Renormalizability

So far only considered 4-dim operators (plus Weinberg)

In QFT, field theories w/ scalars, fermions, and gauge bosons w/ interactions up to dim-4 are <u>renormalizable</u>

- apparently infinite quantities from loop diagrams can be consistently removed

Higher dim operators are <u>non-renormalizable</u>

- need infinite series of increasingly higher-order operators to cancel infinities
- dimensionful couplings $\Rightarrow$ perturbation theory breaks down at coupling scale
- Coefficients relate to measurable quantities $\Rightarrow$ still predictable

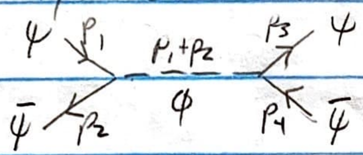
Renormalizable Lagrangian at high energy can generate nonrenormalizable at low

- Useful because full theory is not always well-defined at low energies
- Eg QCD becomes non-perturbative
- Remove heavy particles to make predictions with relevant light ones
- Eg 4-Fermi Theory from last time

Another example: $\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 - y\phi\bar{\psi}\psi$

Suppose $M \gg m$ and we are interested in ψ-$\bar{\psi}$ scattering

Only tree-level diagram is



$$i\mathcal{M} = (-iy)^2\left(\frac{i}{s-M^2}\right)\bar{v}(p_2)u(p_1)\bar{u}(p_3)v(p_4)$$

For CoM energies $\sqrt{s} \ll M$, expand amplitude

$$\frac{1}{s-M^2} = -\frac{1}{M^2}\frac{1}{1-s/M^2} = -\frac{1}{M^2}\left(1+\frac{s}{M^2}+\frac{s^2}{M^4}+\dots\right)$$

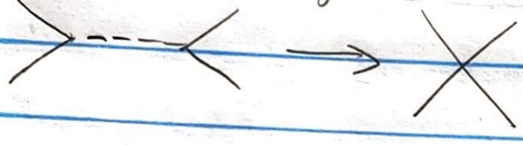
$$\Rightarrow i\mathcal{M} = iy^2\,\bar{v}(p_2)u(p_1)\bar{u}(p_3)v(p_4)\left(\frac{1}{M^2}+\frac{s}{M^4}+\frac{s^2}{M^6}+\dots\right)$$

Convert back to a Lagrangian

First term: $\mathcal{L}_{eff} \supset \frac{y^2}{M^2}\bar{\psi}\psi\bar{\psi}\psi$

Interpretation: at energies much less than M, ϕ particle cannot be produced on-shell

$\Rightarrow$ propagator in original matrix element shrinks to a point



Higher order corrections

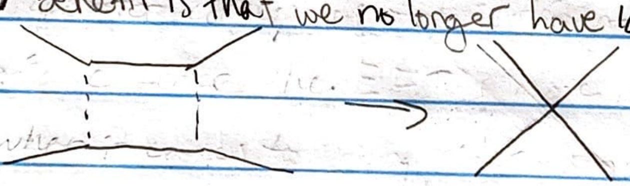
$$\mathcal{L}_{eff} \supset \frac{y^2}{M^2}\bar{\psi}\psi\bar{\psi}\psi + \frac{y^2}{M^4}\partial_\mu\bar{\psi}\partial^\mu\psi\bar{\psi}\psi + \frac{y^2}{M^6}\partial_\mu\bar{\psi}\partial_\nu\psi\partial^\mu\bar{\psi}\partial^\nu\psi + \ldots$$

- Lagrangian no longer contains ϕ

$\Rightarrow$ ϕ particle has been <u>integrated out</u>

- effects are encapsulated in infinite series of operators containing only ψ

(Another benefit is that we no longer have loops involving the massive particle

Eg )

Alternatively, EoM for ϕ is $(\Box + M^2)\phi = -y\bar{\psi}\psi$

"Solve" for ϕ as a formal power series $\phi = -\frac{1}{\Box + M^2}(y\bar{\psi}\psi)$

Expand for small momenta $\phi = -\frac{y}{M^2}\left(1 - \frac{\Box}{M^2} + \frac{\Box^2}{M^4} - \ldots\right)\bar{\psi}\psi$

$\Rightarrow \mathcal{L}_{eff} \supset -y\bar{\psi}\psi\phi \rightarrow y^2\bar{\psi}\psi\left(\frac{1}{M^2} - \frac{\Box}{M^4} + \frac{\Box^2}{M^6} - \ldots\right)\bar{\psi}\psi$

Plugging in ϕ from the EoM is the same as doing the path integral

$$\exp\left[i\int d^4x\,\mathcal{L}_{eff}(x)\right] = \int D\phi\,\exp\left[i\int d^4x\,\mathcal{L}(x,\phi)\right]$$

This is why it's called "integrating" out!

For a cool example of using the path integral method to get an <u>exact</u> result, check out Schwartz 33.3 - 33.4 (no series expansion)

Can also do the procedure in reverse

- For a given higher-dim operator, what additional heavy particles would we have to add to the theory to generate our desired operator after the heavy particles are integrated out?
- Known as UV-completion
- generally not unique, but can point to where to look for new physics responsible for that operator

SMEFT: Standard Model Effective Field Theory

- systematically account for new physics at high energies <u>without specifying UV-completion</u>
- all $SU(3) \times SU(2) \times U(1)$ invariant operators of <u>any</u> dimension built out of SM fields:

$$\mathcal{L}_{SMEFT} = \sum_{d=5}^{\infty} \frac{1}{\Lambda^{d-4}} \sum_{i}^{N^{(d)}} C_i^{(d)} \mathcal{O}_i^{(d)}$$

mass scale where UV completion is required

Wilson coefficients

operators of mass dimension d

- By construction, any measurement of nonzero C_i is proof of BSM physics
- Combinatorics and avoiding double-counting is tricky

$N^{(5)} = 2$ (Weinberg op. and conj.)

$N^{(6)} = 84$, $N^{(7)} = 30$, $N^{(8)} = 993$, ... see arXiv:1512.03433

first worked out correctly in 2010! see arXiv:1008.4884

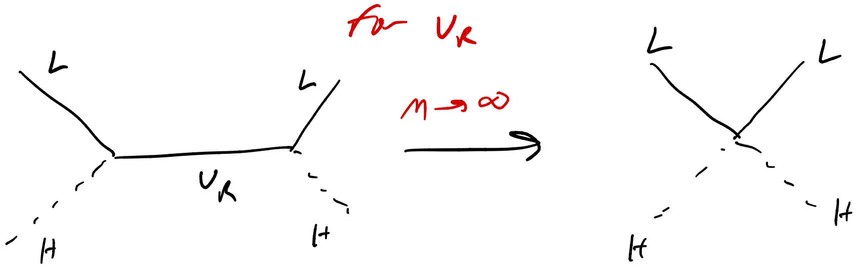
Some examples:

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad y=-\frac{1}{2} \quad y=\frac{1}{2}$

- <u>Weinberg operator</u> $\mathcal{O}^{(5)} = \frac{1}{\Lambda}\epsilon^{\alpha\beta}\left(\epsilon^{ab}L_{a\alpha}H_b\right)\left(\epsilon^{cd}L_{c\beta}H_d\right) + h.c.$

can be UV-completed with a heavy right-handed neutrino ν_R:

$$\mathcal{L} = y_\nu L^\dagger \tilde{H}\nu_R - \underbrace{\frac{M}{2}\epsilon^{\alpha\beta}\nu_{R\alpha}\nu_{R\beta}}_{\text{Majorana mass for } \nu_R}$$



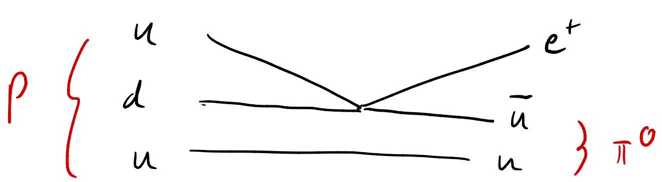
ν_R propagator is $\frac{\not{p}+M}{p^2-M^2} = \frac{-1}{M} + \mathcal{O}\left(\frac{p}{M}\right)$

$\Rightarrow$ identify $\frac{1}{\Lambda} \equiv \frac{1}{M}$, $C^{(5)} = (y_\nu)^2$

If neutrinos get mass from the Weinberg operator, the value of the mass suggests a mass for a new heavy right-handed neutrino: the <u>lighter</u> the SM neutrinos, the <u>heavier</u> ν_R is ("seesaw mechanism"). (Recall from HW 3 $m_\nu \sim \frac{v^2}{\Lambda}$; $m_\nu < 0.9\,eV$ from cosmology $\Rightarrow \Lambda \gtrsim 10^{15}\,GeV$)

- <u>Proton decay</u>. In the SM, protons are absolutely stable because they are the lightest baryon, and baryon number is conserved. But baryon number is an accidental symmetry, and is generically violated in the SMEFT.

Consider $\mathcal{O}^{(6)} = \frac{1}{\Lambda_6^2}\epsilon^{ijk}Q_iQ_jQ_kL$, where ϵ^{ijk} is the color antisymmetric tensor and all SU(2) and fermion indices are contracted with the appropriate $\epsilon^{\alpha\beta}$. This leads to:



$p \to e^+\pi^0$

Experiments such as Super-Kamiokande have been searching for this decay for decades: all null results so far!

$\tau_p > 1.67 \times 10^{34}$ yr from $\pi^0 e^+$ channel.

$\Rightarrow \Gamma_{p \to \pi^0 e^+} < 1.2 \times 10^{-57}$ eV

Let's use this to bound Λ_6:

$$\langle |M|^2 \rangle \simeq \frac{1}{\Lambda_6^4} \times [E]^6$$

to calculate this exactly requires non-perturbative QCD: the largest scale in the problem is m_p, so let's just set $E = m_p$.

$$\Gamma \simeq \frac{1}{2m_p} \frac{1}{8\pi} \frac{m_p^6}{\Lambda_6^4} \simeq \frac{m_p^5}{16\pi \Lambda_6^4}$$

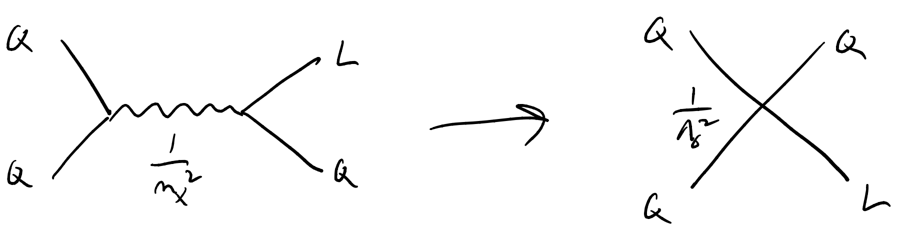
$\Rightarrow \Lambda_6 > 1.0 \times 10^{16}$ GeV!!

What physics could possibly arise at that scale?

Grand Unified Theories (GUTs) try to combine SU(3), SU(2), and U(1) into a single gauge group, where the SM arises from spontaneous symmetry breaking at the <u>GUT scale</u> of $\sim 10^{16}$ GeV.

ex. $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

The analogues of the W/Z are 12 new gauge bosons X, which can mix quarks and leptons.



, so $\frac{1}{\Lambda_6^2} = \frac{1}{m_X^2}$, and

$m_X > 10^{16}$ GeV.

$\Rightarrow$ observation of proton decay would tell us about enormously large energy scales!

"Back-of-the-envelope QFT"

While we're doing tilde-level estimates, it will be good to summarize some rules of thumb for estimating cross sections or decays.

- Phase space: for every extra particle in the final state, factor of $\frac{1}{4\pi^2}$ in rate. Comes from $d\Pi_n = \frac{d^3p_n}{(2\pi)^3}\frac{1}{2E_n}d\Pi_{n-1}$. $\int d^3p_n \sim 4\pi E_n^2$, matrix element has an extra power of E_n to have right dimensions. We saw this for $e^+e^- \to \mu^+\mu^-(\gamma)$: factor of e^2 in $|\mathcal{M}|^2$ combined with $\frac{1}{4\pi^2}$ gives correction $\frac{e^2}{4\pi^2} = \mathcal{O}(\frac{\alpha}{\pi})$
- Loops: for every loop, add a factor $\frac{1}{4\pi^2}$ in $\mathcal{M}$. Here, the 4π's come from $\int \frac{d^4k}{(2\pi)^4}$. We also saw this factor in $g-2$: $\frac{e^2}{4\pi^2} = \frac{\alpha}{\pi}$.

These quick order-of-magnitude estimates are good for guessing the answer <u>before</u> you start a long calculation! Also good for understanding the patterns in rare decay branching ratios [HW]