Colliders and detectors

How do we make elementary particles? \( E = mc^2 \) plus a \( q \).

If you have enough energy, anything that can happen, will happen.

For example, collide electrons and positrons:

\[ \begin{array}{c}
\text{e}^+ \\
\text{e}^-
\end{array} \]

If each beam has energy \( \frac{E}{2} \), then the center-of-mass energy is \( E \); we can create particles with total mass up to \( E \).

QM (really QFT) tells us the probability of making a given set of final-state particles. In particle physics we call this the matrix element \( M_{i\rightarrow f} \), and next week we will see how to calculate it for some specific processes.

Cross sections

Parameterize interaction strength using something with units of area.

\[ \begin{array}{c}
\text{e}^+ \\
\text{e}^-
\end{array} \]

\[ \eta_A \]

\[ \begin{array}{c}
\text{A} \\
\text{B}
\end{array} \]

\[ \eta_B \]

Number of scattered particles proportional to area of scattering target

If we have two colliding beams with cross-sectional area \( A \) and length \( l \),

\[ \text{Scattering rate} = \frac{\text{events}}{\text{time}} = \eta_A \eta_B AL \frac{l}{v_A - v_B} \frac{1}{\sigma} \equiv \sigma_0 \]
L is the luminosity and parameterizes the flux of incoming particles. 

\( \sigma \) is the scattering cross section which parameterizes the interaction strength.

\( N_A, N_B \) are the number densities of particles A and B in the beams.

\( |v_A - v_B| \) is the relative velocity of the two beams. If the beams are relativistic \( (v_A \approx 1, v_B \approx 1) \), this factor is \( |v_A - v_B| = 2 \). Despite appearances, this does not violate the velocity addition rule: it’s formally defined as the “Møller velocity” and ensures the scattering rate is Lorentz-invariant with respect to boosts along the beam axis. (see Peskin & Schroeder Sec. 4.5 if you’re curious)

Fermi’s Golden Rule relates \( \sigma \) to \( M \):

\[
0_{i \rightarrow f} = \left( \frac{1}{(2E_A)(2E_B)|v_A - v_B|} \right)^{\frac{1}{2}} \int |M_{if}|^2 \, d \Pi (2\pi)^4 \delta^4 (p_f - p_i + \frac{\hat{S}}{2} p_i) \]

from relativistic normalization of initial and final states

probabilities are squares of amplitudes

sum over final states: Lorentz-invariant phase space

\( q \)-momentum conservation

Note that \( \sigma \) is not Lorentz-invariant, but transforms like an area: Lorentz-invar, for boosts along beam axis. This is the key observable predicted by QFT: “effective area” of beams of particles A and B, taking into account the fact that some collisions are rarer than others.

Units: \( \sigma \) is usually given in \( \text{[SI prefix] \times barns} \), where

\[
1 \text{ barn} = 10^{-24} \text{ cm}^2
\]

Luminosity is usually quoted in \( \text{[prefix \times barns]}^{-1/8} \), so for example a process with \( \sigma = 1 \text{ fb} = 10^{-15} \text{ barns} \) at the LHC \( (\sim 1 \text{ pb}^{-1/8}) \) has a rate \( R = \mathcal{L} \sigma = 10^{-3} \text{s}^{-1} \).
How do we detect elementary particles?

Two steps: measure an energy and/or momentum and then identify the particle by its mass and electric charge.

Cross-sectional view of the ATLAS detector:

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  total number of photons proportional to particle energy

  Strips of silicon: charged particles deposit small amounts of energy in each pixel, can leave tracks

  Entire detector is immersed in a magnetic field (out of the page in inner region): measure momentum and charge by curvature radius

  \[ R = 3 m \times \frac{p}{q |1B|} \text{[cm]} \]

  If we know E and p \( \Rightarrow \) know m, particle ID
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Detector coordinates and kinematics:

\[ \frac{\theta}{2} \]

interaction point

Basically cylindrical coordinates, but instead of \( \theta \), use pseudorapidity \( \eta = -\ln \tan \frac{\theta}{2} \)

Why this funny variable? 2 related reasons:
- particle production is roughly uniform in \( \eta \)
- behaves nicely under boosts for massless particles (Larkoski 5.3)

Hard to detect particles which go very close to beam direction (how do you avoid the beam?). As a result, often use transverse momentum \( p_T = \sqrt{p_x^2 + p_y^2} = \sqrt{p^2 - p_z^2} \).

Since all 3 components of spatial momentum must be conserved, can infer existence of invisible particles from imbalance in \( p_T \).

\[ p_{T_1} + p_{T_2} > 0 \]

\( R \) must be a particle with \( p_T > 0 \).
To compute cross sections, we need to sum over all final states
intebrate over all 4-momenta consistent with Poincaré
invariance.

Translation invariance \Rightarrow 4-momentum conservation (Noether's
Theorem).

For a process \( P_A + P_B \rightarrow P_1 + P_2 + \cdots P_n \),

\[
\int \prod_n = \int \left( \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4} \right) 2\pi \delta(p_i^0 - m_i^0) \Theta(p_i^0) \left( \prod_{i=1}^{n} \right) (p_A + p_B - \sum p_i) \]

The \( 2\pi \)'s are conventionally attached to \( \prod_n \) but they do
matter – don’t forget them!

This is manifestly Lorentz-invariant because the \( \delta \)-functions enforce
\( p_i^0 = m_i^0 \) for each final-state particle, and \( p_A + p_B - \sum p_i = 0 \)
(the zero 4-vector is also Lorentz-invariant).

We can perform the \( p_i^0 \) integral for each \( i \), using
\( \delta(p_i^0 - m_i^0) = \delta((p_i^0)^2 - p_i^0 - m_i^2) \) and
\[
\int (\delta(x)) = \frac{1}{|\delta'(x)|} \int (x-x_0)
\]

=> \( \delta(p_i^0 - m_i^0) = \frac{1}{2\sqrt{p_i^0 - m_i^0}} \left( \delta(p_i^0 - \sqrt{p_i^2 + m_i^2}) + \delta(p_i^0 + \sqrt{p_i^2 + m_i^2}) \right) \)

=> \( \int d p_i^0 \delta(p_i^0 - m_i^0) \Theta(p_i^0) f(p_i^0) = \frac{1}{2E_i} f(E_i) \) w/ \( E_i = \sqrt{p_i^2 + m_i^2} \)

=> \( \int \prod_n = \left( \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3} \right) \frac{1}{2E_i} (2\pi)^4 \delta^{(4)} (p_A + p_B - \sum p_i) \)

\[
p_i^0 = E_i
\]
For 2-particle phase space, can do most of the integrals. (HW 4. 3-particle phase space.) Consider the process
\[ p_1 + p_2 \rightarrow p_3 + p_4 \] (relabelling to match Schwartz 5.1) in the centre-of-mass frame where \( p_1 + p_2 = (E_m, \vec{0}) \).

\[ p_3 \]
\[ p_1 \rightarrow \]
\[ p_2 \]
\[ \sqrt{p_4} \]

\[ d\Pi_2 = \frac{d^3p_3}{(2\pi)^3} \frac{1}{2E_3} \frac{d^3p_4}{(2\pi)^3} \frac{1}{2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \]

Use \( \delta^4(p_1 - p_3 - p_4) = \delta^4(0 - p_3 - p_4) \) to do \( d^3p_4 \) integral.

Set \( \vec{p}_4 = -\vec{p}_3 \). Then, write \( d^3p_3 = p_3^2 dp_3 d\Omega \), where \( d\Omega \) is the differential solid angle for \( \vec{p}_3 \) in spherical coordinates.

Collecting the \( 2\pi \)'s and relabelling \( p_3 = \vec{p}_f \)

\[ d\Pi_2 = \frac{1}{16\pi^2} d\Omega \int dp_f \frac{p_f^2}{E_3E_4} \delta(E_3 + E_4 - E_m) \]

where \( E_3 = \sqrt{p_f^2 + m_f^2}, \ E_4 = \sqrt{p_f^2 + m_\ell^2} \).

Change variables \( p_f \rightarrow x(p_f) = E_3(p_f) + E_4(p_f) - E_m \)

Jacobian: \( \frac{dx}{dp_f} = \frac{2p_f}{2p_f^2 + m_f^2} + \frac{2p_f}{2p_f^2 + m_\ell^2} = \frac{p_f}{E_3} - \frac{p_f}{E_4} = \frac{E_3 + E_4}{E_3E_4} p_f \)

\( \delta \)-function enforces \( E_3 + E_4 = E_m \), so

\[ d\Pi_2 = \frac{1}{16\pi^2} d\Omega \int_0^\infty dx \frac{p_f(x)}{E_m} \delta(x) = \frac{1}{16\pi^2} d\Omega \frac{|p_f|}{E_m} \theta(E_m - m_f - m_\ell) \]

where \( |p_f| \) is the solution to \( x(p_f) = 0 \)

(usually easier to use Lorentz dot product tricks)