Physics 570 Homework 1

Due on 5pm, Wednesday, September 13, 2017

Problem 1 (20 points)

In the derivation of the neutrino oscillation (see lecture note of Chapter 1), we assume that the neutrino mass eigenstates have identical momenta. Show that the same oscillation expressions can be derived by assuming that the neutrino mass eigenstates have identical energies (rather than identical momenta).

Problem 2 (20 points)

Consider the $\pi^+ \to \mu^+ + \nu_\mu$ decay with π^+ at rest. This process was used to make the direct measurement of muon-neutrino mass (or the upper limit of its mass).

a) Show that

$$E_{\mu^+} = \frac{m_{\pi^+}^2 + m_{\mu^+}^2 - m_{\nu_\mu}^2}{2m_{\pi^+}},$$

where E_{μ^+} is the total energy of μ^+, and m is the mass (Note that we set $c = 1$).

b) Show that

$$P_{\mu^+} = \frac{\sqrt{\lambda(m_{\pi^+}^2, m_{\mu^+}^2, m_{\nu_\mu}^2)}}{2m_{\pi^+}},$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, and P_{μ^+} is the momentum of the muon.

c) For ν_μ of finite but very small mass, show that the fractional shift of the muon momentum would be

$$\frac{\Delta P}{P} = -\frac{m_{\nu_\mu}^2 (m_{\pi^+}^2 + m_{\mu^+}^2)}{(m_{\pi^+}^2 - m_{\mu^+}^2)^2},$$

where $\Delta P = P' - P$, and P, P' are the μ^+ momentum for $m_{\nu_\mu} = 0$ and $m_{\nu_\mu} \neq 0$, respectively.

d) Find $\Delta P/P$ for $m_{\nu_\mu} = 0.19$ MeV, the current upper limit of the ν_μ mass determined from this decay. Note that $m_{\pi^+} = 139.57$ MeV and $m_{\mu^+} = 105.66$ MeV.

Problem 3 (30 points)

The LSND (Liquid Scintillator Neutrino Detector) experiment reported observation of neutrino oscillation for $\bar{\nu}_\mu \to \bar{\nu}_e$. This experiment used neutrinos from a beam dump, where the decay processes $\pi^+ \to \mu^+ + \nu_\mu$ and $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ generated sources of $\nu_\mu, \bar{\nu}_\mu, \nu_e$ (but not $\bar{\nu}_e$).

a) Under the assumption of $\bar{\nu}_e - \bar{\nu}_\mu$ mixing, derive the following expression for the probability of $\bar{\nu}_\mu$ oscillating into $\bar{\nu}_e$:

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = sin^2(2\theta)sin^2(\frac{1.27\ \Delta m^2\ L}{E}),$$

where θ is the mixing angle between $\bar{\nu}_e$ and $\bar{\nu}_\mu$, $\Delta m^2 = m_2^2 - m_1^2$. m_1, m_2 are the masses of the ν_1, ν_2 mass eigenstates, respectively (in unit of ev^2), L is the distance between the neutrino source and the neutrino detector (in unit of meter), and E is the energy of the neutrino (in unit of MeV).

b) LSND observed $P(\bar{\nu}_\mu \to \bar{\nu}_e) \approx 0.31\%$. The mean energy of the $\bar{\nu}_\mu$ beam, $\bar{E}$, is 45 MeV, and the mean distance betwen the beam dump and the detector, $\bar{L}$, is 30 meters. What is the minimal value for the mixing angle θ? What is the minimal value for Δm^2? Find $sin^2 2\theta$ for $\Delta m^2 = 1.0$ ev^2, 0.5 ev^2, and 0.1 ev^2.

Problem 4 (30 points)

Consider the case of neutrino oscillation for three generations (see Chapter I lecture note), derive the following expression:

$$P(\nu_e \to \nu_e) = 1 - sin^2 2\theta_{13} sin^2\Delta_{13} - sin^2\Delta_{12}(C_{13}^4 sin^2 2\theta_{12} + S_{12}^2 sin^2 2\theta_{13})$$
$$+S_{12}^2 sin^2 2\theta_{13}(\frac{1}{2} sin2\Delta_{12} sin2\Delta_{13} + 2sin^2\Delta_{13} sin^2\Delta_{12})$$

The definition of various symbols is given in the lecture note.