Colliders and detectors

How do we make elementary particles? $E = mc^2$ plus QM: if you have enough energy, anything that can happen, will happen, unless forbidden by conservation laws

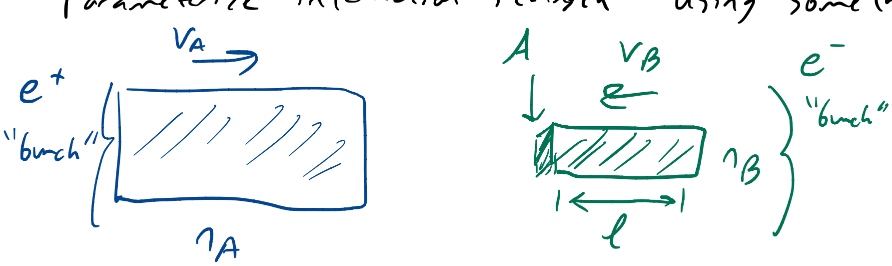
For example, collide electrons and positrons:

If each beam has energy $\frac{E}{2}$, then the center-of-mass energy is E: we can create particles with total mass up to E (with total charge, lepton number, and baryon number 0).

QM (really QFT) tells us the probability of making a given set of final-state particles. In particle physics we call this the matrix element $\mathcal{M}_{i\to f}$, and next week we will see how to calculate it for some specific processes.

Cross sections

Parameterize interaction strength using something with units of area:



number of scattered particles proportional to area of scattering target

If we have two colliding beams with cross-sectional area A and length ℓ,

$$\text{scattering rate} = \frac{\text{events}}{\text{time}} = n_A n_B A \ell \, |v_A - v_B| \, \sigma \equiv \mathcal{L} \sigma$$

$\mathcal{L}$ is the (instantaneous) luminosity and parameterizes the flux of incoming particles.
σ is the scattering cross section which parameterizes the interaction strength.
n_A, n_B are the number densities of particles A and B in the beams.

$|v_A - v_B|$ is the relative velocity of the two beams. If the beams are relativistic ($v_A \approx 1, v_B \approx 1$), this factor is $|v_A - v_B| = 2$. Despite appearances, this does not violate the velocity addition rule: it's the relative velocity of A and B as viewed from the lab, and ensures the scattering rate is Lorentz-invariant with respect to boosts along the beam axis. (see Peskin & Schroeder Sec. 4.5 if you're curious.)

Fermi's Golden Rule relates σ to $\mathcal{M}$:

$$\sigma_{i \to f} = \underbrace{\frac{1}{(2E_A)(2E_B)|v_A - v_B|}}_{\text{from relativistic normalization of initial and final states}} \int \underbrace{|\mathcal{M}_{i\to f}|^2}_{\text{probabilities are squares of amplitudes}} \underbrace{d\Pi}_{\text{sum over final states: Lorentz-invariant phase space}} (2\pi)^4 \underbrace{\delta^4\left(p_A + p_B - \sum_{i=1}^{n} p_i\right)}_{\text{4-momentum conservation}}$$

Note that σ is not Lorentz-invariant, but transforms like an area: Lorentz-invt. for boosts along beam axis. This is the key observable predicted by QFT: "effective area" of beams of particles A and B, taking into account the fact that some collisions are rarer than others.

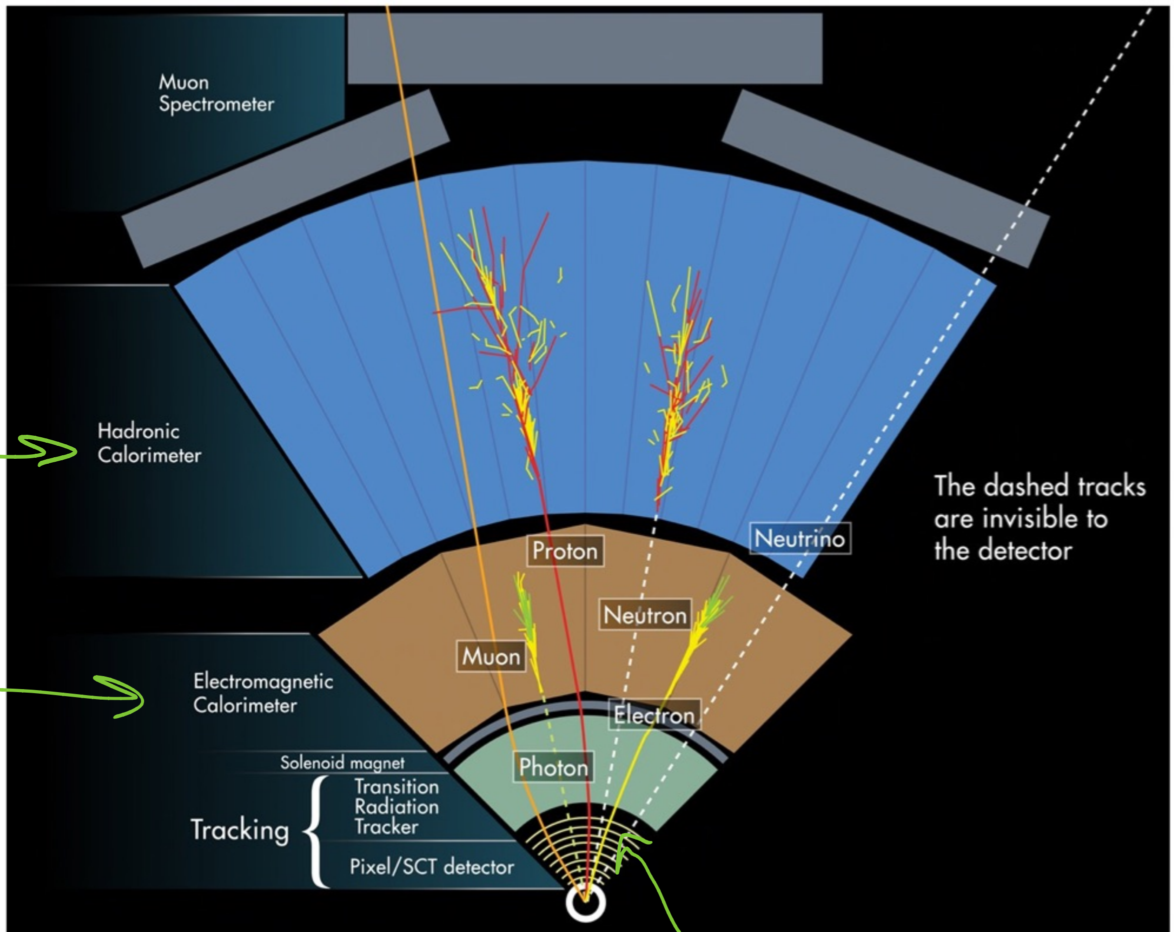
Units: σ is usually given in [SI prefix] × barns, where
$1 \text{ barn} = 10^{-24} \text{ cm}^2$

Luminosity is usually quoted in $[\text{prefix} \times \text{barns}]^{-1}/\text{s}$, so for example, a process with $\sigma = 1 \text{ fb} = 10^{-15}$ barns at the LHC ($\mathcal{L} \sim 1 \text{ pb}^{-1}/\text{s}$) has a rate $R = \mathcal{L}\sigma = 10^{-3}/\text{s}$. Integrated luminosity is $\int \mathcal{L}\, dt$.

How do we detect elementary particles?

Two steps: measure an energy and/or momentum and then identify the particle by its mass and electric charge.

Cross-sectional view of the ATLAS detector:



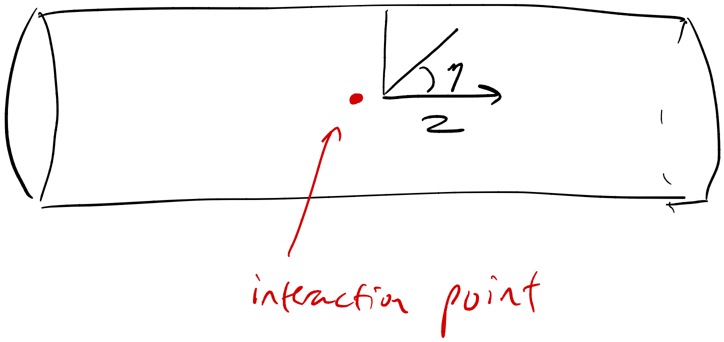
total number of photons proportional to particle energy

Strips of silicon: charged particles deposit small amounts of energy in each pixel, can leave tracks

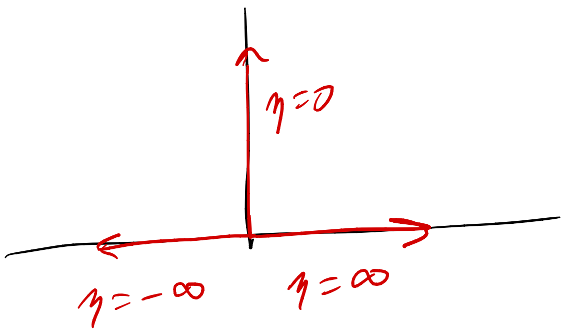
Entire detector is immersed in a magnetic field (out of the page in inner region): measure momentum and charge by curvature radius $R \simeq 3\,m \times \frac{p_\perp [GeV]}{Q\,|B|\,[T]}$

protons, muons, electrons distinguished by where the track stops.

Detector coordinates and kinematics:



Basically spherical coordinates, but instead of θ, use <u>pseudorapidity</u> $\eta \equiv -\ln \tan \frac{\theta}{2}$

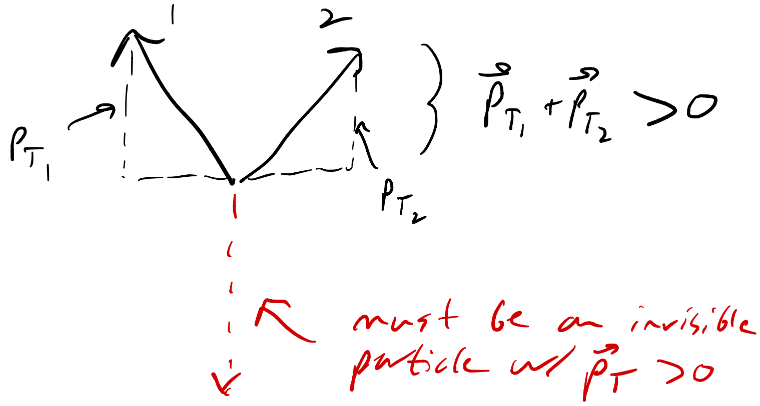


Why this funny variable? 2 related reasons:

- particle production is roughly uniform in η
- behaves nicely under boosts for massless particles (Larkoski 5.3)

Hard to detect particles which go very close to beam direction (how do you avoid the beam?). As a result, often use transverse momentum $p_T = \sqrt{p_x^2 + p_y^2} = \sqrt{p^2 - p_z^2}$.

Since all 3 components of spatial momentum must be conserved, can infer existence of invisible particles from imbalance in p_T.



Phase space

To compute cross sections, we need to sum over all final states
$\Rightarrow$ integrate over all 4-momenta consistent w/ Poincaré invariance

Translation invariance $\Rightarrow$ 4-momentum conservation (Noether's Theorem)

For a process $p_A + p_B \to p_1 + p_2 + \cdots p_n$,

$$\int d\Pi_n = \int \left\{ \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4} 2\pi\, \delta(p_i^2 - m_i^2)\, \Theta(p_i^0) \right\} (2\pi)^4 \delta^{(4)}\left(p_A + p_B - \sum_{i=1}^{n} p_i\right)$$

The 2π's are conventionally attached to $d\Pi_n$ but they do matter — don't forget them!

This is manifestly Lorentz-invariant because the δ-functions enforce $p^2 = m^2$ for each final-state particle, and $p_A + p_B - \sum_{i=1}^{n} p_i = 0$ (the zero 4-vector is also Lorentz-invariant).

We can perform the p^0 integral for each i, using

$\delta(p_i^2 - m_i^2) = \delta((p_i^0)^2 - \vec{p}^{\,2} - m_i^2)$ and

$\delta(f(x)) = \sum_i \frac{1}{|f'(x_i^{(0)})|} \delta(x - x_i^{(0)})$ where $x_i^{(0)}$ are roots of f

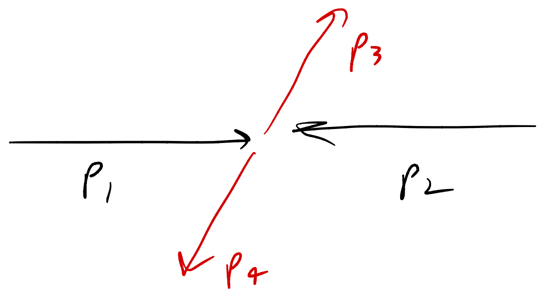
$$\Rightarrow \delta(p_i^2 - m_i^2) = \frac{1}{2\sqrt{\vec{p}_i^{\,2} + m_i^2}} \left\{ \delta\left(p_i^0 - \sqrt{\vec{p}_i^{\,2} + m_i^2}\right) + \cancel{\delta\left(p_i^0 + \sqrt{\vec{p}_i^{\,2} + m_i^2}\right)} \right\}$$

killed by $\Theta(p_i^0)$

$$\Rightarrow \int dp_i^0\, \delta(p_i^2 - m_i^2)\, \Theta(p_i^0)\, f(p_i^0) = \frac{1}{2E_i} f(E_i) \quad \text{w/ } E_i = \sqrt{\vec{p}_i^{\,2} + m_i^2}$$

$$\Rightarrow \int d\Pi_n = \int \left\{ \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \right\} (2\pi)^4 \delta^{(4)} \underbrace{\left(p_A + p_B - \sum_{i=1}^{n} p_i\right)}_{\text{$p_i^0 = E_i$ no longer an integration variable}}$$

For 2-particle phase space, can do most of the integrals. (HW 4: 3-particle phase space.) Consider the process $p_1 + p_2 \to p_3 + p_4$ (relabeling to match Schwartz 5.1) in the center-of-mass frame where $p_1 + p_2 = (E_{cm}, \vec{0})$.



$$d\Pi_2 = \frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_3}\frac{d^3p_4}{(2\pi)^3}\frac{1}{2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

Use $\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4) = \delta^3(\vec{0}-\vec{p}_3-\vec{p}_4)$ to do d^3p_4 integral: sets $\vec{p}_4 = -\vec{p}_3$. Then, write $d^3p_3 = p_3^2\,dp_3\,d\Omega$, where $d\Omega$ is the differential solid angle for $\vec{p}_3$ in spherical coordinates.

Collecting the 2π's and relabeling $p_3 = p_f$

$$d\Pi_2 = \frac{1}{16\pi^2}d\Omega\int dp_f\,\frac{p_f^2}{E_3E_4}\,\delta(E_3+E_4-E_{cm})$$

changed signs for convenience: $\delta(x) = \delta(-x)$

where $E_3 = \sqrt{p_f^2+m_3^2}$, $E_4 = \sqrt{p_f^2+m_4^2}$.

Change variables $p_f \longrightarrow x(p_f) = E_3(p_f) + E_4(p_f) - E_{cm}$

Jacobian: $$\frac{dx}{dp_f} = \frac{2p_f}{2\sqrt{p_f^2+m_3^2}} + \frac{2p_f}{2\sqrt{p_f^2+m_4^2}} = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{E_3+E_4}{E_3E_4}p_f$$

δ-function enforces $E_3 + E_4 = E_{cm}$, so

$$d\Pi_2 = \frac{1}{16\pi^2}d\Omega\int_{m_3+m_4-E_{cm}}^{\infty}dx\,\frac{p_f(x)}{E_{cm}}\,\delta(x) = \frac{1}{16\pi^2}d\Omega\,\frac{|\vec{p}_f|}{E_{cm}}\,\theta(E_{cm}-m_3-m_4)$$

enforces our energy threshold condition from earlier

where $|\vec{p}_f|$ is the solution to $x(p_f) = 0$ (usually easier to use Lorentz dot product tricks)