PHYS 575, HW #8

Due: 4/15/20

1. **Chiral symmetry breaking (20 points).**

   (a) Show that the Lagrangian $\mathcal{L} = \mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + m^2 \mathrm{Tr}(\Sigma^\dagger \Sigma) - \frac{\lambda}{4}\left(\mathrm{Tr}(\Sigma^\dagger \Sigma)\right)^2$ is invariant under $\Sigma \to g_L \Sigma g_R$, where g_L and g_R are arbitrary SU(2) matrices and Σ is a 2×2 matrix-valued scalar field.

   (b) Expand out the chiral Lagrangian $\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$ and show that it contains the expected kinetic term for the pions, $\mathcal{L} \supset \frac{1}{2}(\partial_\mu \pi^0)(\partial^\mu \pi^0) + \partial_\mu \pi^+ \partial^\mu \pi^-$.

2. WZ **scattering (40 points).** From the amplitudes given in Schwartz Sec. 29.2, derive Schwartz eqs. (29.23), (29.24), and (29.26).

3. **Alternative models of symmetry breaking (40 points).** Peskin problem 16.2. (In the 1970's it was not at all clear what the correct group theory structure of symmetry breaking should be; lots of things sort of look right with this attempt, but the prediction for the Weinberg angle in part (g) rules it out.)

4. **Practice with the Abelian Higgs model (Extra credit: 30 points).** Calculate the differential cross section $d\sigma/d\cos\theta$ and the total cross section σ_{tot} for $\sigma A \to \sigma A$ in the Abelian Higgs model. Note that there are four diagrams which need to be added coherently before squaring. You can check your amplitudes against the solution to problem 25.4 in Coleman p. 1057 (what he calls a is what we call v, and his λ differs from ours by a factor of 4).