Electromagnetic Fields in Quantum Mechanics

It is desirable if there are to be any considerations of special relativity, to use a system where the electric and magnetic fields have the same units. Due to Maxwell, Lorentz, and Einstein, electric and magnetic phenomena are not really separate, but are part of one "unified" theory of electromagnetism. Having committed to having $\vec{E}$ and $\vec{B}$ in the same units, the remaining question is where to place certain inevitable factors of 4π. The set of units which results when there is no factor of $1/(4\pi)$ in Coulomb's Law is called Gaussian units. These will be used in Physics 581. The set of units where Coulomb's Law does have a factor of $1/(4\pi)$ is called Heaviside-Lorentz. This is the set of units used in high energy physics and quantum field theory. Below we collect some results for **Gaussian units in vacuum**.

Maxwell Equations

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \tag{1}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{4}$$

Fields from Potentials

$$E = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

Force on a Charge

$$\vec{F} = q\vec{E} + \frac{q}{c}\vec{v} \times \vec{B}$$

Lagrangian for Charge in Electric and Magnetic Fields

$$L = \frac{m}{2}\vec{v} \cdot \vec{v} + \frac{q}{c}\vec{v} \cdot \vec{A} - q\phi$$

Energy Density

$$u = \frac{1}{8\pi}(E^2 + B^2)$$

Coulomb's Law

$$\phi = \frac{q}{r}$$

Fine Structure Constant

$$\alpha_e = \frac{e^2}{\hbar c} = \frac{1}{137\ldots}$$

Bohr Radius

$$a_0 = \frac{\hbar}{m_e c \alpha_e} = \frac{\hbar^2}{m_e e^2}$$

Bohr Magneton

$$\mu_B = \frac{e\hbar}{2m_e c}$$

Comments

1. The annoying factors of ϵ_0 and μ_0 present in standard MKS units have disappeared.
2. In Gaussian units, $\vec{E}$ and $\vec{B}$ are in the same uniits, ϕ and $\vec{A}$ are in the same units, and electric and magnetic dipole moments are in the same units.
3. In Gaussian units, the unit of magnetic field is the **gauss**, where 1 gauss$= 10^{-4}$ tesla.
4. In Gaussian units, the units of charge and electric potential are called "stat-coulomb (sc)" and "stat-volt (sv)," respectively. The conversion factors are $1c = 3 \times 10^9 sc$, and $1v = 1/300sv$. It is rarely necessary to use these factors.
5. The electron volt (eV) is a unit of *energy* and therefore independent of the units of electromagnetic fields. It is the natural unit of energy in atomic physics.