The Standard Model

We have classified spin-0 and spin-$\frac{1}{2}$ fields by their Lorentz reps and internal (gauge) symmetries, through which we introduced spin-1 fields.

Here are the fields which comprise the Standard Model:

spin-$\frac{1}{2}$ | spin-0

gauge fields (spin-1)	$L_f = \begin{pmatrix} \nu_L^f \\ e_L^f \end{pmatrix}$	e_R^f	$Q_f = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix}$	u_R^f	d_R^f	H
$U(1)_Y$	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$
$SU(2)$	✓		✓			✓
$SU(3)$			✓	✓	✓	

Charges/representations

Terminology: L_f, e_R^f are left/right-handed leptons

$Q_f, u_R^f/d_R^f$ are left/right-handed quarks

$f = 1, 2, 3$ are generations or flavors ($f=1$ is electron, electron neutrino, up quark, down quark; $f=2$ is muon, muon neutrino, charm quark, strange quark; $f=3$ is tau, tau neutrino, top quark, bottom quark)

H is the Higgs field

$U(1)_Y$ is hypercharge

$SU(2)$ (sometimes $SU(2)_L$) is the weak force, and only acts on left-handed fermions (and the Higgs)

$SU(3)$ (sometimes $SU(3)_c$) is color, or the strong force

Notation: Anything with a ✓ under $SU(2)$ is a 2-component vector of fields which transforms with $e^{i\alpha^a T^a}$, like Φ we saw earlier (in fact, Φ is H).

Similarly, the quarks are 3-component vectors transforming with 3×3 unitary matrices

$$u_R = \begin{pmatrix} u_R^r \\ u_R^g \\ u_R^b \end{pmatrix}$$ ("red", "green", "blue"), so Q is actually a $3\times2 = \underline{6}$-component field.

The Standard Model consists of (almost) all terms we can write down up to total dimension 4 which are invariant under Lorentz and local $SU(3)\times SU(2)\times U(1)_Y$ symmetry. (You will see the remaining term in HW 3)

Easy stuff first:

$$\mathcal{L}_{kin} = |D_\mu H|^2 - \frac{1}{4}\overset{SU(3)}{G^a_{\mu\nu}G^{\mu\nu a}} - \frac{1}{4}\overset{SU(2)}{W^a_{\mu\nu}W^{\mu\nu a}} - \frac{1}{4}\overset{U(1)_Y}{B_{\mu\nu}B^{\mu\nu}}$$
$$+ \sum_{f=1}^{3}\left\{ i L_f^+ \bar\sigma^\mu D_\mu L_f + i Q_f^+ \bar\sigma^\mu D_\mu Q_f + i e_R^{f+}\sigma^\mu D_\mu e_R^f + i u_R^{f+}\sigma^\mu D_\mu u_R^f + i d_R^{f+}\sigma^\mu D_\mu d_R^f \right\}$$

$$\mathcal{L}_{Higgs} = +m^2 H^+H - \lambda (H^+H)^2$$ (note mass term has wrong sign! Will get to this later in the course)

Since fermions have dimension $\frac{3}{2}$, a fermion-fermion-scalar term (known as a Yukawa term) has dimension 4. What such terms are allowed?

$$\mathcal{L}_{Yukawa} \supset -\underbrace{Y^e_{ij}}_{\substack{3\times 3 \text{ matrix} \\ \text{of numbers}}} L_i^+ H e_R^j - Y^d_{ij} Q_i^+ H d_R^j$$

Consider $L^+ H e_R$ term first:

$SU(3)$: $L_i^+ \to L_i$, $H \to H$, $e_R^j \to e_R^j$ (no transformations, so trivially invariant)

$SU(2)$: $L_i^+ \to L_i^+ U^+$, $H \to UH$, $e_R^j \to e_R$ for some $U \in SU(2)$, so

$$L_i^+ H e_R^j \to L_i^+ (\cancelto{1}{U^+ U}) H e_R^j = L_i^+ H e_R^j, \quad \underline{\text{invariant}}$$

$U(1)_Y$: this group is Abelian, so as a shortcut, can just count charges:

$$\overset{+\frac{1}{2}}{L_i^+}\ \overset{+\frac{1}{2}}{H}\ \overset{-1}{e_R^j} \quad (+\tfrac{1}{2}+\tfrac{1}{2}-1 = 0)$$

So even though L_i and e_R transform differently, H compensates, making it invariant.

Very similar story for second term. Can check $SU(3)$ and $SU(2)$ yourself,

$U(1)_Y$: $$\overset{-\frac{1}{6}}{Q_i^+}\ \overset{+\frac{1}{2}}{H}\ \overset{-\frac{1}{3}}{d_R^j} \quad (-\tfrac{1}{6}+\tfrac{1}{2}-\tfrac{1}{3} = 0)$$

One final trick and we're done! We can make an SU(2)-invariant term without taking Hermitian conjugates.

You will show in HW 3 that $\epsilon^{ab} Q_a H_b$ is invariant under SU(2).

So, defining $\tilde{H} = \epsilon^{ab} H_b^* = \begin{pmatrix} H_2^* \\ -H_1^* \end{pmatrix}$, which has $Y = -\frac{1}{2}$, we can write

$$\mathcal{L}_{\text{Yukawa}} \supset - y^u_{ij} \overset{-\frac{1}{6}}{Q^+_i} \overset{-\frac{1}{2}}{\tilde{H}} \overset{+\frac{2}{3}=0}{u^j_R}$$

That's it!

$$\boxed{\begin{aligned} \mathcal{L}_{SM} &= \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}} \\ &= |D_\mu H|^2 - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ &\quad + \sum_{f=1}^{3} \left\{ i L_f^+ \bar{\sigma}^\mu D_\mu L_f + i Q_f^+ \bar{\sigma}^\mu D_\mu Q_f + i e_R^{f+} \sigma^\mu D_\mu e_R^f + i u_R^{f+} \sigma^\mu D_\mu u_R^f + i d_R^{f+} \sigma^\mu D_\mu d_R^f \right\} \\ &\quad - y^e_{ij} L^+_i H e^j_R - y^d_{ij} Q^+_i H d^j_R - y^u_{ij} Q^+_i \tilde{H} u^j_R + \text{h.c.} \\ &\quad + m^2 H^+ H - \lambda (H^+ H)^2 \end{aligned}}$$

The remaining 11 weeks of the course will be devoted to the physical consequences of this Lagrangian.

To wrap up, a taste of the Higgs mechanism: note that this Lagrangian has no fermion masses (it can't, since all the left- and right-handed fermions have different U(1) charges). But, if we set $H = \begin{pmatrix} 0 \\ v \end{pmatrix}$ with v a constant, then

$$y^e_{11} L^+_1 H e^1_R \to y^e_{11} (\nu_L^+ \;\; e_L^+) \begin{pmatrix} 0 \\ v \end{pmatrix} e_R = v\, y^e_{11} \underbrace{e_L^+ e_R}_{\text{a mass term for the electron!}}$$

More on this, and how electromagnetism emerges from hypercharge, in the weeks to come...