The Standard Model

We have classified spin-0 and spin-$\frac{1}{2}$ fields by their Lorentz reps and internal (gauge) symmetries, through which we introduced spin-1 fields.

Here are the fields which comprise the Standard Model:

spin-$\frac{1}{2}$ | spin-0

gauge fields (spin-1)	$L_f = \begin{pmatrix} \nu_L^f \\ e_L^f \end{pmatrix}$	e_R^f	$Q_f = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix}$	u_R^f	d_R^f	H
$U(1)_Y$	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$
$SU(2)$	✓		✓			✓
$SU(3)$			✓	✓	✓	

Charges/representations

Terminology: L_f, e_R^f are left/right-handed leptons

Q_f, u_R^f/d_R^f are left/right-handed quarks

$f = 1, 2, 3$ are generations or flavors ($f=1$ is electron, electron neutrino, up quark, down quark; $f=2$ is muon, muon neutrino, charm quark, strange quark; $f=3$ is tau, tau neutrino, top quark, bottom quark)

H is the Higgs field

$U(1)_Y$ is hyperchange

$SU(2)$ (sometimes $SU(2)_L$) is the weak force, and only acts on left-handed fermions (and the Higgs)

$SU(3)$ (sometimes $SU(3)_c$) is color, or the strong force

Notation: Anything with a ✓ under $SU(2)$ is a 2-component vector of fields which transforms with $e^{i\alpha^a T^a}$, like Φ we saw earlier (in fact, Φ is H).

Similarly, the quarks are 3-component vectors transforming with 3×3 unitary matrices

$$u_R = \begin{pmatrix} u_R^r \\ u_R^g \\ u_R^b \end{pmatrix}$$ ("red", "green", "blue"), so Q is actually a $3\times 2 = 6$-component field.

The Standard Model consists of (almost) all terms we can write down up to total dimension 4 which are invariant under Lorentz and local $SU(3)\times SU(2)\times U(1)_Y$ symmetry.

Easy stuff first:

$$\mathcal{L}_{kin} = |D_\mu H|^2 - \frac{1}{4}\underbrace{G^a_{\mu\nu}G^{\mu\nu a}}_{SU(3)} - \frac{1}{4}\underbrace{W^a_{\mu\nu}W^{\mu\nu a}}_{SU(2)} - \frac{1}{4}\underbrace{B_{\mu\nu}B^{\mu\nu}}_{U(1)_Y}$$
$$+ \sum_{f=1}^{3}\left\{ iL_f^\dagger\bar\sigma^\mu D_\mu L_f + iQ_f^\dagger\bar\sigma^\mu D_\mu Q_f + ie_R^{f\dagger}\sigma^\mu D_\mu e_R^f + iu_R^{f\dagger}\sigma^\mu D_\mu u_R^f + id_R^{f\dagger}\sigma^\mu D_\mu d_R^f \right\}$$

$$\mathcal{L}_{Higgs} = +m^2 H^\dagger H - \lambda(H^\dagger H)^2$$ (note mass term has wrong sign! Will get to this later in the course)

Since fermions have dimension $\frac{3}{2}$, a fermion-fermion-scalar term (known as a Yukawa term) has dimension 4. What such terms are allowed?

$$\mathcal{L}_{Yukawa} \supset -y^e_{ij}L_i^\dagger H e_R^j - y^d_{ij}Q_i^\dagger H d_R^j + h.c.$$

y_{ij}: 3×3 matrix of numbers

h.c.: Hermitian conjugates: these are needed for Lagrangian to be real, but are often dropped for convenience.

Consider $L^\dagger H e_R$ term first:

$SU(3)$: $L_i^\dagger \to L_i$, $H\to H$, $e_R^j \to e_R^j$ (no transformations, so trivially invariant)

$SU(2)$: $L_i^\dagger \to L_i^\dagger U^\dagger$, $H\to UH$, $e_R^j\to e_R$ for some $U\in SU(2)$, so

$$L_i^\dagger H e_R^j \to L_i^\dagger(\underbrace{U^\dagger U}_{1})He_R^j = L_i^\dagger H e_R^j,$$ <u>invariant</u> (as expected, just like $\Phi^\dagger\Phi$)

$U(1)_Y$: this group is Abelian, so as a shortcut, can just count charges:

$$\underset{+\frac{1}{2}}{L_i^\dagger}\ \underset{+\frac{1}{2}}{H}\ \underset{-1}{e_R^j} \qquad +\tfrac{1}{2}+\tfrac{1}{2}-1=0$$

So even though L_i and e_R transform differently, H compensates, making it invariant.

Very similar story for second term. Can check $SU(3)$ and $SU(2)$ yourself,

$U(1)_Y$:
$$\underset{-\frac{1}{6}}{Q_i^\dagger}\ \underset{+\frac{1}{2}}{H}\ \underset{-\frac{1}{3}}{d_R^j} \qquad -\tfrac{1}{6}+\tfrac{1}{2}-\tfrac{1}{3}=0$$

One final trick and we're done! We can make an $SU(2)$-invariant term <u>without</u> taking Hermitian conjugates.

You will show (HW) that $\epsilon^{ab} Q_a H_b$ (or $\epsilon^{ab} Q_a^\dagger H_b^\dagger$) is invariant under $SU(2)$.

So, defining $\tilde{H} = \epsilon^{ab} H_b^\dagger = \begin{pmatrix} H_2^* \\ -H_1^* \end{pmatrix}$, which has $Y = -\frac{1}{2}$, we can write

$$\mathcal{L}_{\text{Yukawa}} \supset - y^u_{ij}\, \overset{-\frac{1}{6}}{Q^\dagger_i}\, \overset{-\frac{1}{2}}{\tilde{H}}\, \overset{+\frac{2}{3}\,=\,0}{u^j_R}$$

That's it!

$$\boxed{\begin{aligned} \mathcal{L}_{SM} &= \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}} \\ &= |D_\mu H|^2 - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ &\quad + \sum_{f=1}^{3} \left\{ i L_f^\dagger \bar{\sigma}^\mu D_\mu L_f + i Q_f^\dagger \bar{\sigma}^\mu D_\mu Q_f + i e_R^{f\dagger} \sigma^\mu D_\mu e_R^f + i u_R^{f\dagger} \sigma^\mu D_\mu u_R^f + i d_R^{f\dagger} \sigma^\mu D_\mu d_R^f \right\} \\ &\quad - y^e_{ij} L_i^\dagger H e_R^j - y^d_{ij} Q_i^\dagger H d_R^j - y^u_{ij} Q_i^\dagger \tilde{H} u_R^j + \text{h.c.} \\ &\quad + m^2 H^\dagger H - \lambda (H^\dagger H)^2 \end{aligned}}$$

The remaining 11 weeks of the course will be devoted to the physical consequences of this Lagrangian.

To wrap up, a taste of the Higgs mechanism: note that this Lagrangian has no <u>fermion masses</u> (it can't, since all the left- and right-handed fermions have different $U(1)$ charges). <u>But</u>, if we set $H = \begin{pmatrix} 0 \\ v \end{pmatrix}$ with v a constant, then

$$y^e_{11} L_1^\dagger H e_R^1 \rightarrow y^e_{11} \left(\nu_L^\dagger \;\; e_L^\dagger \right) \begin{pmatrix} 0 \\ v \end{pmatrix} e_R = v\, y^e_{11} \underbrace{e_L^\dagger e_R}_{\text{a mass term for the electron!}}$$

More on this, and how electromagnetism emerges from hypercharge, in the weeks to come...