Dark matter

CMB observations tell us 85% of the mass of the universe does not interact with EM or the strong force; can't be neutrinos either (too light), so dark matter (DM) must be some particle beyond the SM.

Model-building for DM

Let's try writing down a Lagrangian that can describe "dark" DM. Only requirements are Lorentz and gauge invariance; at this point, anything goes! One way of organizing: look for renormalizable operators w/ new fields neutral under SM gauge group.

$$\mathcal{O}_{portal} = \{ F_{\mu\nu} F'^{\mu\nu},\ |H|^2 S^2,\ \bar{L}\tilde{H}N \}$$

"portal" to dark sector — dark photon — Higgs portal — RH neutrino portal

By way of example, let's focus on Higgs portal, with a new real scalar S.

$$\mathcal{L}_S = \tfrac{1}{2}\partial_\mu S\, \partial^\mu S - \tfrac{1}{2} m_S^2 S^2 - \lambda_{HS} S^2 H^+ H$$

After EWSB $\left(H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}\right)$, $\mathcal{L} \supset -\frac{\lambda_{HS}}{2} v^2 S^2 - \lambda_{HS} v S^2 h - \frac{\lambda_{HS}}{2} S^2 h^2$.

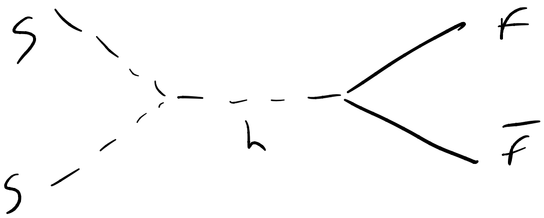
$m_S^2 \to m_S^2 + \lambda_{HS} v^2$, if $m_S \gg v$ this doesn't change the story.

Declare that S has a Z_2 symmetry $S \to -S$ so it's stable ($S|H|^2$ forbidden).

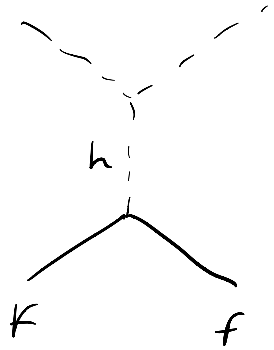
The Cliff notes for DM:

- need to annihilate in early universe to avoid overabundance:
  $SS \to SM\ SM$ $\hookrightarrow$ fixes some relation btwn λ_{HS} and m_S
- Can detect DM by:
  - scattering w/ SM particles ("direct detection")
  - annihilation into SM particles ("indirect detection")
  - making it at a collider ("collider production")

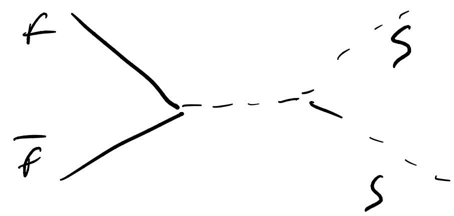
All of these are related by the <u>same</u> Feynman diagram(s):



DM abundance, indirect detection



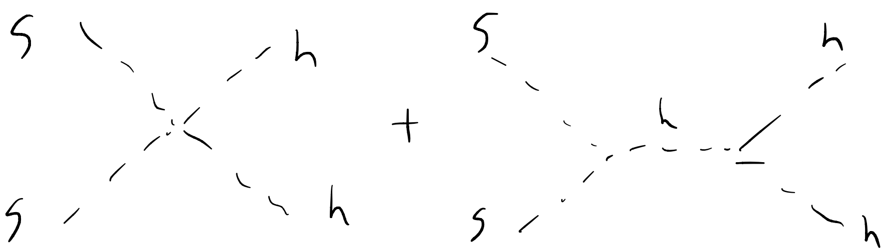
Direct detection



Collider production

Let's compute each in turn.

Suppose $m_S \gg m_h$. One annihilation channel is $SS \to hh$:



For a rough estimate, just use first diagram.

identical particles

$$|M|^2 = 4\lambda_{HS}^2 \text{ (2!2! in Feynman rule } S^2h^2\text{)}, \quad \sigma = \frac{1}{2E_1 2E_2 |v_1 - v_2|} \frac{1}{2} \frac{1}{8\pi} (4\lambda_{HS}^2)$$

For cosmology, the relevant quantity is actually σv_{rel} (really, a thermal average over Boltzmann distribution). When annihilation shuts off, S is just barely relativistic, so $E_1 = E_2 \simeq m_S$

$$\Rightarrow \sigma v_{rel} = \frac{\lambda_{HS}^2}{16\pi m_S^2}$$ (gain a factor of ~4 accounting for $SS \to WW, ZZ$)

To obtain correct amount of DM today ("relic abundance"), need $\sigma v_{rel} \simeq 10^{-26}\ \mathrm{cm^3/s} \Rightarrow \lambda_{HS} = 0.2\left(\frac{m_S}{1\,\mathrm{TeV}}\right)$, very reasonable!

(Note we will violate unitarity for $\lambda_{HS} \gtrsim 4\pi$, so $m_S \lesssim 75$ TeV for this model to be predictive.)

Direct detection

Look for DM scattering off atomic nuclei N. First look at nucleon:

(n = proton, neutron)

What is Higgs coupling to nucleons? $\mathcal{L} \supset h\bar{q}q$, so what we actually want is the matrix element $\langle n|\bar{q}q|n\rangle$ which is necessarily non-perturbative at low energies.

Let's first just parametrize the Higgs-nucleon coupling as f_N: $\mathcal{L}_{eff} = f_n h \bar{n} n$

$$iM = \frac{(-2\lambda_{HS} v)(f_n)}{t - m_h^2} \bar{u}(p_4) u(p_2) \qquad (t = (p_3-p_1)^2 = (p_2-p_4)^2)$$

$$\langle |M|^2 \rangle = \frac{2\lambda_{HS}^2 v^2 f_n^2}{(t-m_h^2)^2} \mathrm{Tr}[(\not{p}_4 + m_n)(\not{p}_2 + m_n)] = \frac{8\lambda_{HS}^2 v^2 f_n^2}{(t-m_h^2)^2}(p_2 \cdot p_4 + m^2)$$

Since DM is non-relativistic, we have to be a little careful with the kinematics:

$$\left.\begin{aligned} &p_1 = (m_S + \tfrac{1}{2} m_S v_{DM}^2, 0, 0, m_S v_{DM}), \quad p_2 = (m_n, 0, 0, 0) \\ &p_4 = (m_n + \tfrac{q^2}{2m}, q\sin\theta, 0, q\cos\theta), \quad p_3 = p_1 + p_2 - p_4 \end{aligned}\right\} \text{ to } \mathcal{O}(v^2)$$

$$\Rightarrow t = (p_2 - p_4)^2 = 2m_n^2 - 2(m_n^2 + \tfrac{q^2}{2}) = -q^2$$

q is the momentum transfer from DM to nucleon.

Since $q_{max} = 2 m_S v_{DM}$, and gravitational measurements tell us $v_{DM} \approx 10^{-3}$, $|q|^2 \ll m_h^2$ and we can approximate the denominator as $\sim m_h^4$.

$$\Rightarrow \langle |M|^2 \rangle = \frac{8\lambda_{HS}^2 v^2 f_n^2}{m_h^4}\left(2m_n^2 + \frac{q^2}{2}\right)$$

Now, can use d^3q instead of d^3p_4 in phase space integral since $\vec{q} = \vec{p}_4 - \vec{p}_2$, $E_4 \simeq m_n$, $E_3 \simeq m_s$

$$\sigma_n = \frac{1}{4 m_s m_n v_{DM}} \int \frac{d^3q}{(2\pi)^3 2m_n} \frac{d^3p_3}{(2\pi)^3 2m_s} (2\pi)^4 \delta(p_1+p_2-p_3-p_4) \langle |M|^2 \rangle$$

$$= \frac{1}{16 m_s^2 m_n^2 v_{DM}} \frac{8\lambda_{HS}^2 v^2 f_n^2}{m_h^4} \int \frac{q^2 dq\, d\cos\theta}{(2\pi)^2} (2\pi)\left(2m_n^2 + \frac{q^2}{2}\right)\delta(E_i - E_f)$$

$$E_i - E_f = \frac{1}{2} m_s v_{DM}^2 - \frac{q^2}{2m_n} - \frac{(m_s \vec{v}_{DM} - \vec{q})^2}{2m_s} = \vec{q}\cdot\vec{v}_{DM} - \frac{q^2}{2m_s} - \frac{q^2}{2m_n}$$

$$= q v_{DM} \cos\theta - \frac{q^2}{2\mu_{sn}}$$

reduced mass

$\Rightarrow$ use δ-function to do $\cos\theta$ integral

$$\delta\left(q v_{DM}\cos\theta - \frac{q^2}{2\mu_{sn}}\right) = \frac{1}{q v_{DM}} \delta\left(\cos\theta - \frac{q}{2 v_{DM} \mu_{sn}}\right)$$

$$\sigma_n = \frac{\lambda_{HS}^2 v^2 f_n^2}{4\pi m_s^2 m_n^2 v_{DM} m_h^4} \int_0^{2\mu_{sn} v_{DM}} q\left(2m_n^2 + \frac{q^2}{2}\right) dq$$

gives $\mathcal{O}(v_{DM}^2)$, subleading

$$= \frac{1}{\pi} \lambda_{HS}^2 f_n^2 \frac{v^2 \mu_{sn}^2}{m_s^2 m_h^4}$$

For $m_s \gg m_n$, $\mu_{sn} \simeq m_n$, and $\sigma_n \simeq \frac{1}{\pi} \lambda_{HS}^2 f_n^2 \frac{v^2 m_n^2}{m_s^2 m_h^4}$

Two remaining ingredients: determine f_n and compute σ_N, the cross section from a nucleus composed of many nucleons.

Higgs couples to all quarks: $f_n = \sum_q \frac{m_q}{m_n} \langle n | \bar{q} q | n \rangle$. Things like $\langle n | u\bar{u} + d\bar{d} | n \rangle$ come from chiral perturbation theory; since Higgs couples more strongly to heavier quarks, dominant contribution is from strange quark content of nucleon, $\langle n | \bar{s} s | n \rangle \simeq 0.5$.

$\Rightarrow$ take $f_n \simeq \frac{m_s}{v}$ ← strange quark mass, not DM mass! $\simeq 10^{-3}$

If nucleus were just a bag of nucleons,

$\sigma_N = \frac{\mu_{SN}^2}{\mu_{Sn}^2} A^2 \sigma_n$. But at large momentum transfer, we lose coherence over nucleons and need to include a nuclear form factor $F_N(q^2)$, which starts to differ from 1 at $q \sim \frac{1}{R_N} \sim$ MeV.

Ignore this for now; take $A = 131$ for xenon, target mass of 1 ton $= 5 \times 10^{27}$ xenon nuclei, suppose $m_S = 1$ TeV $\Rightarrow \lambda_{HS} = 0.2$

$$R = N_{Xe}\, n_{DM}\, \sigma_N\, v_{DM} = 5 \times 10^{27} \left(\frac{0.3\ \text{GeV/cm}^3}{1\ \text{TeV}} \right) \underbrace{(131^2)}_{m_{Xe}/m_n} \underbrace{(131^2)}_{\simeq A} \underbrace{(10^{-3})}_{v_{DM}} \times$$

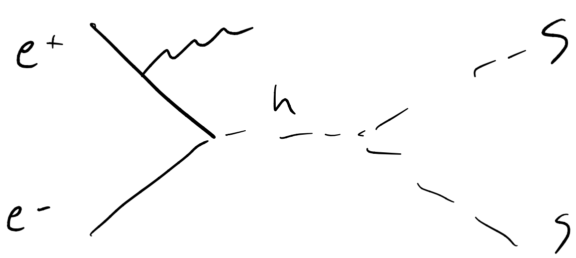
$$\left(\frac{2}{\pi} (0.2)^2 (10^{-6}) \frac{(246\ \text{GeV})^2 (1\ \text{GeV})^2}{(1\ \text{TeV})^2 (125\ \text{GeV})^4} \right)$$

$$\simeq 3 \times 10^{-5}\ \text{Hz} = 1000\ \text{events/yr}$$

Can be probed by XENON-1T!

Collider production

Let's make S at a collider. Can obviously do $f\bar{f} \to h \to SS$, but this is a fully invisible final state, and it's hard to trigger on "nothing." Say we have a lepton collider. We could radiate a photon:

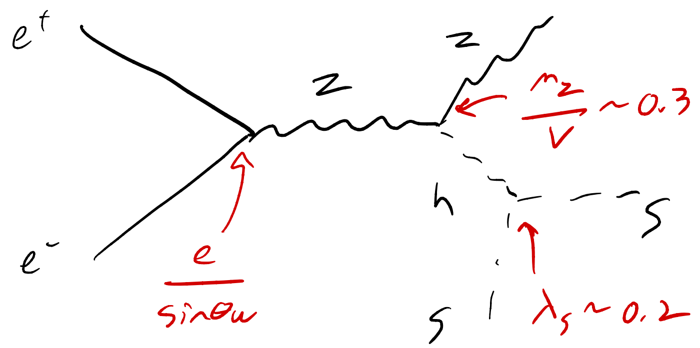


$e^+ e^- \to \gamma +$ invisible

"mono-photon"

But electron Yukawa is small: $y_e = \frac{\sqrt{2}\, m_e}{v} \sim 10^{-5}$.

How do we exploit a large coupling?



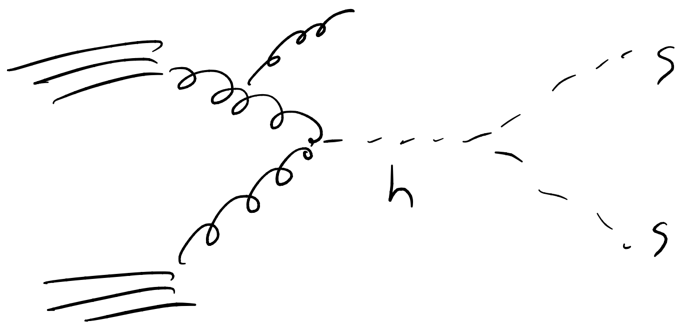
$e^+e^- \to Z + \text{inv.}$

("mono-Z")

Only large couplings!

If Z decays via $\mu^+\mu^-$ or $q\bar{q}$, this is a very clean signal given the large missing energy, but need $\sqrt{s} > 2m_S + m_Z$.

At a hadron collider, want to exploit top quark coupling:



"mono-jet"

Harder because we have to deal with all the soft QCD gunk which did not go into the gluon PDF, but this is a key search strategy at the LHC.