrow \nu + \bar{\nu}$

h) $\rho^0 \rightarrow \pi^0 + \pi^0$

i) $\bar{\nu}_e + n \rightarrow e^- + p$

j) $d + d \rightarrow \pi^0 +^4 He$