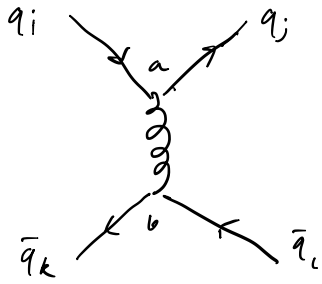


Let's compute the force between $u\bar{d}$ pairs in the color singlet state. 10



$$\propto T_{ji}^a T_{kl}^b f^{ab} = T_{ji}^a T_{kl}^a$$

The sign of the coupling factor tells us if the force is attractive or repulsive; by analogy to QED (and plugging in non-relativistic spinors), positive sign is attractive. Labeling colors $|r\rangle$, $|g\rangle$, and $|b\rangle$, the octet states are linear combinations of states like $|r\bar{g}\rangle$, and the singlet state is $\frac{1}{\sqrt{3}}(|r\bar{r}\rangle + |g\bar{g}\rangle + |b\bar{b}\rangle)$.

If the quarks are in the state $|r\bar{g}\rangle$ ($i=1, k=2$), compute $T_{ji}^a T_{kl}^a$ by summing over outer product of first column times second row of each Gell-Mann matrix:

$$\frac{1}{4} \left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times (1, 0, 0) + \begin{pmatrix} 0 \\ i \\ 0 \end{pmatrix} \times (i, 0, 0) + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times (0, -1, 0) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times (0, 0, 0) + \begin{pmatrix} 0 \\ 0 \\ i \end{pmatrix} \times (0, 0, 0) \right. \\ \left. + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \times (0, 0, 1) + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \times (0, 0, -i) + \frac{1}{3} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times (0, 1, 0) \right]_{jl}$$

$$= \frac{1}{4} \left[\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]_{jl}$$

$$= \begin{pmatrix} 0 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{jl} = -\frac{1}{6} \delta_{ji} \delta_{kl}, \text{ so final state has same color as initial state}$$

(color is conserved). Negative sign means this configuration is repulsive.

A similar computation for $|r\bar{r}\rangle$ gives

$$T_{ji}^a T_{kl}^a = \begin{pmatrix} +\frac{1}{3} & & \\ & +\frac{1}{2} & \\ & & +\frac{1}{2} \end{pmatrix}_{jl}, \text{ and summing over } |r\bar{r}\rangle \rightarrow |r\bar{r}\rangle, |g\bar{g}\rangle, |b\bar{b}\rangle$$

gives a trace: $\frac{1}{3} + \frac{1}{2} + \frac{1}{2} = \frac{4}{3}$. $(\frac{1}{\sqrt{3}})^2 \times 3 \times \frac{4}{3} = +\frac{4}{3}$, so singlet is attractive: $q\bar{q}$ pairs like to form color singlets.