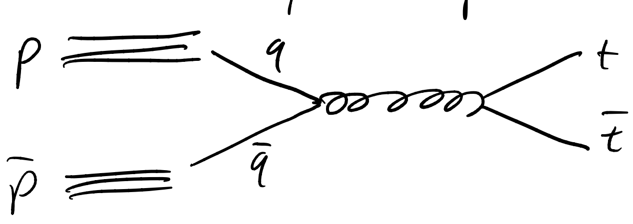
Discovery of the top quark

In 1995, the heaviest known elementary particle, the top quark with $m_t = 173$ GeV, was discovered in $p\bar{p}$ collisions at the Tevatron at Fermilab. The top quark is so heavy that it decays before it hadronizes, so its production and decay can be modeled by free quark processes, simplifying things considerably.

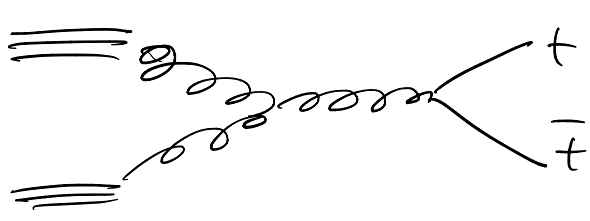
As you will see in detail in PHYS 570, the proton can be modeled with parton distribution functions (PDFs), giving the probability of finding a certain flavor of quark or gluon inside the proton carrying a certain fraction of its energy. This is analogous to the photon splitting function you calculated in HW... except the PDFs are non-perturbative so must be measured. So the cross section for $t\bar{t}$ production is $\sigma(p\bar{p} \to t\bar{t}) = \sum_{i,j = \text{quarks, gluon}} \int_0^1 dx\, d\bar{x}\ f_i(x) f_j(\bar{x})\, \hat{\sigma}(ij \to t\bar{t})$

where x and $\bar{x}$ are the fractions of the proton (antiproton) momentum taken by the parton.

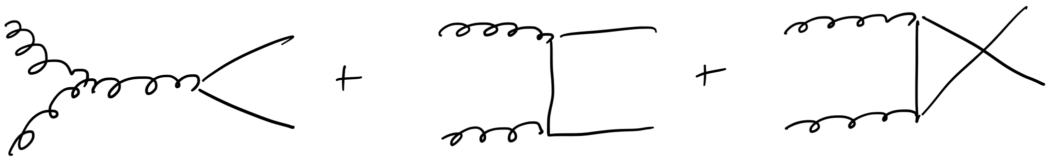
proton is "mostly" up and down quarks, antiproton is "mostly" $\bar{u}$ and $\bar{d}$, so one important process is



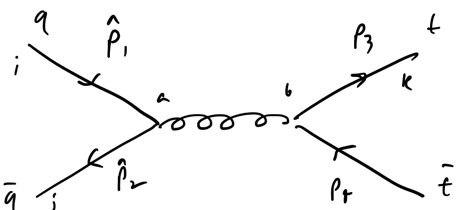
But a nontrivial fraction of the proton is gluons, so the gluon fusion process is also important:



In fact, there are three diagrams which must be added coherently:



This is best done by a computer. You will do this for HW, here we will look at the $q\bar{q}$ annihilation diagram and define some convenient kinematic variables.



$$iM = T^a_{ji} T^b_{kl} (-ig_s)^2 \, \bar{v}_j(\hat{p}_2)\gamma^\mu u_i(\hat{p}_1) \, \frac{-i g_{\mu\nu}\delta^{ab}}{(\hat{p}_1+\hat{p}_2)^2} \, \bar{u}_k(p_3)\gamma^\nu v_l(p_4)$$

$\hat{p}_1$ and $\hat{p}_2$ are parton momenta. This is identical to $e^+e^- \to \mu^+\mu^-$ up to the color factor, which gives the following in the squared matrix element:

$$|M|^2 \propto \left(\frac{1}{3}\right)^2 \sum_{ijkl} T^a_{ji} (T^c_{ji})^* T^b_{kl} (T^d_{kl})^* \delta^{ab}\delta^{cd}$$

avg. over initial colors

sum over final colors

$$= \frac{1}{9}\delta^{ab}\delta^{cd} \sum_{ij} T^a_{ji} T^c_{ij} \sum_{kl} T^b_{kl} T^d_{lk}$$

generators are Hermitian, so $T^* = T^T$

$$= \frac{1}{9}\delta^{ab}\delta^{cd} \, Tr(T^a T^c)\, Tr(T^b T^d)$$

$$= \frac{1}{9} \times \frac{1}{4} \delta^{ab}\delta^{cd}\delta^{ac}\delta^{bd} \quad \left(\frac{1}{4} = T_F^2\right)$$

$$= \frac{1}{9} \times \frac{1}{4} \times \underbrace{\delta^{bc}\delta^{bc}}_{\text{trace of } \mathbb{1}_{8\times 8}}$$

$$= \frac{1}{9} \times \frac{1}{4} \times 8 = \frac{2}{9}$$

$$\Rightarrow \frac{d\hat{\sigma}}{d\cos\theta} = \frac{2}{9} \times \frac{\pi \alpha_s^2}{2\hat{s}}(1+\cos^2\theta),$$ where $\hat{s} = (\hat{p}_1+\hat{p}_2)^2$ is the partonic center of mass energy.

Defining the other partonic Mandelstam variables using

$$p_3 = \left(\frac{\sqrt{\hat{s}}}{2}, \frac{\sqrt{\hat{s}}}{2}\sin\theta, 0, \frac{\sqrt{\hat{s}}}{2}\cos\theta \right)$$
$$p_4 = \left(\frac{\sqrt{\hat{s}}}{2}, -\frac{\sqrt{\hat{s}}}{2}\sin\theta, 0, -\frac{\sqrt{\hat{s}}}{2}\cos\theta \right)$$

} note: we are assuming $\sqrt{\hat{s}} \gg m_t$ so we can approximate top quark as massless; not always a good approximation at e.g. the Tevatron

$$\hat{t} = (p_3 - \hat{p}_1)^2 = -2 p_3 \cdot \hat{p}_1 = -\frac{\hat{s}}{2}(1-\cos\theta)$$
$$\hat{u} = (p_4 - \hat{p}_1) = -2 p_4 \cdot \hat{p}_1 = -\frac{\hat{s}}{2}(1+\cos\theta)$$
$$\hat{t}^2 + \hat{u}^2 = \frac{\hat{s}^2}{2}(1+\cos^2\theta)$$
$$\Rightarrow \frac{d\hat{\sigma}}{d\cos\theta} = \frac{2}{9}\frac{\pi\alpha_s^2}{\hat{s}}\left(\frac{\hat{t}^2+\hat{u}^2}{s^2}\right)$$

$\hat{s}$ is related to $x, \bar{x}$ by $\hat{s} = (\hat{p}_1 + \hat{p}_2)^2 = (xP_1 + \bar{x}P_2)^2 = 2x\bar{x}\,P_1 \cdot P_2 = x\bar{x}s$

$\Rightarrow$ not all center-of-mass energy goes into collision! Beams must be well above threshold of $s_{thresh} = 4m_t^2$ to have any hope of efficiently producing $t\bar{t}$. Indeed, at Tevatron, $\sqrt{s} = 1.8$ TeV, so $s \simeq 27\, s_{thresh}$

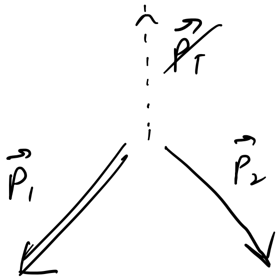
As we anticipated, t decays almost instantaneously. We will see in the coming weeks that the decay is through the weak interaction, $t \to Wq$, where $q = b, s, d$. This means that, though the decay is fast, the width is small compared to the mass since it's proportional to a small coupling. This lets us use the narrow-width approximation and separate production and decay.

The W also decays, 70% of the time to two quarks, and 30% of the time to a lepton plus a neutrino. In HW, you will investigate the fully hadronic decay; here we will look at the channel that the CDF detector used to claim discovery:

$$p\bar{p} \longrightarrow t\ \bar{t}$$
$$t \to W b \to (l\,\nu)\,b$$
$$\bar{t} \to W \bar{b} \to (q\,\bar{q})\,\bar{b}$$

$\Rightarrow$ 4 or more jets plus one lepton plus missing energy from neutrino

More on missing energy: momentum conservation implies $\sum p_f = p_1 + p_2$, but we lose all sorts of particles "down the beampipe" collinear with p_1 and p_2, so it's hard to enforce longitudinal momentum conservation. Transverse is easier: $\sum \vec{p}_T = 0$, so if we measure the total momentum transverse to the beam direction and don't find zero, we know there must be some missing transverse energy $\cancel{E}_T$ from an undetected particle



$\vec{p}_{1T} + \vec{p}_{2T} \neq 0$

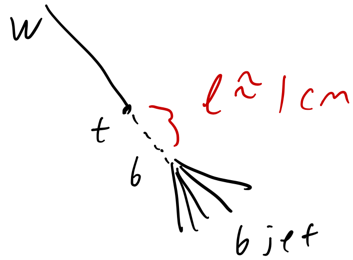
This can be mundane (i.e. neutrino) or exotic (dark matter!) and is often a helpful signature.

Similarly, to reduce QCD backgrounds, usually require a minimum p_T for the jets (this ensures they're not just soft or collinear radiation). This also means we can use events with 4 or more jets and only consider the 4 jets with the largest p_T.

As we discussed last week, in general we can't tell a q vs. $\bar{q}$ jet apart, nor can we identify the flavor of the quark from which the jet arose — with one exception.

Jets arising from b quarks can be "tagged" with some efficiency < 1 because the lifetime of the b quark (or more precisely, hadrons containing b quarks) is "long" on collider scales, $\tau \approx 10^{-12}\,s$, so in the frame of the collider, the mean distance traveled before decaying is $\ell = \gamma c \tau$. We can estimate the boost of the b quark from $t \to Wb$ by assuming the top is produced at rest, and $E_b \approx \frac{m_t}{2} \approx 33 m_b$, so $\gamma \approx 33$.

$\Rightarrow \ell = \gamma c \tau \approx 1\,cm$! This is a measurable distance and gives rise to a displaced vertex.



Let's say we have an event with 1 muon, 4 jets (of which 2 are tagged as b jets), and missing energy. This is a candidate $t\bar{t}$ event. The hypothesized kinematics are

$$p\bar{p} \to t\bar{t} + X$$ (X is all the undetected stuff down the beampipe)

$$t \to b_1 W_1$$
$$\bar{t} \to \bar{b}_2 W_2$$
$$W_1 \to \mu + \nu_\mu$$
$$W_2 \to j_1 + j_2$$

We can measure the full 4-vectors of b_1, b_2, μ, j_1, and j_2. We want to solve for the unknowns $p_{W_1}, p_{W_2}, \vec{p}_{t_1}, \vec{p}_{t_2}, \vec{p}_{\nu_\mu}, m_X, m_t, p_{X_2}$ (18 variables). We have 20 equations (5 4-vector constraints) so this is an overconstrained system and we can check that our solution for m_t is consistent.

Here's how this works: define $\vec{p}_{T\nu}$ as $-(\vec{p}_{T_{b_1}} + \vec{p}_{T_{b_2}} + \vec{p}_{T_\mu} + \vec{p}_{T_{j_1}} + \vec{p}_{T_{j_2}})$, so p_{z_ν} is still unknown. From $W_1 \to \mu + \nu_\mu$, we have

$m_W^2 = m_\mu^2 + 2E_\nu E_\mu - 2p_{z_\mu} p_{z_\nu} - 2\vec{p}_{T_\mu} \cdot \vec{p}_{T_\nu}$. Set $E_\nu = \sqrt{p_{z_\nu}^2 + \vec{p}_{T_\nu}^{\,2}}$, this is now a quadratic equation for p_{z_ν}.

Similarly, let 4-vector of initial $p\bar{p}$ be $P = (\sqrt{s}, 0, 0, 0)$. Then

$P - p_X = p_t + p_{\bar{t}}$, so $s + m_X^2 - 2\sqrt{s}\,E_X = 2m_t^2 + 2\vec{p}_{t_1} \cdot \vec{p}_{t_2}$. Write $E_X = \sqrt{p_{z_X}^2 + m_X^2}$ (X is assumed to carry no transverse momentum), solve for

p_{z_X} by $p_{z_{t_1}} + p_{z_{t_2}} + p_{z_X} = 0$, this becomes a quadratic equation for m_X

$\Rightarrow$ algorithm for kinematic fitting, which gives a best-fit value of m_t.