P and CP violation

The weak interaction is special for two reasons:

- the interactions of the W and Z bosons treat left- and right-handed fermions differently, which violates parity symmetry P.
- the CKM matrix is complex instead of real, which violates a combination of charge conjugation symmetry and parity symmetry called CP.

We will first define how P and CP transformations act on fields, and then examine the phenomenological consequences of the violation of these symmetries by the weak interactions.

Parity transformations

As we briefly discussed many weeks ago, $P: (t, \vec{x}) \to (t, -\vec{x})$ implements spatial inversions and has a representation on 4-vectors as

$$P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}.$$

Note that this matrix has $\det = -1$, so it is not an element of $SO(3,1)$, but rather $O(3,1)$. Because of this, Lorentz-invariant equations of motion do not guarantee invariance under P. However, theories of free bosonic fields (scalars and vectors) are invariant under P (see Schwartz Sec. 11.5). Since $P^2 = 1$, it has eigenvalues ± 1. Spin-0 particles with $P = +1$ are called scalars, and those with $P = -1$ are called pseudoscalars. For example, $P|\pi^0\rangle = -|\pi^0\rangle$, so the pion is a pseudoscalar. If the Lagrangian of a theory is invariant under P, then parity is a multiplicative quantum number: the product of the parities of the initial states equals the product of the parities of the final states.

Similarly for spin-1: $P|V_0(t,\vec{x})\rangle = \pm|V_0(t,-\vec{x})\rangle$, $P|V_i(t,\vec{x})\rangle = \mp|V_i(t,-\vec{x})\rangle$. The parity is determined by the eigenvalue of the spatial component; gauge fields must transform like ∂_μ, so $A_i \to -A_i$ and A_μ has $P = -1$.

(spin-1 particles with $P=+1$ are called pseudovectors or axial vectors)

For fermions, we saw in our discussion of the Lorentz group that P exchanges L and R spinors. In 4-component notation,

$$P: \psi \to \gamma^0 \psi$$

Therefore, we can compute (suppressing the spacetime argument)

$$P: \bar\Psi\psi \to \underbrace{\psi^+\gamma^0}_{\bar\Psi}\gamma^0\gamma^0\psi = \bar\Psi\psi$$

$$P: \bar\Psi\gamma^\mu\psi \to \psi^+\gamma^0\gamma^0\gamma^\mu\gamma^0\psi = \bar\Psi(\gamma^\mu)^+\psi$$

Since $(\gamma^0)^+ = \gamma^0$ and $(\gamma^i)^+ = -\gamma^i$, the time and space components of this combination of spinors transforms just like a vector with $P=-1$.

Therefore,

$P: \bar\Psi \cancel{A}\psi \to \bar\Psi\cancel{A}\psi$ since spatial components are $\underset{\bar\Psi\gamma\psi}{(-1)}\underset{A_\mu}{(-1)} = +1$ and time components are $(+1)(+1) = +1$.

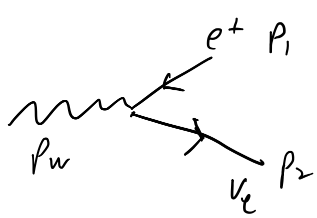
However, inserting a γ^5 changes the signs:

$P: \bar\Psi\cancel{A}\gamma^5\psi \to -\bar\Psi\cancel{A}\gamma^5\psi$. (for Z couplings, this term was what we called c_A)

A Lagrangian that mixes γ^μ with $\gamma^\mu\gamma^5$ (like the weak interaction!) is not symmetric under parity.

Example: polarization in W decay

$W \to e^+\nu_e$:



Ignoring constants: $M \propto \bar u(p_2)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)v(p_1)\,\epsilon_\mu(p_W)$

Say W is initially at rest: $p_W = (m_W, 0, 0, 0)$. Then 3 linearly independent polarization vectors are $\epsilon^x_\mu = (0,1,0,0)$, $\epsilon^y_\mu = (0,0,1,0)$, $\epsilon^z_\mu = (0,0,0,1)$. These satisfy $\epsilon^{(i)}\cdot\epsilon^{(j)} = -\delta^{ij}$, $\epsilon^{(i)}\cdot p_W = 0$.

Define z-axis as direction of outgoing neutrino. In limit of massless neutrinos, neutrino is always left-handed: spin opposite direction of motion, and only top two components of 4-component spinor are nonzero.

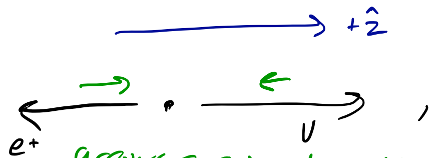
For neutrino energy E_ν, $u(p_2) = \sqrt{2E_\nu}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ (recall there is a very bad typo in Schwartz eq. (11.26)! See errata on Schwartz book website)

The $\begin{pmatrix}0\\1\end{pmatrix}$ in the upper two components specifies spin down along z axis.

Positron moves in $-z$ direction to conserve momentum. Positron spinors can be

$$v^{(1)}(p_1) = \begin{pmatrix}\sqrt{E_e - p_{ze}}\\0\\-\sqrt{E_e+p_{ze}}\\0\end{pmatrix} \text{ or } v^{(2)}(p_1) = \begin{pmatrix}0\\\sqrt{E_e+p_{ze}}\\0\\-\sqrt{E_e-p_{ze}}\end{pmatrix}.$$

Recall $v^{(1)}$ represents a spin-down positron while $v^{(2)}$ represents a spin-up positron.

Let's compute the squared amplitude for 

, i.e. use $v^{(2)}(p_1)$

Compute $\mathcal{M}$ for each W polarization, square, and average.

$$\mathcal{M}_x^{\uparrow} = \underbrace{\sqrt{2E_\nu}\,(0\,0\,0\,1)}_{\bar{u}(p_2) = u^\dagger(p_2)\gamma^0}\ \underbrace{\gamma^1}_{\gamma^\mu \epsilon^x_\mu}\ \underbrace{\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}}_{\frac{1-\gamma^5}{2} \equiv P_L}\ \underbrace{\begin{pmatrix}0\\\sqrt{E_e+p_{ze}}\\0\\-\sqrt{E_e-p_{ze}}\end{pmatrix}}_{v^{(2)}(p_1)} = \sqrt{2E_\nu}\sqrt{E_e+p_{ze}}\left(-(0\,1)\sigma^1, (0\,0)\right)\begin{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\\begin{pmatrix}0\\0\end{pmatrix}\end{pmatrix}$$

$$= 0.$$

Similarly, $\mathcal{M}_y^{\uparrow} = \sqrt{2E_\nu}\sqrt{E_e+p_{ze}}\left(-(0\,1)\sigma^2, (0\,0)\right)\begin{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\\begin{pmatrix}0\\0\end{pmatrix}\end{pmatrix} = 0.$

$$\mathcal{M}_z^{\uparrow} = \sqrt{2E_\nu}\sqrt{E_e+p_{ze}}\left(-(0\,1)\sigma^3, (0\,0)\right)\begin{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\\begin{pmatrix}0\\0\end{pmatrix}\end{pmatrix} = \sqrt{2E_\nu}\sqrt{E_e+p_{ze}}$$

$$\Rightarrow \frac{1}{3}\left(|\mathcal{M}_x^{\uparrow}|^2 + |\mathcal{M}_y^{\uparrow}|^2 + |\mathcal{M}_z^{\uparrow}|^2\right) \propto E_\nu(E_e + p_{ze}).$$

Interpretation: positron spin is an EPR-like measurement of W spin. Along neutrino axis, W could have had spin $-1, 0,$ or 1; only spin-0 has a nonvanishing amplitude, consistent with angular momentum conservation. (Recall $\epsilon^x \pm i\epsilon^y$ have eigenvalues ± 1 under J_z, ϵ^z has eigenvalue 0)

Momentum conservation: $p_{2e} = -E_\nu$. To find E_ν, $p_W = p_1 + p_2 \Rightarrow (p_W - p_2)^2 = p_1^2$,
so $m_W^2 - 2m_W E_\nu = m_e^2$, and $E_\nu = \frac{m_W^2 - m_e^2}{2m_W}$. $E_e = m_W - E_\nu = \frac{m_W^2 + m_e^2}{2m_W}$

$$E_\nu(E_e + p_{2e}) = \left(\frac{m_W^2 - m_e^2}{2m_W}\right)\left(\frac{m_W^2 + m_e^2}{2m_W} - \frac{m_W^2 - m_e^2}{2m_W}\right) \approx \frac{m_W}{2}\frac{m_e^2}{m_W^2}.$$

Note that this vanishes in limit $m_e \to 0$! If we repeated the calculation for the other positron spin, we would find $\langle |\mathcal{M}^\downarrow|^2 \rangle \propto \frac{m_W}{2} \times \mathcal{O}(1)$.
So the relative probability of positron having spin aligned w/direction of motion is $m_W^2/m_e^2 \sim 10^{10}$! Could use this to give an unambiguous definition of "left."
The m_e^2 penalty is known as helicity suppression and we will see it again next lecture.

CP transformations

Another discrete symmetry operation is charge conjugation, denoted C.
Roughly speaking, it takes a spin-up electron to a spin-down positron:

$C: \psi \to -i\gamma_2\psi^*$

Under C, $\bar\psi\psi \to \bar\psi\psi$ and $\bar\psi \not\partial \psi \to \bar\psi \not\partial \psi$ (see Schwartz 11.4), so free Dirac Lagrangian is invariant under charge congugation. For gauge interactions, $\bar\psi\gamma^\mu\psi \to -\bar\psi\gamma^\mu\psi$, so if we define $C: A_\mu \to -A_\mu$, then $\bar\psi \not{A} \psi$ is invariant. This is a bit weird since A is real, but note that $C^2 = 1$, so A_μ is still an eigenstate of C, just with eigenvalue -1.
We can also combine C and P to see under what conditions the SM Lagrangian is invariant under the combined transformation.

Can show the following transformation properties under CP:

$$\bar\psi_i\psi_j(t,\vec{x}) \to +\bar\psi_j\psi_i(t,-\vec{x}) \qquad \bar\psi_i\gamma^5\psi_j(t,\vec{x}) \to -\bar\psi_j\gamma^5\psi_i(t,-\vec{x})$$

$$\bar\psi_i \not{A} \psi_j(t,\vec{x}) \to +\bar\psi_j \not{A} \psi_i(t,-\vec{x}) \qquad \bar\psi_i \not{A}\gamma^5\psi_j(t,\vec{x}) \to \bar\psi_j \not{A}\gamma^5\psi_i(t,-\vec{x})$$

where $A = A, W, Z$ is any vector field.

Consider the part of the SM Lagrangian containing the W:

$$\mathcal{L}_W = \frac{e}{\sqrt{2}\sin\theta_W}\left[\bar u_i V_{ij} \not{W}^+ \left(\frac{1-\gamma^5}{2}\right) d_j + \bar d_i V_{ij}^\dagger \not{W}^- \left(\frac{1-\gamma^5}{2}\right) u_j\right]$$

Under C, complex fields transform to their conjugates, so C takes W^+ to W^-.
By the above, all the fermions transform by changing order but not sign, so

$$\mathcal{L}_W \xrightarrow{CP} \frac{e}{\sqrt{2}\sin\theta_W}\left[\bar{d}_j V_{ij} \cancel{W}^- \left(\frac{1-\gamma^5}{2}\right) u_i + \bar{u}_j V_{ij}^+ \cancel{W}^+ \left(\frac{1-\gamma^5}{2}\right) d_i\right]$$

In matrix form, $\bar{u} V \left(\frac{1-\gamma^5}{2}\right) d \rightarrow \bar{u} (V^T)^+ \left(\frac{1-\gamma^5}{2}\right) d = \bar{u} V^* \left(\frac{1-\gamma^5}{2}\right) d.$

So if $V = V^*$, i.e. if all CKM elements are real, CP is conserved.

However, as discussed last week, V has one complex phase, which is known (as you now can see) as a CP-violating phase.

This is not a basis-independent statement since we can always redefine the quark fields with phase rotations that leave the mass matrix invariant, but determinants are basis-independent:

$$\det\left[Y_u, Y_d\right] = -\frac{16}{v^6}(m_t - m_c)(m_t - m_u)(m_c - m_u)(m_b - m_s)(m_b - m_d)(m_s - m_d)\, J,$$

where J is the Jarlskog invariant $J = \sin\theta_{12}\sin\theta_{23}\sin\theta_{31}\cos\theta_{12}\cos\theta_{23}\cos\theta_{31}^2\sin\delta$

J vanishes if and only if the CP-violating phase $\delta = 0$.

CP violation and $K\bar{K}$ mixing

Let's look at some observable consequences of CP violation.

The lightest mesons containing strange quarks are the neutral kaons $K^0 = \bar{s}d$ and $\bar{K}^0 = \bar{d}s$. P is conserved in the strong interactions, so the parity of the kaon can be determined from its production:

$P|K^0\rangle = -|K^0\rangle$, $P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$. C exchanges particles and antiparticles, so $C|K^0\rangle = |\bar{K}^0\rangle$ and $C|\bar{K}^0\rangle = |K^0\rangle$

$\Rightarrow$ the CP eigenstates are linear combinations:

$$K_1 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0), \quad K_2 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)$$

$CP = -1$ $\qquad CP = +1$

The pions $\pi^0, \pi^\pm$ have $P = -1$. So a neutral state with two pions ($\pi^0\pi^0$, or $\pi^+\pi^-$) has $CP = +1$, and a state with three pions ($\pi^0\pi^0\pi^0$ or $\pi^0\pi^+\pi^-$) has $CP = -1$. If CP were conserved in the Standard Model, K_1 should never decay to $\pi\pi$. Since $m_{K^0} = 498$ MeV and $3m_\pi \approx 405$ MeV, there is a strong phase space suppression for the 3π decay, as well as factors of $\frac{1}{4\pi}$ from the additional $\frac{d^3p}{(2\pi)^3}$.