Physics 570 Homework 10

Due Wednesday, December 6, 2017

Problem 1 (20 points)

Consider a nucleus with a spherically symmetric charge distribution $\rho(r)$, where $\int \rho(r) d^3r = 1$. The form factor in electron scattering off nucleus can be expressed as

$$F(q) = \int \rho(r)\ e^{-i\vec{q}\cdot\vec{r}}\ d^3r$$

a) show that $F(q) = \frac{4\pi}{q} \int_0^\infty \rho(r)\ r\ sin(qr)\ dr$

b) show that $F(q) = 1 - \frac{q^2 <r^2>}{6} + ...$, where $< r^2 >$ is the mean square charge radius ($< r^2 >= 4\pi \int_0^\infty \rho(r) r^4 dr$).

c) Calculate $F(q)$ and $< r^2 >$ for $\rho(r) = \rho_0 e^{-\alpha r}$

d) Calculate $F(q)$ and $< r^2 >$ for $\rho(r) = \rho_0 e^{-\alpha r}/r$

Problem 2 (30 points)

Calculate the differential cross section of the $e^-e^+ \rightarrow \mu^-\mu^+$ reaction in the center-of-mass frame without making any approximations. Show that the total cross section is

$$\sigma(e^-e^+ \rightarrow \mu^-\mu^+) = \frac{4\pi\alpha^2}{3s}\sqrt{\frac{1-M^2/E^2}{1-m^2/E^2}}[1+\frac{1}{2}\frac{M^2}{E^2}][1+\frac{1}{2}\frac{m^2}{E^2}],$$

where E is the electron energy in the center-of-mass frame, m and M are the masses of the electron and muon, respectively.

Problem 3 (25 points)

Prove the Gottfried Sum Rule:

$$\int_0^1\ [F_2^p(x) - F_2^n(x)]/x\ dx = \frac{1}{3},$$

where F_2^p and F_2^n are the proton and neutron structure functions, respectively. Assume charge symmetry for the proton and neutron parton distributions (i.e., up-quark distribution in the proton is the same as the dwon-quark distribution in the neutron, anti-up quark distribution in the neutron is identical to the anti-down quark distribution in the proton, etc.). In addition, assume that the anti-up quark and anti-down quark distributions in the proton are identical. Experimentally, this Sum Rule was found to be broken.

Problem 4 (25 points)

Assume that the momentum distribution of up valence quarks, u_v, in the proton has the form

$$xu_v(x) = a_1 x^{0.5}(1-x)^2,$$

and the momentum distribution of anti-down valence quarks, $\bar{d}_v$, in the antiproton has the form

$$x\bar{d}_v(x) = a_2 x^{0.5}(1-x)^3,$$

where x is the Bjorken variable. Find a_1 and a_2. What is the fraction of proton's momentum carried by the up valence quarks? and by the down valence quarks?