Dark matter

CMB observations tell us 95% of the mass of the universe does not interact with EM or the strong force; can't be neutrinos either (too light), so dark matter (DM) must be some particle beyond the SM.

Model-building for DM

Let's try writing down a Lagrangian that can describe "dark" DM. Only requirements are Lorentz and gauge invariance; at this point, anything goes! One way of organizing: look for renormalizable operators w/ new fields neutral under SM gauge group.

\[ \Theta_{\text{prim}} = \{ F, \bar{F}, H^+ S^-, \bar{H} N^3 \} \]

"portal" to dark sector: dark photon, Higgs portal, RH neutrino portal

By way of example, let's focus on Higgs portal, which has a new scalar $S$.

\[ L = d_s d^* s - \frac{1}{2} m_s^2 s^2 - \lambda_{H S} S^2 S^2 H^+ H \]

After EWSB ($H = \frac{1}{\sqrt{2}} (v + i \phi)$),

\[ L = \frac{\lambda_{H S}}{2} v^2 S^2 - \lambda_{H S} v S^2 H - \frac{\lambda_{H S}}{2} S^2 H^+ H \]

\[ m^2_s \rightarrow m^2_S + \lambda_{H S} v^2, \text{ if } m^2_S \gg v \text{ this doesn't change the story.} \]

Declare that $S$ has a $\mathbb{Z}_2$ symmetry $S \rightarrow -S$ so it's stable ($S H H$ forbidden).

The cliff notes for DM:

- need to annihilate in early universe to avoid overabundance: $S S \rightarrow \text{SM SM}$ fixes some relation b/w $\lambda_{H S}$ and $m_s$
- Can detect DM by:
  - scattering w/ SM particles ("direct detection")
  - annihilation into SM particles ("indirect detection")
  - making it at a collider ("collider production")
All of these are related by the same Feynman diagram(s):

\[ \begin{aligned}
& \text{DM abundance,} \\
& \text{indirect detection}
\end{aligned} \quad \begin{aligned}
& \text{Direct detection} \\
& \text{Collider production}
\end{aligned} \]

Let's compute each in turn.

Suppose \( m_S > m_h \). One annihilation channel is \( SS \rightarrow hh \):

\[ \begin{aligned}
& \text{For a rough estimate, just use first diagram.} \\
& |M|^2 = 4 \lambda_{hs}^2 \ (2! \text{! in Feynman rule } S^2 h) \quad \sigma = \frac{1}{2E_1 E_2 |v_1 - v_2|} \frac{1}{8\pi} \left( 4 \lambda_{hs}^2 \right)
\end{aligned} \]

For cosmology, the relevant quantity is actually \( \sigma v_{\text{rel}} \) (really, a thermal average over Boltzmann distribution). When annihilation shuts off, \( S \) is just barely relativistic, so \( E_1 = E_2 \approx m_S \)

\[ \Rightarrow \sigma v_{\text{rel}} = \frac{\lambda_{hs}^2}{16 \pi m_S^2} \ \text{(gain a factor of } \cdot 4 \text{ accounting for } SS \rightarrow WW, n = 2) \]

To obtain correct amount of DM today ("relic abundance"), need

\[ \sigma v_{\text{rel}} \approx 10^{-26} \text{ cm}^3/\text{s} \Rightarrow \lambda_{hs} = 0.2 \left( \frac{m_S}{1 \text{ TeV}} \right), \text{ very reasonable!} \]

(Note we will violate unitarity for \( \lambda_{hs} > 4\pi \), so \( m_S \leq 75 \text{ TeV} \) for this model to be predictive.)
Direct detection

Look for DM scattering off atomic nuclei. First look at nucleon:

\[
\begin{align*}
S & \rightarrow & - & - & S \\
\pi & \rightarrow & h & & \\
\pi & \rightarrow & \ U(p_4) \ U(p_2) & & (t=(p_3-p_1)^2=(p_2-p_4)^2) \\
\end{align*}
\]

What is Higgs coupling to nucleons? $h \bar{q} q$, so what we actually want is the matrix element $\langle N | \bar{q} q | N \rangle$, which is necessarily non-perturbative at low energies.

Let's first just parametrize the Higgs-nucleon coupling as $F_N$: $\mathcal{L}_{\text{eff}} = F_N h \bar{n} n$

\[
i M = (-2 \lambda_{hs} v)(F_N) \frac{\bar{u}(p_4) u(p_2)}{t-m_h^-} \]

\[
\langle M^2 \rangle = \frac{2 \lambda_{hs}^2 v^2 F_N^2}{(t-m_h^-)^2} \left( \frac{8 \lambda_{hs}^2 v^2 F_N^2}{(t-m_h^-)^2} (p_2 p_4 + m^2) \right)
\]

Since DM is non-relativistic, we have to be a little careful with the kinematics:

\[
p_1 = (m_s + \frac{1}{2} m_v, 0, 0, m_v) \quad p_2 = (m_n, 0, 0, 0) \quad p_3 = (m_n + \frac{q^2}{2 m}, q \sin \theta, 0, q \cos \theta) \quad p_4 = p_1 + p_2 - p_3
\]

\[
\Rightarrow t = (p_2 - p_4)^2 = 2 m_n^2 - 2(m_n^2 + \frac{q^2}{2 m}) = -q^2
\]

$q$ is the momentum transfer from DM to nucleon.

Since $q_{\text{max}} = 2 m_v m_n$, and gravitational measurements tell us $m_v \approx 10^{-3}$, $|q^2| < m_n^2$ and we can approximate the denominator as $m_n^4$. 

\[
\]

\[
\]

\[
\]

\[
\]

\[
\]

\[
\]
Now, can use \( d^3q \) instead of \( d^3p_T \) in phase space integral since \( q = \vec{P}_T - \vec{P}_l, \ E_T \approx m, \ E_l \approx m_s \)

\[
\sigma_n = \frac{1}{4 \pi \frac{m^2}{m^2 + m^2} v \ v_m} \int \frac{d^3q}{(2\pi)^3} \ \frac{d^3p_T}{(2\pi)^3} (2\pi)^4 \delta (p_T + p_L - p_T - p_L) \langle 1m1^+ \rangle
\]

\[
= \frac{1}{16 \pi \frac{m^2}{m^2 + m^2} v \ v_m} \ 8 \ \frac{\lambda_{Hs} \ \frac{2}{3} \ \frac{2}{3}}{m^2 + m^2} \int \frac{q^2 \ dq \ d \cos \theta}{(2\pi)^2} (2\pi) (2m^2 + q^2) \delta (E_i - E_F)
\]

\[
E_i - E_F = \frac{1}{2} m_s v_m - \frac{q^2}{2m} - \left( m_s v_m - \frac{q}{m_s} \right)^2 = \frac{q \cdot v_m - \frac{q^2}{m_s}}{2m_s} = \frac{q \cdot v_m \cos \theta - \frac{q^2}{2m_s}}{2m_s}
\]

\[\Rightarrow\] use \( \delta \)-function to do cos \( \theta \) integral

\[
\delta \left( q \cdot v_m \cos \theta - \frac{q^2}{2m_s} \right) = \frac{1}{q \cdot v_m} \delta (\cos \theta - \frac{q}{2m_s v_m})
\]

\[
\sigma_n = \frac{\lambda_{Hs} \ \frac{2}{3} \ \frac{2}{3}}{4 \pi \frac{m^2}{m^2 + m^2} v \ v_m} \int \frac{2m_s v_m}{(2m^2 + \frac{q^2}{2})} dq
\]

\[
= \frac{1}{\pi} \lambda_{Hs} \ \frac{2}{3} \ \frac{2}{3} \ \frac{v^2 \ m_s^2}{m^2 + m^2}
\]

For \( m_s \gg m \), \( M_n \approx m_T \), and \( \sigma_n \approx \frac{1}{\pi} \lambda_{Hs} \ \frac{2}{3} \ \frac{2}{3} \ \frac{v^2 \ m_s^2}{m^2 + m^2} \)

Two remaining ingredients: determine \( f_n \) and compute \( \sigma_n \) for cross section from a nucleus composed of many nucleons.

Higgs couples to all quarks: \( f_n = \sum \frac{m}{m} \langle n | q \bar{q} | n \rangle \). Things like \( \langle N | u \bar{u} + d \bar{d} | N \rangle \) come from chiral perturbation theory, since Higgs couples more strongly to heavier quarks, dominant contribution is from strange quark content of nucleon, \( \langle n | \bar{s} s | n \rangle \approx 0.5 \).
\[ \Rightarrow \text{take } F_N \sim \frac{m_N^{\text{strange quark mass}}}{m_N^{\text{not on mass}}} \sim 10^{-3} \]

If nucleus were just a bag of nucleons,
\[ \frac{N_n}{A} = \frac{m_N^{\text{strange quark mass}}}{m_N^{\text{not on mass}}} \]
But at large momentum transfer, we lose coherence over nucleons and need to include a nuclear form factor \( F_N(q^2) \), which starts to differ from 1 at \( q^2 \approx \frac{1}{A} \text{meV} \).

Ignore this for now; take \( A=131 \) for xenon, target mass of 1 ton = \( 5 \times 10^{-27} \) xenon nuclei, suppose \( m_N = 1 \text{ TeV} \Rightarrow \lambda_{ds} = 0.2 \)

\[ R = N_{xe} V_0 N \lambda_{ds} \sim 5 \times 10^{-27} \left( \frac{0.3 \text{ GeV}/c^2}{1 \text{ TeV}} \right)^3 (131^2) (131^2) (10^{-3}) \times \]
\[ \left( \frac{1}{\pi} \frac{(0.2)^2 (10^{-2}) (2.6 \text{ GeV}) (1 \text{ TeV})^2}{(1 \text{ TeV})(115 \text{ GeV})^2} \right) \]
\[ \approx 3 \times 10^{-5} \text{ Hz} = 1000 \text{ events/yr} \]
Can be probed by XENON-1T!

**Collider production**

Let's make S at a collider. Can obviously do
\[ FF \rightarrow h \rightarrow SS \] but this is a fully invisible final state and it's hard to trigger on "nothing." Say we have a lepton collider. We could radiate a photon:

\[ e^+ \rightarrow S \rightarrow e^+ e^- \rightarrow \gamma + \text{invisible} \]

But electron Yukawa is small:
\[ y_e = \sqrt{\frac{\alpha}{\pi}} = 10^{-5} \]
How do we exploit a large coupling?
\( e^+e^- \rightarrow Z + \text{inv.} \) ("mono-jet")

Only large couplings!

If \( Z \) decays via \( u\bar{u} \) or \( q\bar{q} \), this is a very clean signal given the large missing energy, but need \( \sqrt{s} > 2m_Z + m_2 \).

At a hadron collider, want to exploit top quark coupling:

Harper because we have to deal with all the soft QCD gluon which did not go into the gluon PDF, but this is a key search strategy at the LHC.