Introduction to supersymmetry

What is supersymmetry? In increasing order of speculativeness,

1. a generalization of the Poincaré algebra, in fact the unique extension of spacetime symmetries
2. a playground for toy models of field theory ("super-Yang-Mills is the harmonic oscillator of QFT")
3. a way to ensure the vacuum energy cancels
4. a doubling of the particle content of the SM that could solve the hierarchy problem and provide a DM candidate.

Will focus mostly on (1)... (4) was very popular 20 years ago, but zero experimental evidence so far, and much of the motivation has been killed by the LHC.

Supersymmetry exchanges bosons and fermions.

$Q\,|\text{boson}\rangle = |\text{fermion}\rangle$, $Q\,|\text{fermion}\rangle = \text{boson}$.

Unlike e.g. Poincaré, where $P_\mu = i\partial_\mu$ is the infinitesimal generator of translations, Q is not an infinitesimal symmetry in the ordinary sense: it is itself a spinor! So its algebra will obey anticommutation relations.

Schematically, we can extend Poincaré as follows:

$\{Q, Q^\dagger\} = P^\mu$ ← Q is "square root of a translation"

$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$

$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$

With just Poincaré, representations were classified by P^2 (and W^2).

In SUSY, reps. are supermultiplets with equal numbers of bosons and fermions (and same mass and gauge charges)

To prove this, consider the operator $\theta = (-1)^{2s}$ where s is the spin operator. $\theta|B\rangle = |B\rangle$ and $\theta|F\rangle = -|F\rangle$.

Therefore, $\theta Q|B\rangle = \theta|F\rangle = -|F\rangle$, but $Q\theta|B\rangle = Q|B\rangle = |F\rangle$, so $\{\theta, Q\} = 0$ and likewise $\{\theta, Q^\dagger\} = 0$. Now, let $|i\rangle$ be all states w/same eigenval. of P^μ,

$$\sum_i \langle i|\theta P^\mu|i\rangle = \sum_i \langle i|\theta Q Q^\dagger|i\rangle + \sum_i \langle i|\theta Q^\dagger Q|i\rangle$$

$$= \sum_i \langle i|\theta Q Q^\dagger|i\rangle + \sum_{i,j} \langle i|\theta Q^\dagger|j\rangle\langle j|Q|i\rangle$$

$$\sum_{i,j} \langle i|\theta Q^\dagger|j\rangle\langle j|Q|i\rangle = \sum_j \langle j|Q\theta Q^\dagger|j\rangle = -\sum_j \langle j|\theta Q Q^\dagger|j\rangle$$

$$= 0$$

But $\sum_i \langle i|\theta P^\mu|i\rangle = p^\mu \mathrm{Tr}[(-1)^{2s}] = n_B - n_F$, so $\boxed{n_B = n_F}$

Every boson has a fermionic partner of the same mass and charge!
(Not our universe.)

Simplest example of a supermultiplet is a single Weyl fermion ($n_F = 2$) and a complex scalar ($n_B = 2$).

$$\mathcal{L}_{WZ} = \partial^\mu \phi^* \partial_\mu \phi + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

This is the (massless, non-interacting) Wess-Zumino model.

Looks kind of boring... but what if we propose a transformation $\delta\phi = \epsilon\psi$, $\delta\phi^* = \epsilon^\dagger \psi^\dagger$?

Here, ϵ is an infinitesimal <u>fermionic</u> parameter, and $\epsilon\psi$ means $\epsilon^{\alpha\beta}\epsilon_\alpha \psi_\beta$ (the Lorentz-invt. spinor contraction).

Note also $[\epsilon] = -\frac{1}{2}$. We have $\delta\mathcal{L}_{scalar} = \epsilon\partial^\mu\psi\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi$.

If we want $\mathcal{L}_{WZ}$ to be invariant up to a total derivative, we need $\delta\psi$ to contain $\partial_\mu\phi$. Try $\delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha \partial_\mu\phi$.

$$\Rightarrow \delta\mathcal{L}_{fermion} = \epsilon\sigma^\mu\bar{\sigma}^\nu\partial_\nu\psi\partial_\mu\phi^* - \psi^\dagger\bar{\sigma}^\nu\sigma^\mu\epsilon^\dagger\partial_\mu\partial_\nu\phi$$

After some Pauli matrix identities we get

$$\delta\mathcal{L}_{fermion} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi + \partial_\mu(\text{stuff})$$

so $\delta\mathcal{L} = 0$ up to total derivatives.

Still need to check that the SUSY algebra is <u>closed</u>: two transformations should give another.

$$\{\delta_{\epsilon_1}, \delta_{\epsilon_2}\}\phi \equiv \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) - \delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) = i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\phi$$

$\{Q, Q\}$ as anticipated! P_μ

More subtle for ψ:

$$\{\delta_{\epsilon_1}, \delta_{\epsilon_2}\}\psi_\alpha = i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\psi_\alpha \underbrace{- i\epsilon_{1\alpha}\epsilon_2^\dagger\bar{\sigma}^\mu\partial_\mu\psi + i\epsilon_{2\alpha}\epsilon_1^\dagger\bar{\sigma}^\mu\partial_\mu\psi}$$

Only vanish if equations of motion are satisfied!
$\bar{\sigma}^\mu\partial_\mu\psi = 0$

What's going on? On-shell, ψ has 2 d.o.f., which matches 2 d.o.f. in ϕ. But off-shell, ψ is a 2-component complex field with <u>4</u> d.o.f., and Weyl eqn. projects out half.

Can fix this with a Lagrange multiplier. Add to the action $\mathcal{L}_{aux} = F^* F$ where F is a complex <u>auxiliary</u> field. E.o.M. is $F = 0$, so F does nothing. But if we define

$\delta\psi_\alpha \to \delta\psi_\alpha + \epsilon_\alpha F$ along with $\delta F = i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi$, then

$\delta\mathcal{L} = 0$ up to $\partial_\mu(\ldots)$ <u>and</u> $\{\delta_{\epsilon_1}, \delta_{\epsilon_2}\} = i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)$

on all fields $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$. F has restored d.o.f. matching off-shell with 2 more bosonic d.o.f.

Can now restore indices:

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu.$$

A taste of SUSY pheno:

The minimal supersymmetric SM (MSSM) contains SUSY partners for all SM fields, plus an extra Higgs doublet. Instead of H and $\tilde{H}$, we have H_d and H_u which separately give mass to up and down quarks.

Can define a new discrete symmetry called R-parity, $P_R = (-1)^{3(B-L)+2s}$. $P_R = +1$ for all fields in SM, but $P_R = -1$ for SUSY partners, so if this is a symmetry of the MSSM Lagrangian the following are true:

- SUSY particles are always pair-produced
- Lightest SUSY particle is stable: if neutral, can be DM
- forbids leading contributions to dim-6 operator $p \to e^+ \pi^0$.

Note that we don't see a boson w/charge -1 and mass 511 keV, so if SUSY exists, it must be broken, making SUSY partners heavier. Haven't seen any yet, so $M_{SUSY} \gtrsim 1$ TeV.

Finally, SUSY can be extended from a global to a local symmetry => supergravity. Multiplet containing the graviton has a fermionic partner $\psi_{\mu\alpha}$, the spin-$\frac{3}{2}$ gravitino. Could also be DM but hard not to make too many of them in early universe