Colliders and detectors

How do we make elementary particles? $E = mc^2$ plus QM: if you have enough energy, anything that can happen, will happen

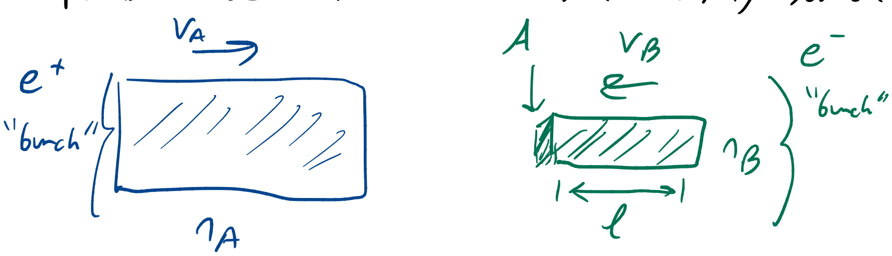
For example, collide electrons and positrons:

If each beam has energy $\frac{E}{2}$, then the center-of-mass energy is E: we can create particles with total mass up to E.

QM (really QFT) tells us the probability of making a given set of final-state particles. In particle physics we call this the matrix element $M_{i \to f}$, and next week we will see how to calculate it for some specific processes.

Cross sections

Parameterize interaction strength using something with units of area



number of scattered particles proportional to area of scattering target

If we have two colliding beams with cross-sectional area A and length ℓ,

$$\text{scattering rate} = \frac{\text{events}}{\text{time}} = n_A n_B A \ell \, |v_A - v_B| \, \sigma \equiv \mathcal{L} \sigma$$

$\mathcal{L}$ is the luminosity and parameterizes the flux of incoming particles.
σ is the scattering cross section which parameterizes the interaction strength.
n_A, n_B are the number densities of particles A and B in the beams.

$|v_A - v_B|$ is the relative velocity of the two beams. If the beams are relativistic ($v_A \approx 1$, $v_B \approx 1$), this factor is $|v_A - v_B| = 2$. Despite appearances, this does not violate the velocity addition rule: it's formally defined as the "Møller velocity" and ensures the scattering rate is Lorentz-invariant with respect to boosts along the beam axis. (see Peskin & Schroeder Sec. 4.5 if you're curious.)

Fermi's Golden Rule relates σ to $\mathcal{M}$:

$$\sigma_{i \to f} = \underbrace{\frac{1}{(2E_A)(2E_B)|v_A - v_B|}}_{\text{from relativistic normalization of initial and final states}} \int \underbrace{|\mathcal{M}_{i\to f}|^2}_{\text{probabilities are squares of amplitudes}} \underbrace{d\Pi}_{\text{sum over final states: Lorentz-invariant phase space}} \underbrace{(2\pi)^4 \delta^4\left(p_A + p_B - \sum_{i=3}^{n} p_i\right)}_{\text{4-momentum conservation}}$$

Note that σ is not Lorentz-invariant, but transforms like an area: Lorentz-invt. for boosts along beam axis. This is the key observable predicted by QFT: "effective area" of beams of particles A and B, taking into account the fact that some collisions are rarer than others.

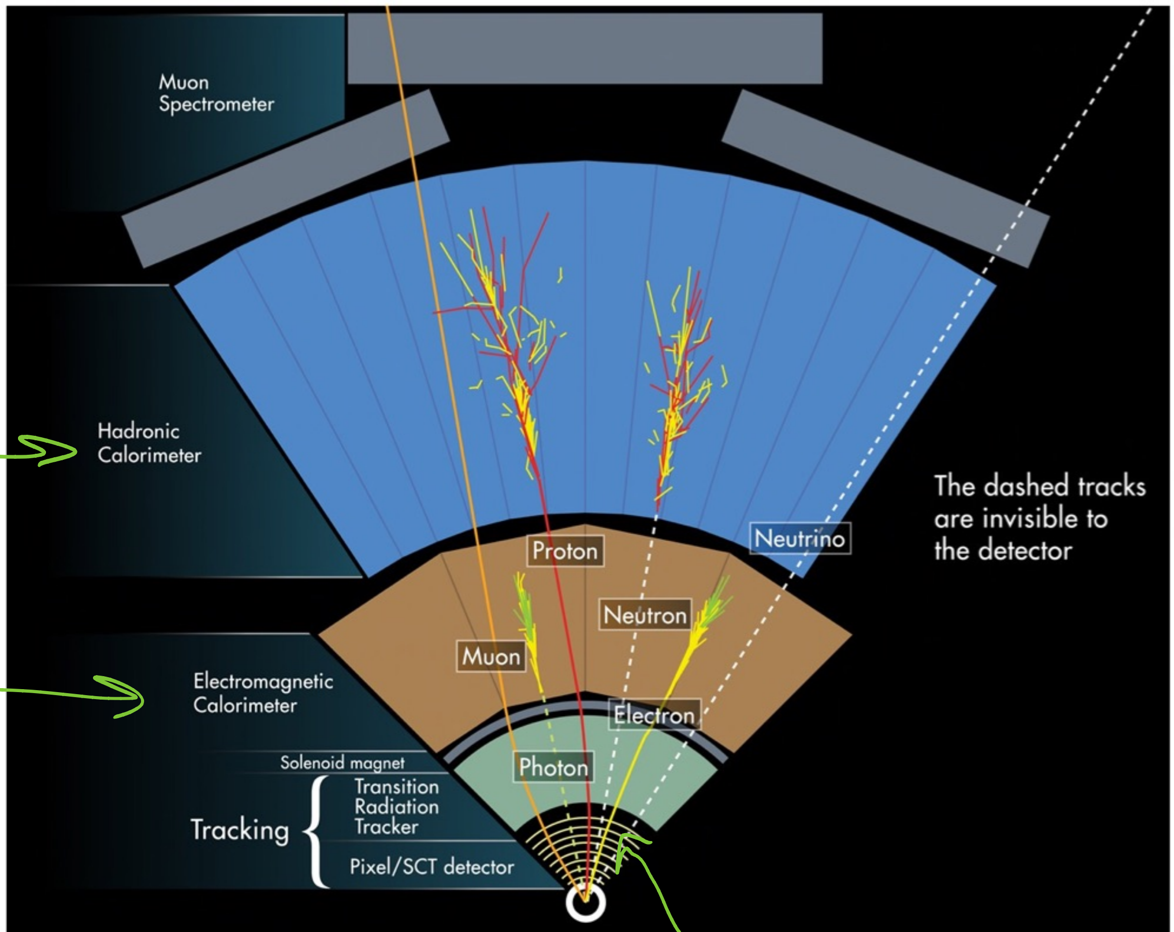
Units: σ is usually given in [SI prefix] × barns, where
1 barn $= 10^{-24}\,\text{cm}^2$
Luminosity is usually quoted in $[\text{prefix} \times \text{barns}]^{-1}/\text{s}$, so for example, a process with $\sigma = 1\,\text{fb} = 10^{-15}$ barns at the LHC ($\mathcal{L} \sim 1\,\text{pb}^{-1}/\text{s}$) has a rate $R = \mathcal{L}\sigma = 10^{-3}/\text{s}$.

How do we detect elementary particles?

Two steps: measure an energy and/or momentum and then identify the particle by its mass and electric charge.

Cross-sectional view of the ATLAS detector:

total number of photons proportional to particle energy



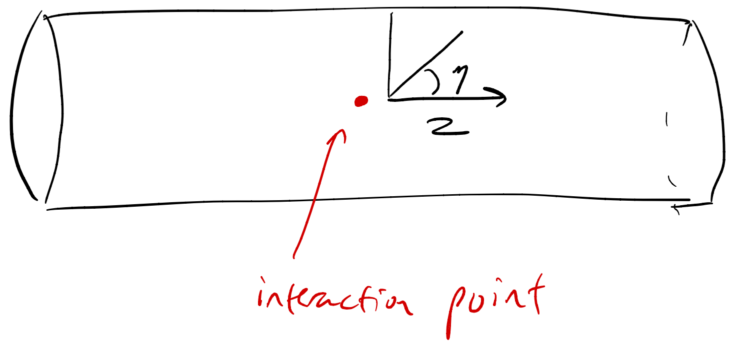
Strips of silicon: charged particles deposit small amounts of energy in each pixel, can leave tracks

Entire detector is immersed in a magnetic field (out of the page in inner region): measure momentum and charge by curvature radius

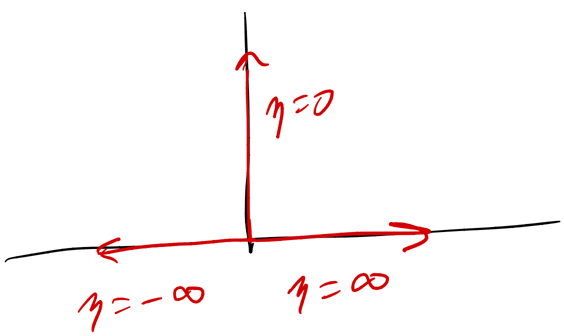
$$R \simeq 3\,m \times \frac{p_\perp[GeV]}{Q\,|B|[T]}$$

If we know E and p => know m, particle ID

Detector coordinates and kinematics:



Basically cylindrical coordinates, but instead of θ, use pseudorapidity $\eta \equiv -\ln \tan \frac{\theta}{2}$

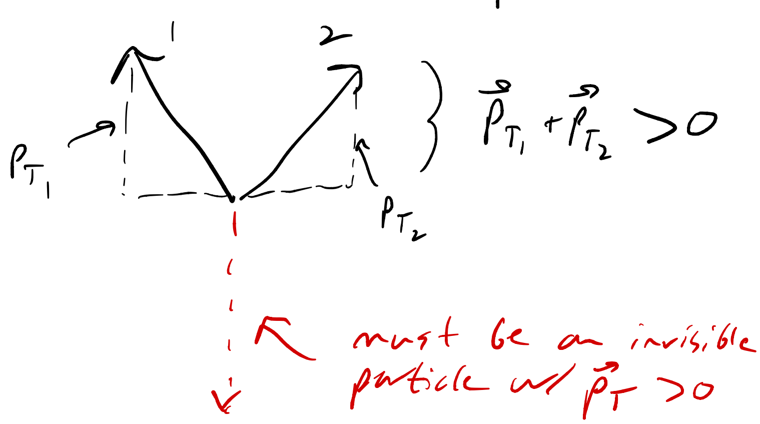


Why this funny variable? 2 related reasons:

- particle production is roughly uniform in η
- behaves nicely under boosts for massless particles (Larkoski 5.3)

Hard to detect particles which go very close to beam direction (how do you avoid the beam?). As a result, often use transverse momentum $p_T \equiv \sqrt{p_x^2+p_y^2} = \sqrt{p^2-p_z^2}$.

Since all 3 components of spatial momentum must be conserved, can infer existence of invisible particles from imbalance in p_T.



Phase space

To compute cross sections, we need to sum over all final states

$\Rightarrow$ integrate over all 4-momenta consistent w/ Poincaré invariance

Translation invariance $\Rightarrow$ 4-momentum conservation (Noether's Theorem)

For a process $p_A + p_B \to p_1 + p_2 + \cdots p_n$,

$$\int d\Pi_n = \int \left\{ \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4} 2\pi\, \delta(p_i^2 - m_i^2)\, \theta(p_i^0) \right\} (2\pi)^4 \delta^{(4)}\left(p_A + p_B - \sum_{i=1}^{n} p_i\right)$$

The 2π's are conventionally attached to $d\Pi_n$ but they do matter — don't forget them!

This is manifestly Lorentz-invariant because the δ-functions enforce $p^2 = m^2$ for each final-state particle, and $p_A + p_B - \sum_{i=1}^{n} p_i = 0$ (the zero 4-vector is also Lorentz-invariant).

We can perform the p^0 integral for each i using

$$\delta(p_i^2 - m_i^2) = \delta\left((p_i^0)^2 - \vec{p}^{\,2} - m_i^2\right) \quad \text{and}$$

$$\delta(f(x)) = \frac{1}{|f'(x_0)|} \delta(x - x_0)$$

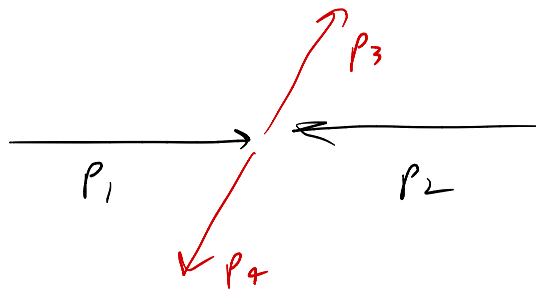
killed by $\theta(p_i^0)$

$$\Rightarrow \delta(p_i^2 - m_i^2) = \frac{1}{2\sqrt{\vec{p}_i^{\,2} + m_i^2}} \left\{ \delta\left(p_i^0 - \sqrt{\vec{p}_i^{\,2} + m_i^2}\right) + \cancel{\delta\left(p_i^0 + \sqrt{\vec{p}_i^{\,2} + m_i^2}\right)} \right\}$$

$$\Rightarrow \int dp_i^0\, \delta(p_i^2 - m_i^2)\, \theta(p_i^0)\, f(p_i^0) = \frac{1}{2E_i} f(E_i) \quad \text{w/ } E_i = \sqrt{\vec{p}_i^{\,2} + m_i^2}$$

$$\Rightarrow \int d\Pi_n = \int \left\{ \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \right\} (2\pi)^4 \delta^{(4)} \underbrace{\left(p_A + p_B - \sum_{i=1}^{n} p_i\right)}_{p_i^0 = E_i}$$

For 2-particle phase space, can do most of the integrals. (HW 4: 3-particle phase space.) Consider the process $p_1 + p_2 \to p_3 + p_4$ (relabeling to match Schwartz 5.1) in the center-of-mass frame where $p_1 + p_2 = (E_{cm}, \vec{0})$.



$$d\Pi_2 = \frac{d^3 p_3}{(2\pi)^3}\frac{1}{2E_3}\frac{d^3 p_4}{(2\pi)^3}\frac{1}{2E_4}(2\pi)^4 \delta^{(4)}(p_1+p_2-p_3-p_4)$$

use $\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4) = \delta^3(\vec{0}-\vec{p}_3-\vec{p}_4)$ to do d^3p_4 integral: sets $\vec{p}_4 = -\vec{p}_3$. Then, write $d^3p_3 = p_3^2\, dp_3\, d\Omega$, where $d\Omega$ is the differential solid angle for $\vec{p}_3$ in spherical coordinates.

Collecting the 2π's and relabeling $p_3 = p_f$

$$d\Pi_2 = \frac{1}{16\pi^2} d\Omega \int dp_f \frac{p_f^2}{E_3 E_4}\delta(E_3+E_4-E_{cm})$$

changed signs for convenience: $\delta(x) = \delta(-x)$

where $E_3 = \sqrt{p_f^2+m_3^2}$, $E_4 = \sqrt{p_f^2+m_4^2}$.

Change variables $p_f \longrightarrow x(p_f) = E_3(p_f) + E_4(p_f) - E_{cm}$

Jacobian: $\frac{dx}{dp_f} = \frac{2p_f}{2\sqrt{p_f^2+m_3^2}} + \frac{2p_f}{2\sqrt{p_f^2+m_4^2}} = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{E_3+E_4}{E_3E_4}p_f$

δ-function enforces $E_3 + E_4 = E_{cm}$, so

$$d\Pi_2 = \frac{1}{16\pi^2}d\Omega\int_{m_3+m_4-E_{cm}}^{\infty} dx\, \frac{p_f(x)}{E_{cm}}\delta(x) = \frac{1}{16\pi^2}d\Omega\frac{|\vec{p}_f|}{E_{cm}}\theta(E_{cm}-m_3-m_4)$$

enforces our energy threshold condition from earlier

where $|\vec{p}_f|$ is the solution to $x(p_f) = 0$

(usually easier to use Lorentz dot product tricks)