We have classified spin-$0$ and spin-$\frac{1}{2}$ fields by their Lorentz reps and internal (gauge) symmetries through which we introduced spin-$1$ fields.

Here are the fields which comprise the Standard Model:

|\[ L^e = (\begin{pmatrix} e^e_R \\ e^e_L \end{pmatrix}, e^\mu_R, e^\mu_L) \quad Q^q = (\begin{pmatrix} u^q_R \\ u^q_L \end{pmatrix}, u^d_R, u^d_L) \quad H \]

\[ \begin{array}{cccccc}
\text{gauge/} \\
\text{fields} \\
\text{(spin-1)}
\end{array}
\begin{array}{ccccccc}
U(1)_y \\
SU(2) \\
SU(3)
\end{array}
\begin{array}{cccccc}
-\frac{1}{2} & -1 & \frac{2}{3} & -\frac{1}{3} & \frac{1}{2} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}

\begin{align*}
\text{Charges/representations} \\
\text{Terminology: } L^e, e^\mu_R \text{ are left/right-handed leptons} \\
Q^q, u^d_R \text{ are left/right-handed quarks} \\
\{F = 1, 2, 3\} \text{ are \underline{generations} (or \underline{flavors})} \\
\text{\underline{H}} \text{ is the Higgs field} \\
U(1)_y \text{ is hypercharge} \\
SU(2) \text{ (sometimes SU(2)_L) is the weak force, and only acts on} \\
\text{left-handed fermions (and the Higgs)} \\
SU(3) \text{ (sometimes SU(3)_c) is color, or the strong force}
\end{align*}

\text{Notation: anything with a } \checkmark \text{ under SU(2) is a 2-component vector of fields which transforms with } \epsilon^\mu_{\alpha\beta}, \text{ like } \psi \text{ we saw earlier (in fact, } \psi \text{ is } H). \\
\text{Similarly, the quarks are 3-component vectors transforming with } 3 \times 3 \text{ unitary matrices}

\[ u^q = \begin{pmatrix} u^q_R \\ u^q_L \\ u^q_c \end{pmatrix}, \] ("red", "green", "blue"), so } Q \text{ is actually a } 3 \times 2 = 6 \text{-component field.}
The Standard Model consists of (almost) all terms we can write down up to total dimension 4 which are invariant under Lorentz and local SU(3) x SU(2) x U(1) Y symmetry.

**Easy stuff first:**

\[ L_{\text{kin}} = \frac{1}{4} G_{\mu \nu} G^{\mu \nu} - \frac{1}{4} W_{\mu \nu} W^{\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} + \sum_{f=1}^{3} \left( i \bar{L}^f \gamma^\mu D_\mu L^f + i \bar{Q}^f \gamma^\mu D_\mu Q^f + i e^f \gamma^\mu D_\mu \bar{e}^f \right) \]

**Lagrangian:**

\[ L_{\text{kin}} = m^2 H^\dagger H - \lambda (H^\dagger H)^2 \]

(Notice mass term has wrong sign! Will get to this later in the course)

Since fermions have dimension \( \frac{3}{2} \), a fermion-fermion-scalar term (known as a Yukawa term) has dimension 4. What such terms are allowed?

\[ L_{\text{Yukawa}} = - \sum_{f=1}^{3} Y_{ij} L^i \dagger H e_j \bar{L}^j \]

Consider \( L^i H e_j \) term first:

**SU(3):** \( L^i \rightarrow L^i, \ H \rightarrow H, \ e_j \rightarrow e_j \) (no transformations, so trivially invariant)

**SU(2):** \( L^i \rightarrow U^i L^i, \ H \rightarrow U H, \ e_j \rightarrow U e_j \) for some \( U \in SU(2) \), so

\[ L^i H e_j \rightarrow L^i (U^i U) H e_j = L^i H e_j, \ \text{invariant (as expected, just like } \Phi^+ \Phi) \]

**U(1) Y:** this group is Abelian, so as a shortcut, can just count charges:

\[ + \frac{1}{2} \ y - 1 = 0 \]

\[ L^i H e_j \]

So even though \( L^i \) and \( e_j \) transform differently, \( H \) compensates, making it invariant.

Very similar story for second term. Can check \( SU(3) \) and \( SU(2) \) yourself.

**U(1) Y:**

\[ - \frac{1}{6} + \frac{1}{2} - \frac{5}{3} = 0 \]

\[ Q^i H d^j \]
One final trick and we're done! We can make an SU(2) invariant term without taking Hermitian conjugates. You will show (HW) that $E^{ab} Q_a H_b$ (or $E^{ab} Q_a^+ H_b^+$) is invariant under SU(2).

So, defining $\tilde{H} = E^{ab} H_b^+ = \begin{pmatrix} H_2^+ \\ -H_1^+ \end{pmatrix}$, which has $\text{v} = -\frac{1}{2}$, we can write

$$L_{\text{known}} \rightarrow -Y_{ij} \tilde{Q}_i H_u^j$$

That's it!

\[
L_{\text{SM}} = L_{\text{kinetic}} + L_{\text{known}} + L_{\text{Higgs}}
= \frac{1}{4} D_{\mu} H^+ D^\mu H - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}
+ \frac{1}{2} \sum_{i=1}^{2} \left( i L^+_i c^+_i D_L c_L^i \right) - Y_{ej} L^+_j H e^j - Y_{ij} \tilde{Q}_i H d^j - Y_{ij} \tilde{Q}_i H u^j + \text{h.c.}
+ m^2 H^+ H - \lambda (H^+ H)^2
\]

The remaining 11 weeks of the course will be devoted to the physical consequences of this Lagrangian.

To wrap up, a taste of the Higgs mechanism: note that this Lagrangian has no fermion masses (it can't, since all the left- and right-handed fermions have different U(1) charges). But, if we set $H = (0$) with $V$ a constant, then

$$Y_{ej} L^+_j H e^j \rightarrow Y_{ej} (v^+ e^+_L)(0) e_R = V Y_{ej} e^+_L e_R$$

a mass term for the electron!

More on this, and how electromagnetism emerges from hypercharge, in the weeks to come...