Physics 570 Homework 3

Due Wednesday, September 27, 2017

Problem 1 (30 points)

In an earlier experiment, it was reported that a new particle, X, was found in the following reaction:

$$K^+ + d \to p + X$$

where d and p represents deuteron and proton, respectively. This reaction proceeds via strong interaction, which conserves quark flavors, isospin, parity, and so on.

a) What is the quark content of X? (The quark content of K^+ is $u\bar{s}$).

b) What are the possible values for the isospin of the particle X?

c) The following decays have been observed for the particle X:

$$X \to K^+ + n \quad \text{and} \quad X \to K^\circ + p,$$

where both decays proceed via strong interaction. What are the possible values for R, which is the ratio of the rates of these two decays?

$$R = \Gamma(X \to K^+ + n) \;/\; \Gamma(X \to K^\circ + p)$$

From the measurement of R, can the experimenters determine the isospin of the particle X?

Isospin of some particles:

p, n form isospin doublets ($I = 1/2$)

K^+, K° form isospin doublets ($I = 1/2$)

deuteron is an isospin singlet ($I = 0$)

Problem 2 (30 points)

a) Consider the following three reactions at a given center-of-mass energy:

(a) $\pi^+ + p \to K^+ + \Sigma^+$

(b) $\pi^- + p \to K^\circ + \Sigma^\circ$

(c) $\pi^- + p \to K^+ + \Sigma^-$

Through which isospin channels can these reactions proceed? Find the ratio of the cross sections for these reactions by assuming that one or the other isospin channel dominates.

b) At a given center-of-mass energy, find the ratio of cross sections for

(a) $p + d \rightarrow {}^3He + \pi^\circ$

(b) $p + d \rightarrow {}^3H + \pi^+$

c) Assuming isospin conservation, are the following reactions allowed?

(a) $d + d \rightarrow {}^4He + \omega$

(b) $d + d \rightarrow {}^4He + \eta$

(c) $d + d \rightarrow {}^4He + \rho^\circ$

(d) $p + d \rightarrow {}^3He + \eta$

(e) $p + p \rightarrow d + \rho^+$

Note that d and p represents deuteron and proton, respectively. The isospin of some particles are listed at the end.

Problem 3 (20 points)

Problem 9.3 of Thomson

Problem 4 (20 points)

Show that isospin conservation implies that the angular distribution of neutral pions produced in the reaction $n + p \rightarrow d + \pi^0$ should be symmetric with respect to 90° in the center-of-mass system.

(Actually, an experiment has clearly shown an asymmetry, providing evidence for isospin violation. See Phys. Rev. Lett. 91 (2003) 142302.)

Isospin of some particles:

$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$ form isospin quartets ($I = 3/2$)

$\Sigma^+, \Sigma^\circ, \Sigma^-$ form isospin triplets ($I = 1$)

π^+, π°, π^- form isospin triplets ($I = 1$)

$\rho^+, \rho^\circ, \rho^-$ form isospin triplets ($I = 1$)

η is an isospin singlet ($I = 0$)

ω is an isospin singlet ($I = 0$)

^3He,^3H form isospin doublets ($I = 1/2$)

^4He is an isospin singlet ($I = 0$)

Clebsch-Gordon coefficients $< JM|J_1, m_1, J_2, m_2 >$

$< 1, 1|1/2, 1/2, 1/2, 1/2 >= 1$

$< 1, 0|1/2, 1/2, 1/2, -1/2 >= \sqrt{1/2}$

$< 1, 0|1/2, -1/2, 1/2, 1/2 >= \sqrt{1/2}$

$< 1, -1|1/2, -1/2, 1/2, -1/2 >= 1$

$< 0, 0|1/2, 1/2, 1/2, -1/2 >= \sqrt{1/2}$

$< 0, 0|1/2, -1/2, 1/2, 1/2 >= -\sqrt{1/2}$

$< 3/2, 3/2|1/2, 1/2, 1, 1 >= 1$

$< 3/2, 1/2|1/2, -1/2, 1, 1 >= \sqrt{1/3}$

$< 3/2, 1/2|1/2, 1/2, 1, 0 >= \sqrt{2/3}$

$< 3/2, -1/2|1/2, 1/2, 1, -1 >= \sqrt{1/3}$

$< 3/2, -1/2|1/2, -1/2, 1, 0 >= \sqrt{2/3}$

$< 3/2, -3/2|1/2, -1/2, 1, -1 >= 1$

$< 1/2, 1/2|1/2, 1/2, 1, 0 >= -\sqrt{1/3}$

$< 1/2, 1/2|1/2, -1/2, 1, 1 >= \sqrt{2/3}$

$< 1/2, -1/2|1/2, -1/2, 1, 0 >= \sqrt{1/3}$

$< 1/2, -1/2|1/2, 1/2, 1, -1 >= -\sqrt{2/3}$