Physics 570 Homework 7

Due Wednesday, Nov. 1, 2017

Problem 1 (20 points)

How does $\bar{\psi}\sigma^{\mu\nu}\psi$ transform under proper Lorentz transformation? How does each of the six components of $\bar{\psi}\sigma^{\mu\nu}\psi$ transform under space inversion?

Problem 2 (25 points)

a) The operators

$$P_R \equiv \tfrac{1}{2}(1+\gamma^5), \quad P_L \equiv \tfrac{1}{2}(1-\gamma^5)$$

are the right-hand and left-hand projection operators. Show that they have the appropriate properties to be projection operators, namely,

$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = 0.$$

b) The operators to project out the positive and negative energy states for the Dirac spinors are

$$\Lambda_+ \equiv \tfrac{\not{p}+m}{2m}, \quad \Lambda_- \equiv \tfrac{-\not{p}+m}{2m}.$$

Show that they have the appropriate properties to be projection operators, namely,

$$\Lambda_-^2 = \Lambda_-, \quad \Lambda_+^2 = \Lambda_+, \quad \Lambda_+ + \Lambda_- = 1, \quad \Lambda_+\Lambda_- = 0.$$

Problem 3 (25 points)

In terms of a four-component spinor ψ, the right- and left-handed helicity states are defined as

$$\psi_R = \tfrac{1}{2}(1+\gamma_5)\psi,$$
$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\psi.$$

Show that

$$\bar{\psi}\gamma_\mu\psi = \bar{\psi}_L\gamma_\mu\psi_L + \bar{\psi}_R\gamma_\mu\psi_R,$$
$$\bar{\psi}\gamma_\mu\gamma_5\psi = \bar{\psi}_L\gamma_\mu\gamma_5\psi_L + \bar{\psi}_R\gamma_\mu\gamma_5\psi_R,$$
$$\bar{\psi}m\psi = \bar{\psi}_L m\psi_R + \bar{\psi}_R m\psi_L.$$

This exercise shows that the vector and axial vector currents do not connect spinors of different chirality, while the mass m (or scalar potential) flips helicity.

Problem 4 (30 points)

Define

$$\psi_L = \frac{1-\gamma_5}{2}\psi; \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

a) Show that the charge conjugate of ψ_L is right-handed and the charge conjugate of ψ_R is left-handed.

b) Under time-reversal transformation, what are the handedness of ψ'_L and ψ'_R?

c) Under parity transformation, what are the handedness of ψ'_L and ψ'_R?