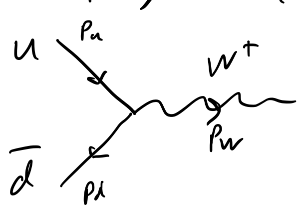
Discovery of the W/Z and Higgs

The predictions of electroweak symmetry breaking were confirmed in spectacular fashion with the discovery of the W and Z bosons at CERN in 1983, and the discovery of the Higgs boson in 2012. Today we will survey these processes, which took place at proton-proton colliders, and additionally examine the precision electroweak tests that can take place at electron-positron colliders. Throughout, we will exploit the simplifications of the narrow-width approximation to factorize production and decay:

$$\sigma(\text{initial state} \longrightarrow X \rightarrow \text{final state}) \simeq \sigma(\text{initial state} \rightarrow X) \times Br(X \rightarrow \text{final state})$$

W production in $p\bar{p}$ collisions

From the W coupling to quarks, the following diagram exists:

$i\mathcal{M}_{u\bar{d}\rightarrow W^+} =$  $= \frac{ig}{\sqrt{2}} V_{ud}^* \bar{v}(p_d)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u(p_u)\,\epsilon^*_\mu(p_W)$

(mnemonic: u-type arrows going in to a vertex get CKM*; this will be important later)

As we saw when we discussed QCD, we need to weight this matrix element by the parton distribution function of the proton, which counts quarks. At energies >100 GeV, the proton's quark content is mostly u and d valence quarks, so this diagram suffices.

This is very similar to the $t \rightarrow bW$ diagram we computed last time. Indeed, all that changes is $V_{tb} \rightarrow V_{ud}$ and a $\bar{v}$ instead of a $\bar{u}$ spinor. But since the only difference is the sign of the quark mass term in the trace, and the terms proportional to m_t vanished, we can just borrow the result from last time, with a slightly different prefactor:

$$\langle|\mathcal{M}|^2\rangle = \frac{g^2}{12}|V_{ud}|^2\left(p_u\cdot p_d + \frac{2(p_u\cdot p_W)(p_d\cdot p_W)}{m_W^2}\right)$$

average over spins and colors: 2x2 for spins, only 3 colors since W doesn't change quark color

This time, we have $p_u + p_d = p_W$. Defining $(p_u+p_d)^2 = \hat{s}$, the dot products are $p_u\cdot p_d = \frac{\hat{s}}{2}$, $p_u\cdot p_W = p_d \cdot p_W = \frac{m_W^2}{2}$, so $\langle|\mathcal{M}|^2\rangle = \frac{g^2}{12}|V_{ud}|^2\left(\frac{\hat{s}}{2} + \frac{m_W^2}{2}\right)$

$\sigma(u\bar{d} \to W^+) = \frac{1}{2\hat{s}} \int d\Pi_1 \langle |M|^2 \rangle$ where

$$\int d\Pi_1 = \int \frac{d^3 p_W}{(2\pi)^3 2E_W} (2\pi)^4 \delta^{(4)}(p_u + p_d - p_W) = 2\pi\delta(\hat{s} - m_W^2)$$

(as we've alluded to before, 1-particle phase space has one unresolved δ-function)

Therefore we can set $\hat{s} = m_W^2$ in the matrix element, giving

$$\sigma(u\bar{d} \to W^+) = \frac{1}{\cancel{2\hat{s}}} \cancel{2}\pi \left(\frac{g^2}{12} |V_{ud}|^2 (\cancel{m_W^2}) \right) \delta(\hat{s} - m_W^2)$$

$$= \frac{\pi g^2}{12} |V_{ud}|^2 \delta(\hat{s} - m_W^2)$$

$$= \frac{\pi^2 \alpha_W}{3} |V_{ud}|^2 \delta(\hat{s} - m_W^2) \text{ where } \alpha_W = \frac{g^2}{4\pi} \text{ (weak "fine-structure constant")}$$

Integrating over PDF's,

$$\sigma(p\bar{p} \to W^+) = \int dx_1 dx_2 \left[f_u(x_1) f_{\bar{d}}(x_2) \sigma(u(x_1 P_1) \bar{d}(x_2 P_2) \to W^+) + 1 \leftrightarrow 2 \right]$$

where P_1 and P_2 are the initial $p/\bar{p}$ 4-momenta:

$$P_1 = \left(\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\right), \quad P_2 = \left(\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\right)$$ (s not $\hat{s}$! Protons have the full center of mass energy)

$$\Rightarrow p_W = x_1 P_1 + x_2 P_2 = \left((x_1 + x_2)\frac{\sqrt{s}}{2}, 0, 0, (x_1 - x_2)\frac{\sqrt{s}}{2}\right).$$

This looks more symmetric if we parameterize p_W in terms of rapidity Y:

$$p_W = (\sqrt{\hat{s}}\cosh Y, 0, 0, \sqrt{\hat{s}}\sinh Y) \quad \text{where } p_W^2 = \hat{s}$$ (which we leave free for now)

Change variables $(x_1, x_2) \to (\hat{s}, Y)$:

$$(x_1 + x_2)\frac{\sqrt{s}}{2} + (x_1 - x_2)\frac{\sqrt{s}}{2} = \sqrt{\hat{s}}(\cosh Y + \sinh Y)$$ (equating $p_W^0 + p_W^3$ in both coordinates)

$$\Rightarrow x_1 = \frac{\sqrt{\hat{s}}}{\sqrt{s}} e^Y, \quad \text{similarly } x_2 = \frac{\sqrt{\hat{s}}}{\sqrt{s}} e^{-Y}$$

$$\frac{\partial(x_1, x_2)}{\partial(\hat{s}, Y)} = \begin{vmatrix} \frac{e^Y}{2\sqrt{\hat{s}}\sqrt{s}} & \frac{e^{-Y}}{2\sqrt{\hat{s}}\sqrt{s}} \\ \frac{\sqrt{\hat{s}} e^Y}{\sqrt{s}} & \frac{-\sqrt{\hat{s}} e^{-Y}}{\sqrt{s}} \end{vmatrix} = \frac{1}{s}$$

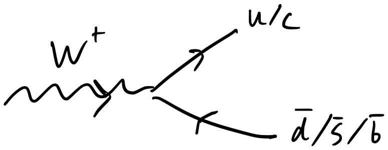
$$\Rightarrow dx_1 dx_2 \, \delta(\hat{s} - m_W^2) = \frac{1}{s} d\hat{s} \, dY \, \delta(\hat{s} - m_W^2)$$

$$\Rightarrow \sigma(p\bar{p} \to W^+) = \frac{\pi^2 \alpha_W}{3s} |V_{ud}|^2 \int dY \left[f_u\left(\frac{m_W}{\sqrt{s}} e^Y\right) f_{\bar{d}}\left(\frac{m_W}{\sqrt{s}} e^{-Y}\right) + f_{\bar{d}}\left(\frac{m_W}{\sqrt{s}} e^Y\right) f_u\left(\frac{m_W}{\sqrt{s}} e^{-Y}\right) \right]$$

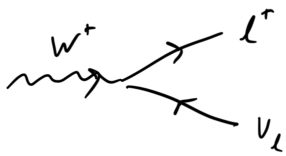
Note that once we know the W exists, this process can be used to measure the PDF's!

W decays

Two kinds of decay processes, which look very different at colliders:



"hadronic" (note $m_W < m_t$, so W can't decay to top quarks)



"leptonic" ($\ell = e, \mu, \tau$)

These matrix elements are very similar to the production matrix element: only differences are color sums and CKM elements.

Hadronic: sum $\to$ average over W spins $\quad 1 \to \frac{1}{3}$

average $\to$ sum over quark spins: $\quad \frac{1}{4} \to 1$

average $\to$ sum over quark colors: $\quad \frac{1}{3} \to 3$

$\Rightarrow$ overall factor of 12 in the matrix element

$$\langle |M| \rangle^2 = g^2 |V_{CKM}|^2 m_W^2 = 4\pi \alpha_W |V_{CKM}| m_W^2$$

So e.g.

$$\Gamma_{W\to c\bar{s}} = \frac{1}{2m_W} \frac{1}{8\pi} 4\pi\alpha_W |V_{cs}|^2 m_W^2 = \frac{\alpha_W M_W}{4} |V_{cs}|^2$$

2-body phase space for $m_W \gg m_c, m_s$: $\frac{1}{16\pi^2}\frac{p_f}{m_W}\int d\Omega = \frac{1}{16\pi^2}\frac{1}{2}(4\pi) = \frac{1}{8\pi}$

However, there are also QCD corrections from quarks emitting final-state gluons,

so

$$\Gamma_{W\to \text{jets}} = \frac{\alpha_W M_W}{4}\left(1 + \frac{\alpha_s(m_W)}{\pi}\right)\Big[\underbrace{|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2}_{=1 \text{ by unitarity}} + \underbrace{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2}_{=1}\Big]$$

$$= 1.399 \text{ GeV}$$

$\Gamma_{tot} = 2.085$ GeV from PDG, so predict $Br(W\to \text{jets}) = \dfrac{\Gamma_{W\to\text{jets}}}{\Gamma_{tot}} = 67.1\%$

Experimentally, $Br(W\to \text{jets}) = 67.41\%$; not bad! (QCD corrections important)

For leptonic decay, no sum over colors: $\Gamma_{W\to e\bar{\nu}} = \dfrac{\alpha_W M_W}{12}$, equal for e, μ, τ up to phase space effects for nonzero m_ℓ. Again, well-supported by data.

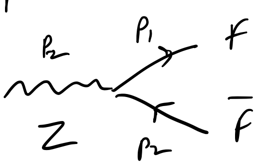
Even without knowing W mass precisely, can predict ratios of branching ratios:

$$\frac{Br(W^+\to e^+\nu_e)}{Br(W^+\to \text{hadrons})} = \frac{1/3}{\left(1+\frac{\alpha_s}{\pi}\right)\sum_{\substack{i=u,c\\ j=d,s,b}}|V_{ij}|^2} = \frac{1}{6\left(1+\frac{\alpha_s}{\pi}\right)} \quad \text{since } \sum_{\substack{i=u,c\\ j=d,s,b}}|V_{ij}|^2 = 2$$

Measurement of W mass is a little tricky; for 2-jet events, $(p_{j_1}+p_{j_2})^2 = m_W^2$, but lots of QCD background. Instead, use transverse mass derived from leptonic decays (important implications for recent CDF W-mass result!)

Z boson decays

For HW, you will calculate the Z production cross section at e^+e^- colliders; here we will focus on the decay modes. The Z boson couples to all SM fermions:



$$iM = \frac{ig}{\cos\theta_W}\bar{v}(p_2)\left(T^3\gamma^\mu P_L - Q\sin^2\theta_W\gamma^\mu\right)u(p_1)\epsilon_\mu(p_2)$$
$$= \frac{ig}{2\cos\theta_W}\bar{v}(p_2)\left(T^3\gamma^\mu(1-\gamma^5) - 2Q\sin^2\theta_W\gamma^\mu\right)u(p_1)\epsilon_\mu(p_2)$$
$$\equiv \frac{ig}{2\cos\theta_W}\bar{v}(p_2)\left(c_V\gamma^\mu - c_A\gamma^\mu\gamma^5\right)u(p_1)\epsilon_\mu(p_2)$$

Here, $c_V \equiv T^3 - 2Q\sin^2\theta_W$ and $c_A \equiv T^3$ are "vector" and "axial-vector" couplings.

This way of writing things makes spinor products in 4-component notation easier:

$$[\bar{u}\gamma^\mu\gamma^5 v]^\dagger = v^\dagger\gamma^5\underbrace{(\gamma^\mu)^\dagger\gamma^0}u = v^\dagger\gamma^5\gamma^0\gamma^\mu u = -\bar{v}\gamma^5\gamma^\mu u = +\bar{v}\gamma^\mu\gamma^5 u$$

For example, for $f=e$, $T^3 = -\frac{1}{2}$ and $Q=-1$, $c_V = -\frac{1}{2}+2\sin^2\theta_W$, $c_A = -\frac{1}{2}$.

$$\text{So } \langle |M_{Z\to f\bar{f}}|^2\rangle = \frac{N_c}{3}\frac{g^2}{4\cos^2\theta_W}\sum_{\text{spins}}\bar{v}(p_2)(c_V\gamma^\mu - c_A\gamma^\mu\gamma^5)u(p_1)\bar{u}(p_1)(c_V\gamma^\nu - c_A\gamma^\nu\gamma^5)v(p_2)\epsilon^*_\mu(p_2)\epsilon_\nu(p_2)$$

w/ factor of $N_c = 3$ if f is a quark

$$= \frac{N_c}{3}\frac{g^2}{4\cos^2\theta_W}\text{Tr}\left[\not{p}_1\gamma^\nu(c_V - c_A\gamma^5)\not{p}_2\gamma^\mu(c_V - c_A\gamma^5)\right]\left(-\eta_{\mu\nu} + \frac{p_{2\mu}p_{2\nu}}{m_Z^2}\right) \quad (\text{setting } m_f = 0)$$
$$= \frac{N_c}{3}\frac{g^2}{4\cos^2\theta_W}\text{Tr}\left[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu(c_V - c_A\gamma^5)(c_V - c_A\gamma^5)\right]\left(-\eta_{\mu\nu} + \frac{p_{2\mu}p_{2\nu}}{m_Z^2}\right)$$
$$= \frac{N_c}{3}\frac{g^2}{4\cos^2\theta_W}\text{Tr}\left[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu(c_V^2 + c_A^2 - 2c_Vc_A\gamma^5)\right]\left(-\eta_{\mu\nu} + \frac{p_{2\mu}p_{2\nu}}{m_Z^2}\right)$$

As with top quark decay, the γ^5 trace is proportional to the antisymmetric tensor $\epsilon^{\mu\nu\alpha\beta}$, so it vanishes when contracted with the polarization sum. The 4-vector products are identical to previous calculations, so we can just skip to the answer:

$$\langle |M| \rangle^2 = \frac{N_c g^2}{3\cos^2\theta_W}\left(p_1 \cdot p_2 + \frac{2(p_1\cdot p_Z)(p_2\cdot p_Z)}{m_Z^2}\right)(c_V^2+c_A^2)$$

$$= \frac{N_c g^2 m_Z^2}{3\cos^2\theta_W}(c_V^2+c_A^2)$$

$$\Gamma_{Z\to f\bar{f}} = \frac{1}{2m_Z}\frac{1}{8\pi}\langle |M_{Z\to e^+e^-}|^2\rangle = \frac{N_c \alpha_W m_Z}{12\cos^2\theta_W}(c_V^2+c_A^2)$$

As with W's, this predicts:

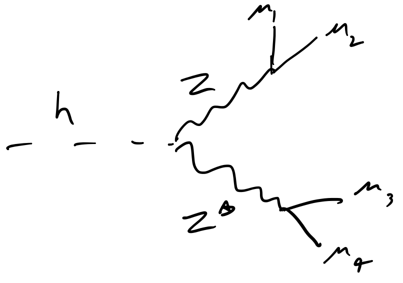
- equal branching fractions into $e/\mu/\tau$, up to mass effects (HW)
- hadronic decays enhanced by a factor of 3 for color, but also $c_V^2+c_A^2$ is different! In the end, 70% to hadrons vs. 30% to charged leptons + neutrinos.
- Decay products are polarized! Indeed, W decay products are fully polarized (in massless approximation), since W only couples to L spinors, but Z decays are partially polarized, depending on fermion (HW)
- Easy to reconstruct mass of Z at e^+e^- collider: look for events with $\mu^+\mu^-$, $m_Z^2 = (p_{\mu^+}+p_{\mu^-})^2$ (see plot on course webpage)

Discovery of the Higgs

Finally, let's examine the last piece of the Standard Model. For HW, you will calculate $H \to b\bar{b}$ and $H \to WW, ZZ$. Since Higgs couplings are proportional to mass, we should try to produce it and detect it with the heaviest initial- and final-state particles possible. However, perversely, $m_h < 2m_t$ and $m_h < 2m_W$, so decays into on-shell tops or gauge bosons are kinematically forbidden. Even worse, $m_b \simeq 0.02\, m_t$, so decays to b's are smaller by $\sim 10^4$, and 2-jet events have an enormous QCD background! To find the Higgs at the LHC, experimentalists and theorists had to get creative.

Two strategies:

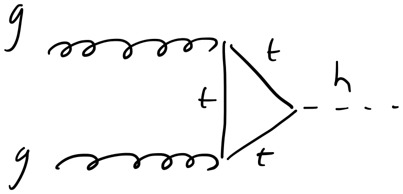
1) off-shell gauge bosons, $H \to Z Z^* \to \mu^+ \mu^- \mu^+ \mu^-$

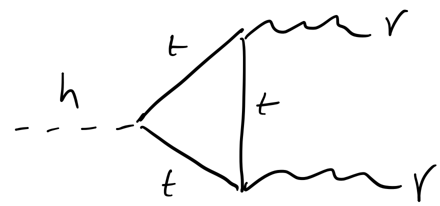


Here, one Z can be on-shell, so $(p_{\mu_1} + p_{\mu_2})^2 = m_Z^2$, and together, $(p_{\mu_1} + p_{\mu_2} + p_{\mu_3} + p_{\mu_4})^2 = m_h^2$

Indeed, this "golden channel" confirmed the initial Higgs discovery, and with more data became the best channel to study the Higgs.

2) photon and gluon couplings





The photon and gluon are massless, so there is no coupling to the Higgs in the Lagrangian. However, such a coupling does exist at 1-loop, much like the anomalous magnetic moment diagram we studied. Calculating these diagrams is beyond the scope of this course, but note that they are both proportional to $\frac{m_t}{v}$ when the loop consists of top quarks. This lets us exploit the large coupling to tops as a virtual particle. Indeed, in the first Higgs discovery analysis in 2012, the Higgs was mostly produced via gluon fusion (left diagram) and detected via the diphoton channel (right diagram), through a small bump in the invariant mass distribution $m_{\gamma\gamma}^2 \equiv (p_{\gamma_1} + p_{\gamma_2})^2$ at $m_h^2 \simeq (125 \text{ GeV})^2$.