# PHYS 575, HW #5

Due: 3/11/20

**Note:** In Schwartz Ch. 20, the notation $e_R$ is used for the electromagnetic coupling constant. The distinction between $e$ and $e_R$, where $R$ stands for "renormalized," is beyond the scope of this course, so just read $e_R$ as $e$. Please note that $e_R$ is *not* the right-handed electron spinor!

1. **Dirac equation and magnetic moments (25 points).** Schwartz problem 10.1 (a)-(e).

2. **Electron magnetic moment: the details (30 points).**

   (a) Prove the identity

   $$\frac{1}{ABC} = 2\int_0^1 dx\, dy\, dz \delta(x+y+z-1)\frac{1}{(xA+yB+zC)^3}.$$

   You can do this by direct computation, or more cleverly (and generally) following Peskin & Schroeder: first prove $\frac{1}{AB} = \int_0^1 dx\, dy\, \delta(x+y-1)\frac{1}{(xA+yB)^2}$, differentiate $n$ times with respect to $B$ to get a formula for $\frac{1}{AB^n}$, then use this to prove the general formula

   $$\frac{1}{A_1 A_2 \cdots A_n} = \int_0^1 dx_1 \cdots dx_n \delta(\sum x_i - 1)\frac{(n-1)!}{(x_1A_1 + x_2A_2 + \cdots + x_nA_n)^n}$$

   by induction on $n$.

   (b) Perform the change of variables $k' = k + yp - zq$ in the numerator $N^\mu = \gamma^\nu(\not{p} + \not{k} + m)\gamma^\mu(\not{k} + m)\gamma_\nu$, and contract with $\bar{u}(q_2)$ on the left and $u(q_1)$ on the right to show that the coefficient of $i\bar{u}(q_2)\sigma^{\mu\nu}p_\nu u(q_1)$ is $-2mz(1-z)$.

   (c) Do the final Feynman parameter integral: show that

   $$\int_0^1 dx\, dy\, dz \frac{z}{1-z}\delta(x+y+z-1) = 1/2.$$

   (I encourage you to do this by hand rather than just asking Mathematica to do it: there are factors of 2 which show up very often in these calculations, and it is good to know where they appear for the next time you encounter such an integral.)

3. **Final-state radiation: the details (30 points).**

   (a) Derive the limits of integration for 3-body phase space in terms of $x_1$ and $\beta \equiv m_\gamma^2/Q^2$ pretending that the final-state photon has a mass $m_\gamma$ (Schwartz problem 20.1).

   (b) Perform the Dirac traces to derive Schwartz eq. (20.43).

   (c) Repeat part (a), but instead set $\beta = 0$ and restore a finite muon mass $m_\mu$. Which singularities survive?

4. **Experimental resolutions at BaBar (15 points).** Estimate $Q$, as well as $E_{\text{res}}$ and $\theta_{\text{res}}$ for a 10 GeV final-state photon, at the electron-positron experiment BaBar. Cite references to justify the values you took. Plug these values into Schwartz (20.55): at this experiment, do we have to worry about the perturbation expansion breaking down due to large logarithms?