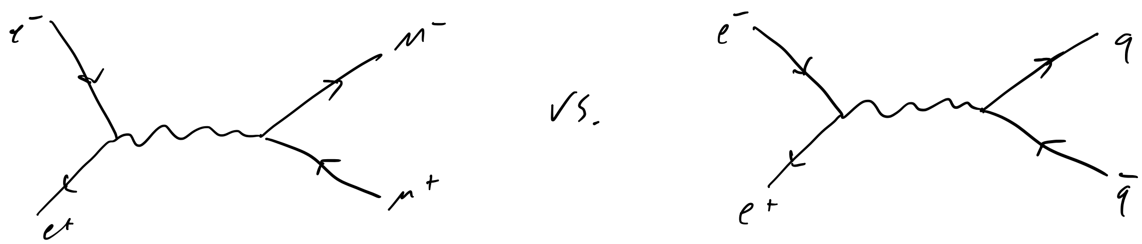
Moving to even higher energies: $e^+e^- \to$ hadrons.

Some jargon: "hadrons" = any strongly-interacting particles. Pions, kaons, protons, neutrons, ... these are what are actually observed in experiments. Free quarks are not observed! (More on this after the break.)

We will compute $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ as a function of $\sqrt{s} = E_{cm}$, approximating the numerator by $\sigma(e^+e^- \to q\bar{q})$. In the following weeks we will discuss the transition from quarks to hadrons.



In limit where all particles are massless, these diagrams are identical up to $e \to Q_i e$.

$$\Rightarrow \sigma(e^+e^- \to \text{all quarks}) = 3 \times \sum_i Q_i^2 \, \sigma(e^+e^- \to \mu^+\mu^-)$$

quarks are a 3-component vector under $SU(3)_c$

$m_u \simeq 2$ MeV, $m_d \approx 5$ MeV, $m_s \simeq 100$ MeV, but $m_c \simeq 1.5$ GeV, so for $\sqrt{s} = 1$ GeV, not enough energy to produce $c\bar{c}$.

$$\Rightarrow R(\sqrt{s} = 1\text{ GeV}) = 3\Big(\underbrace{\left(\tfrac{2}{3}\right)^2}_{q=u} + \underbrace{\left(-\tfrac{1}{3}\right)^2}_{q=d} + \underbrace{\left(-\tfrac{1}{3}\right)^2}_{q=s}\Big) = 2$$

Well-matched by experiment! Experimental confirmation that quarks have 3 colors.

To see what happens around 3 GeV, we need to include masses.

Let's just look at the quark half of the diagram, which by now should be familiar:

$$Q^{\mu\nu} = \sum_{s_1 s_2} \bar{u}_{s_1}(p_3)\gamma^\mu v_{s_2}(p_4)\bar{v}_{s_2}(p_4)\gamma^\nu u_{s_1}(p_3) = \mathrm{Tr}[(\not{p}_3 + m_c)\gamma^\mu(\not{p}_3 - m_c)\gamma^\nu]$$

We previously computed the 4-γ term the 2γ term is

$$-m_c^2\,\mathrm{Tr}(\gamma^\mu\gamma^\nu) = -4m_c^2\eta^{\mu\nu}$$

$$\Rightarrow Q^{\mu\nu} = 4\left(p_3^\mu p_4^\nu + p_3^\nu p_4^\mu - \eta^{\mu\nu}(p_3\cdot p_4 + m_c^2)\right)$$

From previous week, $L^{\mu\nu} = 4(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \eta^{\mu\nu} p_1\cdot p_2)$ (still ignoring m_e)

$$\Rightarrow \langle|M|\rangle^2 = \frac{8e^4 \overset{\swarrow Q_c}{(\tfrac{2}{3})^2}}{q^4}\left[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) + m_c^2\, p_1\cdot p_2\right]$$

only new term

Taking same kinematics as before, but with m_c included:

$p_1 = (\frac{E}{2}, 0, 0, \frac{E}{2})$, $p_2 = (\frac{E}{2}, 0, 0, -\frac{E}{2})$, $p_3 = (E_3, |\vec{p}_3|\sin\theta, 0, |\vec{p}_3|\cos\theta)$, $p_4 = (E_3, -\vec{p}_3)$

$p_1\cdot p_3 = p_2\cdot p_4 = \frac{E}{2}(E_3 - |\vec{p}_3|\cos\theta)$, $p_1\cdot p_4 = p_2\cdot p_3 = \frac{E}{2}(E_3 + |\vec{p}_3|\cos\theta)$, $p_1\cdot p_2 = \frac{E^2}{2}$

Conservation of energy gives $E_3 = \frac{E}{2}$, so

$$\langle|M|\rangle^2 = \frac{8e^4(\frac{2}{3})^2}{E^4}\left(\frac{E^2}{4}\left(\frac{E}{2} - |\vec{p}_3|\cos\theta\right)^2 + \frac{E^2}{4}\left(\frac{E}{2} + |\vec{p}_3|\cos\theta\right)^2 + \frac{E^2}{2}m_c^2\right)$$

$$= \frac{8e^4(\frac{2}{3})^2}{E^4}\left(2\frac{E^4}{16} + 2\frac{E^2}{4}|\vec{p}_3|^2\cos^2\theta + \frac{E^2}{2}m_c^2\right)$$

Using $|\vec{p}_3|^2 = \frac{E^2}{4} - m_c^2$,

$$\langle|M|\rangle^2 = e^4\left(\tfrac{2}{3}\right)^2\left(1 + \cos^2\theta + (1 - \cos^2\theta)\frac{4m_c^2}{E^2}\right)$$

Note: kinematics requires $E^2 > 4m_c^2$ to have enough energy to produce $c\bar{c}$, but the matrix element doesn't know about this! It remains positive even for unphysical kinematics.

Phase space enforces kinematics:

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{64\pi^2 E^2}\frac{|\vec{p}_3|}{|\vec{p}_1|}\langle|M|\rangle^2\,\theta(E - 2m_c) \quad \text{(see Schwartz 5.1.2)}$$

Here, $|\vec{p}_1| = \frac{E_1}{2}$, $|\vec{p}_3| = \frac{E}{2}\sqrt{1-\frac{4m_c^2}{E^2}}$

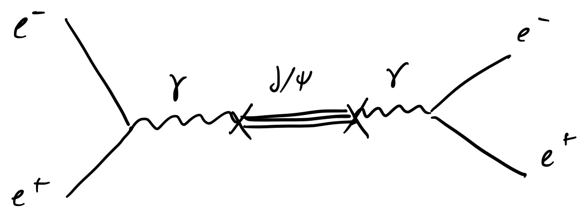
$$\Rightarrow \sigma_{e^+e^-\to c\bar{c}} = \underbrace{\frac{4\pi\alpha^2}{3E^2}}_{\sigma_0 \text{ for high-energy}} \left(\frac{2}{3}\right)^2 \underbrace{\sqrt{1-\frac{4m_c^2}{E^2}}}_{\text{threshold behavior}} \underbrace{\left(1+\frac{2m_c^2}{E^2}\right)}_{\text{goes to 1 for } E \gg m_c}$$

Resonances

In fact, what happens is the cross section jumps by many orders of magnitude at $E = 3.096900$ GeV. We interpret this as the formation of a bound state of $c\bar{c}$, called the J/ψ.

By the helicity analysis from week 6, for $E \gg m_e$, e^+ and e^- must have total spin 1. Therefore this new particle has spin-1.

Eventually it decays, with a rate Γ. Often, unstable particles have multiple decay modes, so we will often speak of the partial width Γ_f to a particular final state f: $\Gamma_{tot} = \sum_f \Gamma_f$. Let's redraw the diagram for e^+e^- annihilation including the J/ψ:



There are two new ingredients: the propagator for the J/ψ and the coupling between the photon and the J/ψ. To determine these, we need to know how to write down Lagrangians for massive spin-1 particles. This is actually considerably easier than massless spin-1, since there is a third physical polarization vector, $\epsilon_\mu^L = \left(\frac{p_z}{m}, 0, 0, \frac{E}{m}\right)$ for $p^\mu = (E, 0, 0, p_z)$.

$\Rightarrow$ we don't need gauge invariance! All we need is $\partial_\mu A^\mu = 0$, which is implied from the equations of motion of $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$ (see Schwartz 8.2), where m is mass of J/ψ.

Let's write C_μ and $C_{\mu\nu}$ for the J/ψ to not confuse it with the photon.

Claim: $\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \underbrace{- \frac{1}{4}C_{\mu\nu}C^{\mu\nu} + \frac{1}{2}m^2 C_\mu C^\mu}_{\text{gives propagator for } J/\psi} + \underbrace{\kappa F_{\mu\nu}C^{\mu\nu}}_{\text{gives Feynman rule for } \gamma/J \text{ vertex}}$

First, Feynman rule: $= i\kappa(ip_\mu)(ip^\alpha)\delta_\mu^\alpha \delta_\nu^\beta$

$\underbrace{p \quad p}_{\text{these are the same by momentum conservation}}$

Propagator for a stable spin-1 particle of mass m is $\dfrac{-i\eta^{\mu\nu}}{p^2-m^2}$.

For an <u>unstable</u> particle, this is modified to $\dfrac{-i\eta^{\mu\nu}}{p^2-m^2+im\Gamma}$

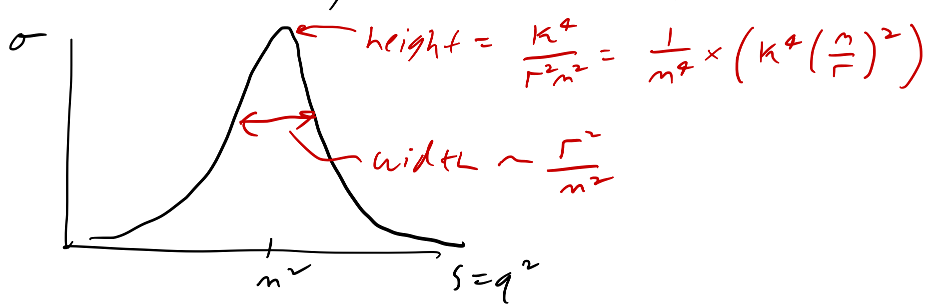
We will not derive this (you'll do this in QFT), but we'll show that it's well supported by data.

$$\Rightarrow \quad i\mathcal{M} = (\text{electrons})^\mu \times \frac{-i}{\cancel{q^2}}(-i\kappa \cancel{q^2})\left(\frac{-i\eta_{\mu\nu}}{q^2-m^2+im\Gamma}\right)(-i\kappa\cancel{q^2})\left(\frac{-i}{\cancel{q^2}}\right)(\text{electrons})^\nu$$

$$= -i\kappa^2(\text{electrons}) \cdot \frac{1}{q^2-m^2+im\Gamma} \cdot (\text{electrons})$$

$$\langle |\mathcal{M}| \rangle^2 = \frac{\kappa^4}{(q^2-m^2)^2+m^2\Gamma^2} \times (\text{things we've already calculated})$$

The first factor is known as a Breit-Wigner distribution, and tells us the energy dependence of the cross section. For $q^2 \ll m^2$ or $q^2 \gg m^2$, this reduces to the usual $\frac{1}{E^4}$ we've seen before. But for $q^2 \approx m^2$ and $\Gamma \ll m$, there is a huge enhancement:



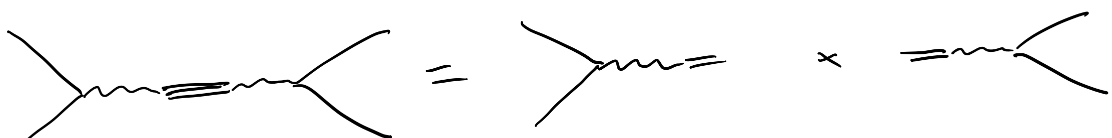
Experimentally, $\Gamma = 93$ keV, so for $m \approx 3$ GeV, $\frac{\Gamma}{m} = 3 \times 10^{-5}$

$\Rightarrow \sigma_{e^+e^-}$ enhanced by $\kappa^4 \times 10^9$! Measuring height of peak lets us determine κ. This effect (enhancement of annihilation cross section through production of a long-lived particle) is known as a resonance.

We can be a little more quantitative using the narrow-width approximation. Let's say we want to calculate $\sigma_{e^+e^- \to J/\psi \to \mu^+\mu^-}$. In the limit $\frac{\Gamma}{m} \to 0$, the Breit-Wigner becomes infinitely narrow.

$$\int_0^\infty ds \; \frac{1}{(s-m^2)^2 + m^2\Gamma^2} = \frac{\pi}{m\Gamma}, \text{ so } \frac{1}{(s-m^2)^2 + m^2\Gamma^2} \to \frac{\pi}{m\Gamma}\,\delta(s-m^2)$$

This allows us to factorize the matrix element:



Very similar to $e^+e^- \to \gamma^* \to \mu^+\mu^-\gamma$ from last week! In more detail,

$$\sigma(e^+e^- \to J/\psi \to \mu^+\mu^-) = \frac{1}{2s}\int d\Pi_{\mu^+\mu^-} \langle |M| \rangle^2$$

$$= \frac{1}{2s}\int d\Pi_{\mu^+\mu^-} |M(e^+e^- \to J/\psi)|^2 \frac{\pi}{m\Gamma}\delta(s-m^2)|M(J/\psi \to \mu^+\mu^-)|^2$$

$$= \underbrace{\frac{\pi}{s}|M(e^+e^- \to J/\psi)|^2 \delta(s-m^2)}_{\text{interpret this as } \sigma(e^+e^- \to J/\psi)\text{; note "1-body phase space" has one delta function remaining}} \times \frac{\frac{1}{2m}\int d\Pi_{\mu^+\mu^-} |M(J/\psi \to \mu^+\mu^-)|^2}{\Gamma}$$

$$= \frac{\Gamma(J/\psi \to \mu^+\mu^-)}{\Gamma_{tot}} = Br(J/\psi \to \mu^+\mu^-)$$

branching ratio

In HW you will investigate this in more detail.

Aside: Since $m \gg m_e, m_\mu$, $M(J/\psi \to e^+e^-) = M(J/\psi \to \mu^+\mu^-)$ since only dependence on flavor comes from masses. So we predict $Br(J/\psi \to e^+e^-) = Br(J/\psi \to \mu^+\mu^-)$; indeed, this is what the data shows.

Away from resonance, can also look at $\frac{d\sigma}{d\cos\theta}$ for $e^+e^- \to jj$. Assuming jets follow directions of progenitor quarks (more on this after break), expect this to follow $1+\cos^2\theta$ distribution. This is precisely what is seen in data, lending more support to the SM prediction that quarks have spin-$\frac{1}{2}$.