Basic Electroweak processes + Neutrino oscillations.

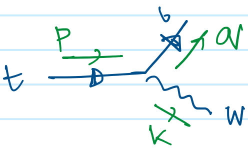
Wednesday, April 14, 2021 2:49 PM

Top Quark decay.

$$\mathcal{L} \supset \frac{e}{\sqrt{2}\sin\theta_W} W_\mu^+ \bar{t}_L \gamma^\mu V^{3j} \underbrace{d_{jL}}_{\substack{j=1,2,3\\ d,s,b.}} + h.c.$$

expt : $V^{33} \gg V^{32}, V^{31}$

so for now consider decay to b quarks only & then extend.

$i\mathcal{M}_{t\to bW}$:  $= \frac{ie}{\sqrt{2}\sin\theta_W} V_{tb}\, \bar{u}(q)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u(p)\,\varepsilon_\mu^*(k)$

* need to be careful with γ^5 hermition!

$$\left[\bar{u}(q)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u(p)\right]^* = u^\dagger(p)\left(\frac{1-\gamma^5}{2}\right)\gamma^{\mu\dagger}\gamma^0 u(q) = \bar{u}(p)\left(\frac{1+\gamma^5}{2}\right)\gamma^\mu u(q)$$

$\gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^{\mu\dagger}$ & $\gamma^5\gamma^0 = -\gamma^0\gamma^5$

$$\Rightarrow |\mathcal{M}|^2 = \frac{1}{2}\frac{e^2|V_{tb}|^2}{8\sin^2\theta_W} \mathrm{Tr}\left[(\not{q}+m_b)\gamma^\mu(1-\gamma_5)(\not{p}+m_t)(1+\gamma_5)\gamma^\nu\right]\left(-\eta^{\mu\nu}+\frac{k_\mu k_\nu}{M_W^2}\right)$$

Here I used $\sum_\gamma \varepsilon_\mu^{*\gamma}\varepsilon_\nu^\gamma = -\eta^{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}$

Simplifying approximation : $m_b \Rightarrow 0$

as $m_b \approx 4\,\text{GeV} \ll \underset{m_t.}{173\,\text{GeV}}$

$$\Rightarrow \mathrm{Tr}\left[\not{q}\gamma^\mu(1-\gamma_5)(\not{p}+m_t)(1+\gamma_5)\gamma^\nu\right] = \mathrm{Tr}\left[\not{q}\gamma^\mu(1-\gamma^5)(1-\gamma^5)(\not{p}+m_t)\gamma^\nu\right]$$

γ_5 crosses γ^μ Thrice $= 2\mathrm{Tr}\left[\not{q}\gamma^\mu(1-\gamma^5)(\not{p}+m_t)\gamma^\nu\right]$

Use γ_5 algebra trick : 1) $\mathrm{Tr}(\gamma^5) = 0$

2) $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5) = 0$

3) $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) = -4i\varepsilon^{\mu\nu\rho\sigma}$

last day ended here.

$$\Rightarrow 2\mathrm{Tr}\left[\not{q}\gamma^\mu(1-\gamma_5)(\not{p}+m_t)\gamma^\nu\right] = 2\left(4(q^\mu p^\nu + q^\nu p^\mu - \eta^{\mu\nu} q\cdot p) + \underbrace{4i\varepsilon^{\alpha\mu\beta\nu} q_\alpha p_\beta}_{\text{this goes to zero}}\right)$$

because $\left(-\eta_{\mu\nu}+\frac{k_\mu k_\nu}{m_W^2}\right)$ is symm in $\mu\nu$

Plugging back in $|\mathcal{M}|^2$

\+ using $p = (m_t, \vec{0})$, $q = (|\vec{q}|, \vec{q})$, $k = \left(\sqrt{M_W^2+|\vec{q}|^2}, -\vec{q}\right)$

where $|\vec{q}| + \sqrt{m_W^2+|\vec{q}|^2} = m_t$

\+ using $\Gamma = \frac{|\vec{q}|}{8\pi m_t^2}\langle|\mathcal{M}|^2\rangle$

we obtain

$$\boxed{\Gamma_{t\to bW} = \frac{e^2|V_{tb}|^2}{64\pi\sin^2\theta_W}\frac{m_t^3}{m_W^2}\left(1-\frac{m_W^2}{m_t^2}\right)^2\left(1+2\frac{M_W^2}{m_t^2}\right)}$$

* Note $\langle |M|^2 \rangle = |M|^2$ because $p\cdot q$, $q\cdot P$, $K\cdot q$, etc all are independent of θ.

for e.g. $q\cdot K = |\vec{q}|\sqrt{M_W^2 + |\vec{q}|^2} + |\vec{q}|^2 = \frac{1}{2}(M_t^2 - M_W^2)$

$\Rightarrow \Gamma_{t \to any} = (|V_{tb}|^2 + |V_{td}|^2 + |V_{ts}|^2) \times \ldots$

Plugging in $e = 0.303$, $\sin^2\theta_W = 0.231$ $\sum |V_{td_j}|^2 = 0.78$
$m_t = 173$ GeV, & $m_W = 80.4$ GeV

$\Rightarrow \Gamma_{t,tot} = 1.38$ GeV

Experimentally, $\Gamma_{t,tot} = 1.42^{+0.19}_{-0.15}$ GeV, so matches within error bars!

Although note that both 'e' & '$\sin^2\theta_W$' are functions of CM energy. Here we were rough and used 'e' value $E_{CM} = 0$ & '$\sin^2\theta_W$' value at $E_{CM} = M_Z$. This detail becomes important for precision measurement.

Regardless, the variation is tiny and doesn't change the fact that Γ_{top} is HUGE! The lifetime is $\tau = \Gamma^{-1} \sim 5\times10^{-25}$ s

which is shorter than the time it takes for hadronization. Hence top quark is the closest thing to free quark in SM.

So weak interaction isn't that weak at high energies.

(HW: more practice on Z & higgs decay using same technique)

Neutrino Oscillations.

14 April 2021 16:24

Nuclear reactions such as $4p \to {}^4He + 2e^+ + 2\nu_e$ produce neutrinos in the Sun.

Using the so-called Standard Solar Model (SSM), one can exactly predict the amount of ν_e produced in the sun & hence predict ν_e flux on earth.

BUT : Super Kamiokande expt measured the ν_e flux on Earth & found it to be $\sim 1/3$ of expected. (in 1998)

Even more surprisingly : In 2001, the Nobel prize winning SNO expt. measured flux from $\nu_e + \nu_\mu + \nu_\tau$ and found the total flux to match with what was expected. of ν_e.

It is impossible for sun to produce ν_μ, ν_τ directly because the energies in sun is smaller than M_μ, M_τ, & ν_μ, ν_τ can only be produced in conjunction with μ^-, τ^-. Yet we see sun to shine $\nu_\mu + \nu_\tau$ twice as brighter than ν_e ..... How come?

Ans : neutrino oscillations due to neutrino masses.

To see how neutrino masses can generate oscillation recall

$$\mathcal{L} \supset \bar{e}_{Li} \not{W} \nu_{Li} - M_{ij} \bar{e}_{Li} e_{Rj} + h.c \leftarrow \text{pre-lepton mass diagonalization post-symmetry breaking}$$

we diagonalize M_{ij} by rotating $\bar{e}_L \to \bar{e}_L V$, $e_R \to U e_R$. such that $VmU = \underset{\text{diagonal}}{M}$

then $$\mathcal{L} \supset \bar{e}_{Li} \not{W} (V\nu_L)_i - M_i \bar{e}_{Li} e_{Ri} + h.c$$

define as new ν_L & everything is diagonal & happy.

But suppose we also have a neutrino mass matrix.

$$\mathcal{L} \supset \bar{e}_{Li} \not{W} (V\nu_L)_i - M_i \bar{e}_{Li} e_{Ri} - (m_\nu)_{ij} \bar{\nu}_{Ri} \nu_{Lj} + h.c$$

then diagonalizing $m_\nu = U' M_\nu V'$ & redefining $\bar{\nu}_R \to U' \nu_R$ gives then diagonalizing $M_\nu = V' M_\nu V'$ & redefining $\bar\nu_R \to V' \nu_R$ gives

$$\mathcal{L} \supset \bar e_{Li} W (\underbrace{V \nu_L}_{})_i - M_i \bar e_{Li} e_{Ri} - M_{\nu i} \bar\nu_{Ri} (\underbrace{V' \nu_L}_{})_i$$

only one of them can be absorbed in new ν_L!

By choosing to keep interaction with W_μ diagonal, we obtain a $\underbrace{\text{non-diagonal mass-matrix}}$.

Implication?

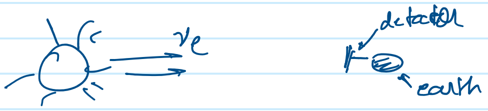
consider simple 2 generation scenario:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

flavour basis (i.e $e W \nu$ is diagonal)

mass basis (i.e. $M_{\nu ij}$ is diagonal)

And consider case of sun:



(not to scale!)

As sun can only produce e^- & not μ^-, we know the emmitted neutrinos have to be in flavour basis & must be ν_e

Then as ν_e propogates in space time, at some coordinate x^μ we have $|\nu_e(x)\rangle = e^{-iP_1\cdot x}\cos\theta\,|\nu_1\rangle + e^{-iP_2\cdot x}\sin\theta\,|\nu_2\rangle$

& Note above we have evolved $|\nu_i(x)\rangle = e^{-iP_i\cdot x}|\nu_i(0)\rangle$ as away from any interaction, particles just evolve as plane waves.

In the language of QM above is equivalent to $|\psi_i(t)\rangle = e^{-iE_i t}|\psi_i(0)\rangle$ where $|\psi_i\rangle$ is eigenstate of Free hamiltonian.

rewriting: $|\nu_e(x)\rangle = e^{-iP_1\cdot x}\left(\cos\theta\,|\nu_1\rangle + e^{i(P_1-P_2)\cdot x}\sin\theta\,|\nu_2\rangle\right)$

let $x = (T, 0, 0, L)$ when $x=0$ at sun.

using the fact that avg neutrino velocity $\approx \bar v = \dfrac{|\vec P_1 + \vec P_2|}{E_1+E_2} \approx \dfrac{|\vec P_1| + |\vec P_2|}{E_1+E_2}$ as $\vec P_1 \parallel \vec P_2$

we have $T = \dfrac{L}{\bar v} = L\left(\dfrac{E_1+E_2}{|\vec P_1|+|\vec P_2|}\right) \Rightarrow x^\mu = \dfrac{L}{|\vec P_1|+|\vec P_2|} \times (P_1^\mu + P_2^\mu)$

$P_1^\mu = (E_1, 0, 0, |\vec P_1|)$
$P_2^\mu = (E_2, 0, 0, |\vec P_2|)$

$$\Rightarrow \quad x^\mu = \frac{L}{|\vec{p}_1| + |\vec{p}_2|} \times (p_1^\mu + p_2^\mu)$$

$p_1^\mu = (E_1, 0, 0, |\vec{p}_1|)$
$p_2^\mu = (E_2, 0, 0, |\vec{p}_2|)$

Then
$$(p_1 - p_2)\cdot x = \frac{L}{|\vec{p}_1| + |\vec{p}_2|}(p_1 - p_2)\cdot(p_1 + p_2) = \frac{L}{|\vec{p}_1| + |\vec{p}_2|}(m_1^2 - m_2^2) \approx \frac{L}{2E}(m_1^2 - m_2^2)$$

assumed relativistic neutrinos. True for any expts or astrophysics. $E \sim MeV \gg eV$.

$$\Rightarrow |\nu_e(x)\rangle = e^{-ip_1\cdot x}\left(\cos\theta\,|\nu_1\rangle + \exp\left(i\frac{L\Delta m_{12}^2}{2E}\right)\sin\theta\,|\nu_2\rangle\right)$$

Note we only made $p_1 \approx p_2$ approx in the end in denominator because we wanted to track the mass difference Δm_{12}^2 in numerator.

Finally as detectors on earth also measure ν_e or ν_μ, we take the overlap:
$$\langle \nu_e | \nu_e(x)\rangle = e^{-ip_1\cdot x}\left(\cos\theta\langle\nu_e|\nu_1\rangle + \exp\left(i\frac{L\Delta m_{12}^2}{2E}\right)\sin\theta\,\langle\nu_e|\nu_2\rangle\right)$$

$$\langle \nu_e | \nu_e(x)\rangle = e^{-ip_1\cdot x}\left(\cos^2\theta + \exp\left(i\frac{L\Delta m_{12}^2}{2E}\right)\sin^2\theta\right)$$

So detection probabilities are
$$P(\nu_e \to \nu_e) = |\langle\nu_e|\nu_e(x)\rangle|^2 = 1 - \sin^2 2\theta\,\sin^2\left(\frac{L}{4E}\Delta m_{12}^2\right)$$

Illy
$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta\,\sin^2\left(\frac{L}{4E}\Delta m_{12}^2\right)$$

Note for $\Delta m_{12}^2 \to 0$ $\quad P(\nu_e) = 1$ & $P(\nu_\mu) = 0$

Hence observation of neutrino oscillation is evidence for neutrino mass!

☆ Note the above formalism would give incorrect result for neutrino oscillation of solar neutrinos.

↳ Because 1) we did not consider ν_τ

2) we did not take into account matter effects as neutrinos propagate in sun.

Very interesting effect but no time to explore.

In expts on Earth one can setup neutrino production & detection expts to probe neutrino oscillation.

For that we want to maximize.
$$\sin^2\left(\frac{L}{4E}\Delta m_{12}^2\right) = \sin^2\left(1.27\times10^3\,\frac{\Delta m_{12}^2}{eV^2}\,\frac{L/km}{E/MeV}\right)$$

So a detector $\sim 1\,km$ away is most sensitive to $\Delta m_{12}^2 \sim 10^{-3}\,eV^2$.

This drives design considerations.

Importance of coherence for neutrino oscillations!

14 April 2021 16:25

(In class I was not able to go into details & only gave a rough outline.
Here I have discussed it in bit more technical detail if anyone is curious.

For even more curious peeps: refer to arXiv:0706.1216)

The reason we observe neutrino oscillation is because ν_e is not a mass eigenstate.

Recall $\mathcal{L} \supset \bar{e}_{Li} \not{W} (V \nu_L)_i - M_i \bar{e}_{Li} e_{Ri} - M_{\nu i} \bar{\nu}_{Ri} (V' \nu_L)_i + h.c$

we made this part diagonal at expense of this part

But this is very unusual. In all other scenarios we strive to make mass matrix diagonal

└ For e.g in the quark case we make the mass matrix of both u_i & d_i diagonal & instead have non-diagonal interaction with W_μ.

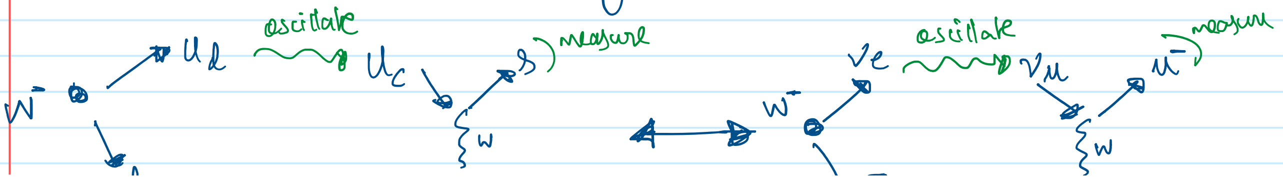
$$\mathcal{L} \supset \bar{d}_{Li} \not{W} \underbrace{V_{ij}}_{\text{not diagonal.}} u_{Lj} - M_{d_i} \bar{d}_{Li} d_{Ri} - \underbrace{M_{u_i}}_{\text{diagonal}} \bar{u}_{Ri} u_{Ri} + h.c$$

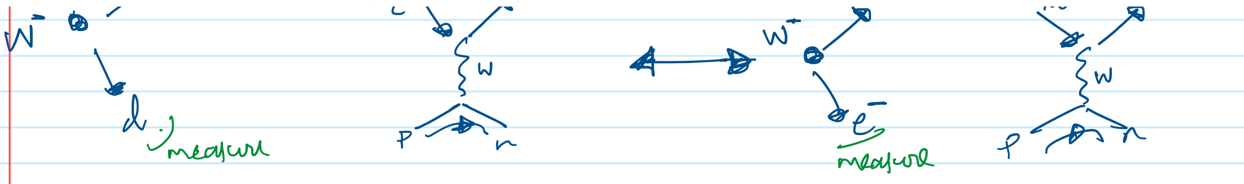
To understand why mass basis is not a favourable basis for neutrinos, we shall try to see what happens when we consider up quarks in flavour basis:

$$|u_d\rangle = \cos\tilde{\theta}\, |u\rangle + \sin\tilde{\theta}\, |c\rangle$$
$$|u_c\rangle = -\sin\tilde{\theta}\, |u\rangle + \cos\tilde{\theta}\, |c\rangle$$

And we turn off strong interactions for simplicity.

So we set up a following 'up quark oscillation' expt:





Coherence at Emmission:

for W boson to emit a coherent state with particles with different masses superimposed (u & s quark or ν_1 & ν_2)

one would require : $\underbrace{\sigma_m^2}_{\text{uncertainity in mass of emmited particle}} > \underbrace{\Delta m^2}_{\text{difference in } m^2 \text{ of 2 particles: } m_c^2 - m_u^2 \text{ or } m_1^2 - m_2^2}$

as $M^2 = E^2 - p^2 \rightarrow \sigma_{m^2} = \sqrt{\sigma_{E^2}^2 + \sigma_{p^2}^2} \Rightarrow \sigma_m^2 = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2}$

note $\sigma_p = \frac{1}{\sigma_x} = \frac{1}{v\sigma_t} = \frac{E}{p}\Gamma$ & $\sigma_E = \frac{1}{\sigma_t} = \Gamma$

coherence of emmission requires $\frac{\Delta m^2}{2\sqrt{2}E\Gamma} < 1$

For W-decay : $\Gamma = 230$ MeV , $E = \frac{M_W}{2} = 40$ GeV.

$\therefore$ For quarks : $\frac{1 \text{ GeV}^2}{40\text{GeV} \times 230\text{MeV}} \ll 1$ & for neutrinos : $\frac{10^{-3}\text{eV}^2}{40\text{GeV} \times 230\text{MeV}} \ll 1$

Hence u_d can be emmitted coherently & so can $\nu_{e,\mu,\tau}$!

Coherence at propagation:

Particles with different masses but same energy travel at different speeds $\Rightarrow$ wavepacket will separate eventually $\Rightarrow$ decoherence.

if $\underbrace{\Delta v}_{\text{speed difference}} \times \underbrace{t}_{\text{time elapsed}} > \underbrace{\sigma_x}_{\text{initial wavepacket separation.}}$ then $|u_d\rangle$ or $|\nu_e\rangle$ become incoherent.

note $\sigma_x = v\sigma_t \approx c\Gamma^{-1}$, $\frac{\Delta v}{c} = \frac{p_1}{E_1} - \frac{p_2}{E_2} \approx \underbrace{\frac{m_2^2 - m_1^2}{2E^2}}_{\text{using } p \approx E - \frac{m^2}{2E}}$,

distance away from W⁻ after which they decohere.

$t \approx \frac{l_{coh}}{c}$

$$\rightarrow \boxed{l_{coh} = \left(\frac{4E^2}{\Delta m^2} \frac{\text{MeV}}{\Gamma}\right) \times 10^{-11} \text{cm.}}$$

for $\Delta m^2 = m_c^2 - m_u^2 \sim 1\text{GeV}^2 \rightarrow l_{coh} \sim 10^{-10}$ cm (interatomic distance)

for $\Delta m^2 = m_{\nu_2}^2 - m_{\nu_1}^2 \sim 10^{-5}\text{eV}^2 \rightarrow l_{coh} \sim 10^{11}$ cm (Diameter of sun)

Hence practically speaking, any other emmitted particle quickly decohere but neutrinos are so light that different mass eigenstates remain coherent for macroscopic distances to exhibit oscillations!

Furthermore:

Any realistic expt we can come up with only probes flavour of neutrinos because all neutrino interactions are through $W^{\pm}$ Z_0.

There is no practical way we can directly measure neutrino mass.

However for charged leptons/quarks we can easily measure their mass by the arc they form in magnetic fields, etc.

Thus both the fact that neutrino mass-eigenstates remain coherent at macroscopic distances + only exhibit interactions via flavour eigenstates.

allow for the interesting observation of oscillations.