Dark matter

CMB observations tell us 95% of the mass of the universe does not interact with EM or the strong force; can't be neutrinos either (too light), so dark matter (DM) must be some particle beyond the SM.

Model-building for DM

Let's try writing down a Lagrangian that can describe "dark" DM. Only requirements are Lorentz and gauge invariance; at this point, anything goes! One way of organizing: look for renormalizable operators w/new fields, neutral under SM gauge group.

\[ \mathcal{O}_{\text{prim}} = \mathcal{F}_\nu, \mathcal{F}^\mu, \mathcal{H}^2 \mathcal{S}^2, \mathcal{L}N \]

"portal" to dark sector: dark photon/Higgs portal/RH neutrino portal

By way of example, let's focus on Higgs portal, which has a new scalar \( S \).

\[ \mathcal{L} = \mathcal{D}_a S \mathcal{D}^a S - \frac{1}{2} m_S^2 S^2 - \lambda_{hh} S^2 H^+ H \]

After EWSB \( (H = \frac{1}{\sqrt{2}} (\Phi + i \chi)) \), \[ \mathcal{L} = \frac{\lambda_{hh}}{2} V S^2 - \lambda_{hs} V S^2 h - \frac{\lambda_{hs}}{2} S^2 h^2 \]

\[ m_S^2 \to m_\gamma^2 + \lambda_{hs} V^2 \text{ if } m_S \gg V \text{ this doesn't change the story.} \]

Declare that \( S \) has a \( Z_2 \) symmetry \( S \to -S \) so it's stable \( (S H H^2 \text{ forbidden}) \).

The cliff notes for DM:

- need to annihilate in early universe to avoid overabundance:
  \[ SS \to \text{SM SM} \rightarrow \text{fixes some relation between } \lambda_{hh} \text{ and } m_S \]
- can detect DM by:
  - scattering w/SM particles ("direct detection")
  - annihilation into SM particles ("indirect detection")
  - making it at a collider ("collider production")
All of these are related by the same Feynman diagram(s):

\[ S \rightarrow f + f \]

DM abundance, indirect detection

\[ S \rightarrow h + f \]

Direct detection

\[ S \rightarrow f + f \]

Collider production

Let's compute each in turn.

Suppose \( m_S > m_h \). One annihilation channel is \( S S \rightarrow h h \) :

\[ S \rightarrow h + S \]

\[ S \rightarrow h + h \]

For a rough estimate, just use first diagram. \( \Gamma \sim 4 \lambda_{hs}^2 \) (2!2! in Fermion rule \( S^2 h^2 \)), \( \sigma = \frac{1}{2E_i E_f |V_{hf}|^2} \sim \frac{1}{8\pi} (4 \lambda_{hs}^2) \)

For cosmology, the relevant quantity is actually \( \sigma \langle v \rangle \) (really, a thermal average over Boltzmann distribution). When annihilation shuts off, \( S \) is just barely relativistic, so \( E_f = E_i \sim m_S \)

\[ \Rightarrow \sigma \langle v \rangle \sim \frac{\lambda_{hs}^2}{16\pi m_S^2} \text{ (gain a factor of } \sim 4 \text{ accounting for } SS \rightarrow WW, ZZ) \]

To obtain correct amount of DM today ("relic abundance"), need

\[ 0.01 \text{ cm}^3/\text{s} \Rightarrow \lambda_{hs} \sim 0.2 \left( \frac{m_S}{1 \text{ TeV}} \right) \text{, very reasonable!} \]

(Note we will violate unitarity for \( \lambda_{hs} > 4\pi \), so \( m_S \leq 75 \text{ TeV} \) for this model to be predictive.)
Direct detection

Look for DM scattering off atomic nuclei. First look at nucleon:

\[ S = \frac{\hbar}{\mu} N \]

What is Higgs coupling to nucleons? \( \mathcal{L} \supset h \bar{q} q \), so what we actually want is the matrix element \( \langle N | \bar{q} q | N \rangle \), which is necessarily non-perturbative at low energies.

Let's first just parametrize the Higgs-nucleon coupling as \( \mathcal{F}_N \): \n
\[
\mathcal{L}_{\text{eff}} = \mathcal{F}_N \sum_{\text{nucleon}} (N\bar{q}q) 
\]

\[
\mathcal{M} = (-2\lambda_{hs} V)(\mathcal{F}_N) \frac{-\bar{u}(\not{q}) u(\not{p})}{t-m_n^2} \quad (t=(p_3-p_1)^2=(p_2-p_4)^2)
\]

\[
\langle |\mathcal{M}|^2 \rangle = \frac{2\lambda_{hs}^2 v^2 f_N^2}{(t-m_n^2)^2} \quad \text{Tr} \left[ (\not{q}+m_n)(\not{q}+m_n) \right] = \frac{8 \lambda_{hs}^2 v^2 f_N^2}{(t-m_n^2)^2} \quad (p_2-p_4+m^2)
\]

Since DM is non-relativistic, we have to be a little careful with the kinematics:

\[
p_1 = (m_1 + \frac{1}{2} m_{\text{DM}}, 0, 0, m_{\text{DM}}), \quad p_2 = (m_1, 0, 0, 0) \\
p_3 = (m_1 + \frac{q^2}{2m}, q \sin \theta, 0, q \cos \theta), \quad p_4 = (p_1 + p_2 + p_3)
\]

\[ t = (p_2-p_4)^2 = 2m_n^2 - 2(m_n^2 + \frac{q^2}{2}) = -q^2 \]

\( q \) is the momentum transfer from DM to nucleon.

Since \( q_{\text{max}} = 2 m_{\text{DM}} \), and gravitational measurements tell us \( v_{\text{DM}} \approx 10^{-3} \),

\[ q^2 \ll m_n^2 \] and we can approximate the denominator as \( \approx m_n^2 \).
Now, can use $d^3q$ instead of $d^3p_T$ in phase space integral since $\vec{q} = \vec{p}_T - \vec{p}_\nu$, $E_T \approx m_\nu$, $E_\nu \approx m_\nu$.

$$\sigma_n = \frac{1}{4 m_\nu m_\nu v m_n} \int \frac{d^3q}{(2\pi)^2} \frac{d^3p_T}{m_\nu m_n} \frac{2}{m_\nu} \delta^4(p_T + p_\nu - p_\nu - q) \langle \ell m_\nu^+ \rangle$$

$$= \frac{1}{16 m_\nu m_\nu v m_n} \frac{8 \lambda_{H_s}^2 \nu \bar{\nu}}{m_H^+} \int \frac{q^2 dq dN_\nu}{(2\pi)^2} \delta^4(p_T + p_\nu - p_\nu - q) \delta^4(E_\nu - E_T)$$

$$E_\nu - E_T = \frac{1}{2} m_\nu v^2 m_n - \frac{q^2}{2 m_\nu} - \frac{(m_\nu v_\nu - q)^2}{2 m_\nu} = q \cdot v_\nu - \frac{q^2}{2 m_\nu} - \frac{q^2}{2 m_\nu}$$

$$= q v_\nu \cos \theta - \frac{q^2}{2 m_\nu} \text{ recoil mass}$$

$\Rightarrow$ use $\delta$-function to do $\cos \theta$ integral

$$\delta(q v_\nu \cos \theta - \frac{q^2}{2 m_\nu}) = \frac{1}{q v_\nu} \delta(\cos \theta - \frac{q}{2 m_\nu v_\nu})$$

$$\sigma_n = \frac{\lambda_{H_s}^2 \nu \bar{\nu}}{4 \pi m_\nu m_\nu v m_n} \int_0^{2 m_\nu v m_n} q \left(\frac{q^2}{2 m_\nu} + \frac{q^2}{2 m_\nu}\right) dq$$

$$= \frac{1}{\pi} \frac{\lambda_{H_s}^2 \nu \bar{\nu}}{m_n^2 m_H^+} \frac{\nu^2 m_n^2}{m_\nu m_n}$$

For $m_\nu \gg m_n$, $m_n \approx m_\nu$, and $\sigma_n \approx \frac{1}{\pi} \frac{\lambda_{H_s}^2 \nu \bar{\nu}}{m_n^2 m_H^+}$.

Two remaining ingredients: determine $f_\nu$, and compute $\sigma_\nu$, the cross section from a nucleus composed of many nucleons.

Higgs couples to all quarks: $f_\nu = \sum \frac{m_\nu}{m_n} \langle \nu | q \bar{q} | n \rangle$. Things like $\langle N | \bar{u} u + \bar{d} d | N \rangle$ come from chiral perturbation theory, since Higgs couples more strongly to heavier quarks, dominant contribution is from strange quark content of nucleon, $\langle N | \bar{u} u + \bar{d} d | N \rangle \approx 0.5$. 

\[ \Rightarrow \]
\[ \Rightarrow \text{take } F_N \approx \frac{m_s}{v} \left[ \text{strange quark mass} \right] \left[ \text{not on mass!} \right] \approx 10^{-3} \]

If nucleus were just a bag of nucleons,
\[ O_N = \frac{A}{m_{A^{-}}} \text{A}^{0} \text{N}. \] But at large momentum transfer, we lose coherence over nucleons and need to include a nuclear form factor \( F_N(q^2) \), which starts to differ from 1 at \( q \approx \frac{1}{A} \text{meV} \).

Ignore this for now; take \( A = 131 \) for xenon, target mass of 1 ton = 5 \( \times \) 10\(^{23} \) xenon nuclei, suppose \( m_{\text{xf}} = 1 \text{ TeV} \Rightarrow \lambda_{\text{NS}} = 0.2 \)

\[ R = N_{\text{xe}}^{10^7} 0_{N/V_{\text{m}}} = 5 \times 10^{27} \left( \frac{0.3 \text{ GeV}/c^2}{1 \text{ TeV}} \right) (131^2)(131^2) (10^{-3}) \times \]
\[ \left( \frac{1}{10^2} (10^{-2}) \frac{(246 \text{ GeV})^2}{(10^2)(115 \text{ GeV})^2} \right) \]
\[ \approx 3 \times 10^{-5} \text{ Hz} = 1000 \text{ events/yr} \]

Can be probed by XENON-1T!

**Collider production**

Let's make \( S \) at a collider. Can obviously do \( FF \rightarrow h \rightarrow SS \), but this is a fully invisible final state and it's hard to trigger on "nothing." Say we have a lepton collider. We could radiate a photon:

\[ e^+ + e^- \rightarrow S \]
\[ e^+ + e^- \rightarrow V + \text{invisible} \]

But electron Yukawa is small:
\[ y_e = \frac{\sqrt{2} m_e}{v} \approx 10^{-5} \]

How do we exploit a large coupling?
If $Z$ decays via $u \bar{u}$ or $q \bar{q}$, this is a very clean signal given the large missing energy, but need $\sqrt{s} > 2m_Z + m_H$.

At a hadron collider, want to exploit top quark coupling:

\[ e^+ e^- \rightarrow Z + \text{inv.} \]  
\[ ("\text{mono-}Z") \]

Only large couplings!

"mono-jet"

Harder because we have to deal with all the soft QCD glue which did not go into the gluon PDF, but this is a key search strategy at the LHC.