

# Colliders and detectors



How do we make elementary particles?  $E = mc^2$  plus QM!

if you have enough energy, anything that can happen, will happen, unless forbidden by conservation laws

For example, collide electrons and positrons:

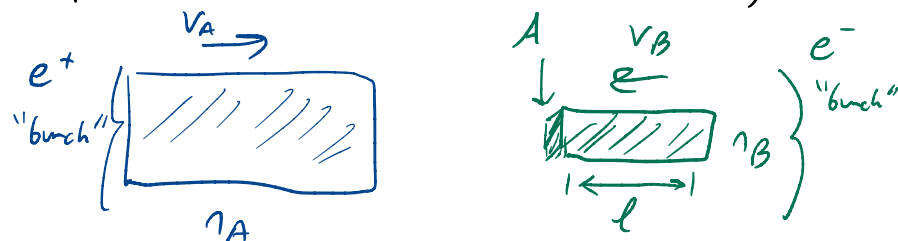


If each beam has energy  $\frac{E}{2}$ , then the center-of-mass energy is  $E$ : we can create particles with total mass up to  $E$  (with total charge, lepton number, and baryon number 0).

QM (really QFT) tells us the probability of making a given set of final-state particles. In particle physics we call this the matrix element  $M_{i \rightarrow f}$ , and next week we will see how to calculate it for some specific processes.

## Cross sections

Parameterize interaction strength using something with units of area



number of scattered particles proportional to area of scattering target

If we have two colliding beams with cross-sectional area  $A$  and length  $l$ ,

$$\text{scattering rate} = \frac{\text{events}}{\text{time}} = n_A n_B A l |v_A - v_B| \sigma \equiv L \sigma$$

$\mathcal{L}$  is the luminosity and parameterizes the flux of incoming particles.

$\sigma$  is the scattering cross section which parameterizes the interaction strength.

$n_A, n_B$  are the number densities of particles A and B in the beams.

$|v_A - v_B|$  is the relative velocity of the two beams. If the beams are relativistic ( $v_A \approx 1, v_B \approx 1$ ), this factor is  $|v_A - v_B| = 2$ . Despite appearances, this does not violate the velocity addition rule: it's formally defined as the "Møller velocity" and ensures the scattering rate is Lorentz-invariant with respect to boosts along the beam axis. (see Peskin & Schroeder Sec. 4.5 if you're curious.)

Fermi's Golden Rule relates  $\sigma$  to  $M$ :

$$\sigma_{i \rightarrow f} = \frac{1}{(2E_A)(2E_B)|v_A - v_B|} \int |M_{i \rightarrow f}|^2 d\pi (2\pi)^4 \delta^4(p_A + p_B - \sum_{i=1}^n p_i)$$

↑
↑
↑

from relativistic normalization of initial and final states

probabilities are squares of amplitudes

Sum over final states: Lorentz-invariant phase space

4-momentum conservation

Note that  $\sigma$  is not Lorentz-invariant, but transforms like an area: Lorentz-invariant for boosts along beam axis. This is the key observable predicted by QFT: "effective area" of beams of particles A and B, taking into account the fact that some collisions are rarer than others.

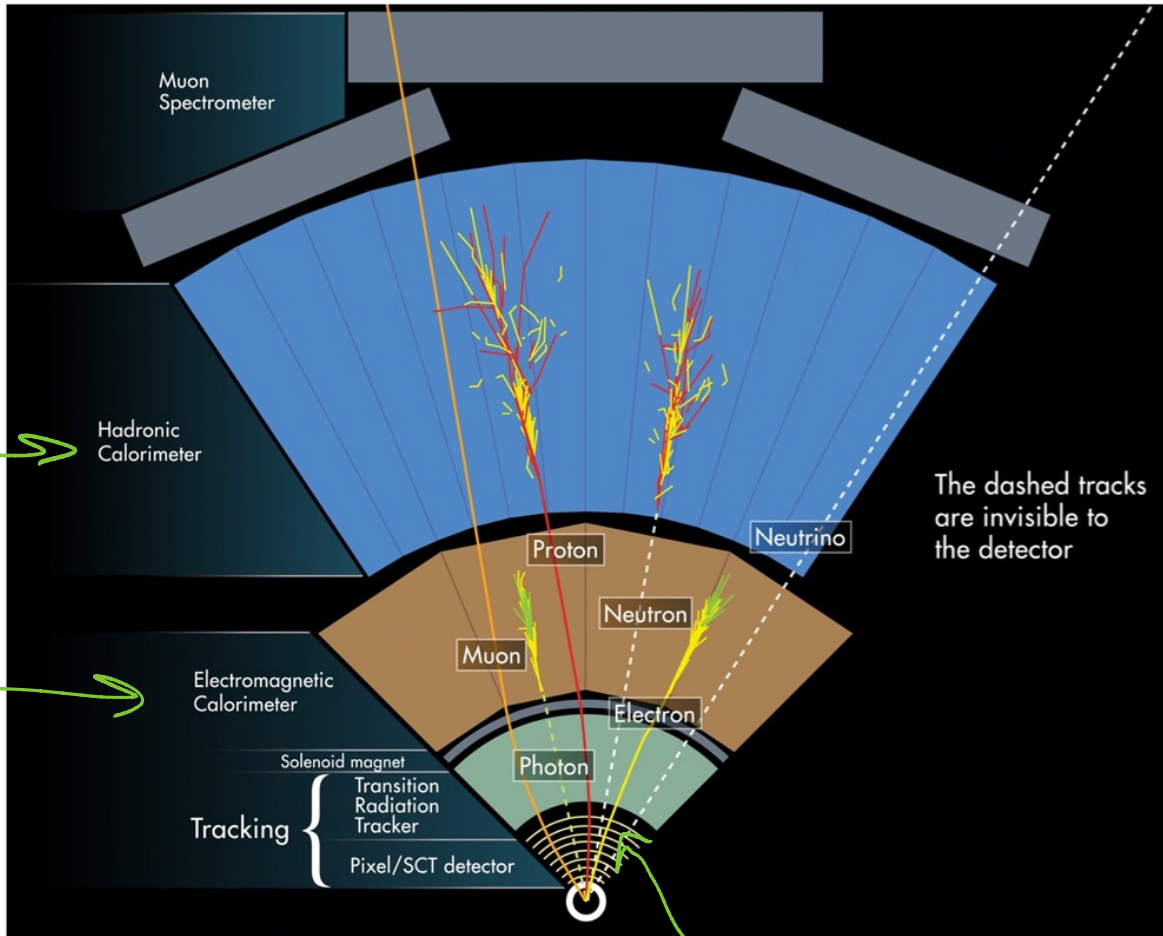
Units:  $\sigma$  is usually given in [SI prefix] × barns, where 1 barn =  $10^{-24} \text{ cm}^2$

Luminosity is usually quoted in [prefix × barns]<sup>-1</sup>/s, so for example, a process with  $\sigma = 1 \text{ fb} = 10^{-15} \text{ barns}$  at the LHC ( $\mathcal{L} \sim 1 \text{ pb}^{-1}/\text{s}$ ) has a rate  $R = \mathcal{L}\sigma = 10^{-3}/\text{s}$ . Integrated luminosity is  $\int \mathcal{L} dt$ .

# How do we detect elementary particles?

Two steps: measure an energy and/or momentum, and then identify the particle by its mass and electric charge.

Cross-sectional view of the ATLAS detector:



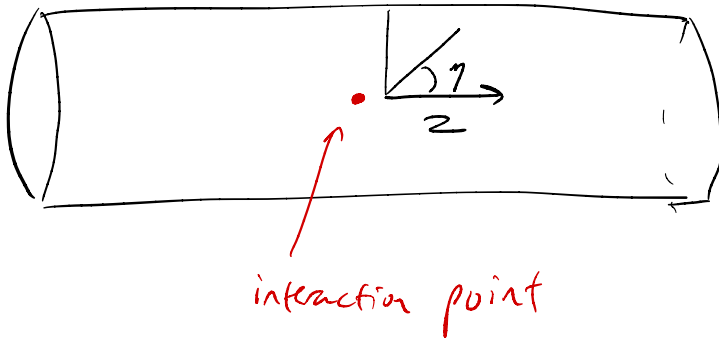
total number of photons proportional to particle energy

The dashed tracks are invisible to the detector

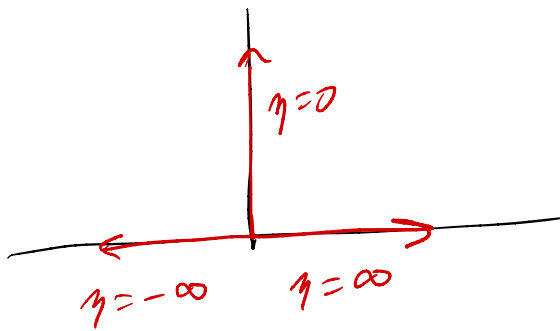
strips of silicon: charged particles deposit small amounts of energy in each pixel, can leave tracks

Entire detector is immersed in a magnetic field (out of the page in inner region); measure momentum and charge by curvature radius  $R \approx 3 \text{ m} \times \frac{p_{\perp} [\text{GeV}]}{Q |B| [\text{T}]}$   
protons, muons, electrons distinguished by where the track stops

# Detector coordinates and kinematics:



Basically spherical coordinates, but instead of  $\theta$ , use pseudorapidity  $\eta \equiv -\ln \tan \frac{\theta}{2}$

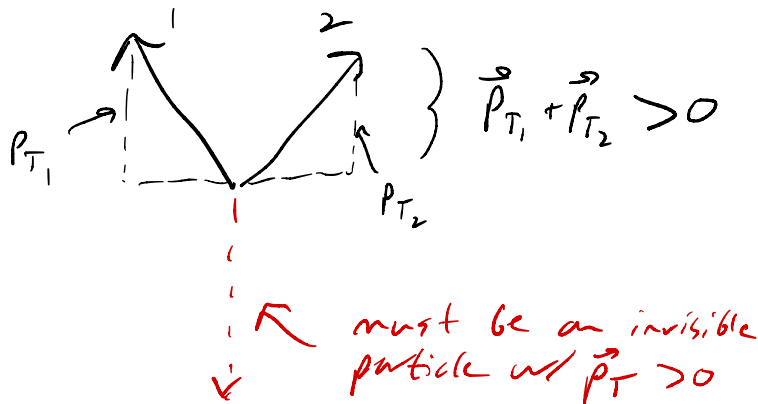


Why this funny variable? 2 related reasons:

- particle production is roughly uniform in  $\eta$
- behaves nicely under boosts for massless particles (Larkoski 5.3)

Hard to detect particles which go very close to beam direction (how do you avoid the beam?). As a result, often use transverse momentum  $p_T \equiv \sqrt{p_x^2 + p_y^2} = \sqrt{p^2 - p_z^2}$ .

Since all 3 components of spatial momentum must be conserved, can infer existence of invisible particles from imbalance in  $p_T$ .



# Phase space

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To compute cross sections, we need to sum over all final states  
 $\Rightarrow$  integrate over all 4-momenta consistent w/ Poincaré invariance

Translation invariance  $\Rightarrow$  4-momentum conservation (Noether's Theorem)

For a process  $p_A + p_B \rightarrow p_1 + p_2 + \dots + p_n$ ,

$$\int d\pi_n = \int \left\{ \prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^4} 2\pi \delta(p_i^2 - m_i^2) \theta(p_i^0) \right\} (2\pi)^4 \delta^{(4)}(p_A + p_B - \sum_{i=1}^n p_i)$$

The  $2\pi$ 's are conventionally attached to  $d\pi_n$  but they do matter - don't forget them!

This is manifestly Lorentz-invariant because the  $\delta$ -functions enforce  $p^2 = m^2$  for each final-state particle, and  $p_A + p_B - \sum_{i=1}^n p_i = 0$  (the zero 4-vector is also Lorentz-invariant).

We can perform the  $p^0$  integral for each  $i$ , using

$$\delta(p_i^2 - m_i^2) = \delta((p_i^0)^2 - \vec{p}^2 - m_i^2) \text{ and}$$

$$\delta(f(x)) = \sum_i \frac{1}{|f'(x_i^{(0)})|} \delta(x - x_i^{(0)}) \text{ where } x_i^{(0)} \text{ are roots of } f \text{ killed by } \theta(p_i^0)$$

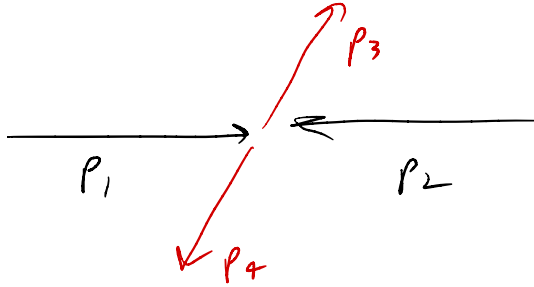
$$\Rightarrow \delta(p_i^2 - m_i^2) = \frac{1}{2\sqrt{\vec{p}_i^2 + m_i^2}} \left\{ \delta(p_i^0 - \sqrt{\vec{p}_i^2 + m_i^2}) + \delta(p_i^0 + \sqrt{\vec{p}_i^2 + m_i^2}) \right\}$$

$$\Rightarrow \int d p_i^0 \delta(p_i^2 - m_i^2) \theta(p_i^0) f(p_i^0) = \frac{1}{2E_i} f(E_i) \text{ w/ } E_i = \sqrt{\vec{p}_i^2 + m_i^2}$$

$$\Rightarrow \int d\pi_n = \int \left\{ \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \right\} (2\pi)^4 \delta^{(4)}(p_A + p_B - \sum_{i=1}^n p_i)$$

$p_i^0 = E_i$  no longer an integration variable

For 2-particle phase space, can do most of the integrals. (HW 4: 3-particle phase space.) Consider the process  $p_1 + p_2 \rightarrow p_3 + p_4$  (relabeling to match Schwartz 5.1) in the center-of-mass frame where  $p_1 + p_2 = (E_{cm}, \vec{0})$ .



$$d\pi_2 = \frac{d^3 p_3}{(2\pi)^3} \frac{1}{2E_3} \frac{d^3 p_4}{(2\pi)^3} \frac{1}{2E_4} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4)$$

Use  $\delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) = \delta^3(\vec{0} - \vec{p}_3 - \vec{p}_4)$  to do  $d^3 p_4$  integral:

Set  $\vec{p}_4 = -\vec{p}_3$ . Then, write  $d^3 p_3 = p_3^2 dp_3 d\Omega$ , where  $d\Omega$  is the differential solid angle for  $\vec{p}_3$  in spherical coordinates.

Collecting the  $2\pi$ 's and relabeling  $p_3 = p_f$

$$d\pi_2 = \frac{1}{16\pi^2} d\Omega \int dp_f \frac{p_f^2}{E_3 E_4} \delta(E_3 + E_4 - E_{cm})$$

changed signs for convenience:  $\delta(x) = \delta(-x)$

where  $E_3 = \sqrt{p_f^2 + m_3^2}$ ,  $E_4 = \sqrt{p_f^2 + m_4^2}$ .

Change variables  $p_f \rightarrow x(p_f) = E_3(p_f) + E_4(p_f) - E_{cm}$

Jacobian:  $\frac{dx}{dp_f} = \frac{2p_f}{2\sqrt{p_f^2 + m_3^2}} + \frac{2p_f}{2\sqrt{p_f^2 + m_4^2}} = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{E_3 + E_4}{E_3 E_4} p_f$

$\delta$ -function enforces  $E_3 + E_4 = E_{cm}$ , so

$$d\pi_2 = \frac{1}{16\pi^2} d\Omega \int_{m_3 + m_4 - E_{cm}}^{\infty} dx \frac{p_f(x)}{E_{cm}} \delta(x) = \frac{1}{16\pi^2} d\Omega \frac{|\vec{p}_f|}{E_{cm}} \theta(E_{cm} - m_3 - m_4)$$

where  $|\vec{p}_f|$  is the solution to  $x(p_f) = 0$  (usually easier to use Lorentz dot product tricks)

enforces our energy threshold condition from earlier