Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

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Outline of Presentation

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We will first introduce the classical Hall effect and then the Quantum Hall effect as motivation for the Quantum Spin Hall Effect.
Classical Hall Effect

- Occurs when an electric current $I$ in a conductor is exposed to a perpendicular magnetic field.
- The Lorentz force causes a drift current perpendicular to $I$, causing a build up of charge on the sides.

$$R_H = \frac{V_H}{I} = -\frac{1}{n_e}$$

(Hall coefficient)

Where $n$ is the charge carrier density.
Quantizing the Hall Effect

Consider an electron gas confined to two dimensions (x-y direction) subjected to a strong magnetic field in the z-direction.

- The gas experiences a quantization of its energy into “Landau Levels” given by:

\[ \epsilon_{n,k} = \frac{\hbar}{\pi} \left( n + \frac{1}{2} \right) \]

where \( n \) is the quantized number, \( k \) the 2-d wave vector number and \( \omega_c \) is the cyclotron frequency given by \( \frac{eB}{m_*} \) with \( m_* \) the reduced mass of the electron.
Energy Levels in the Quantum Hall Effect

• Landau levels are degenerate with degeneracy given by:

\[ d = \frac{L_x L_y}{2\pi l_B^2} = \frac{L_x L_y eB}{2\pi \hbar}, \quad l_B = \sqrt{\frac{\hbar}{eB}} \]

In particular, the degeneracy of the Landau level is proportional to B. Hence at high enough B, electrons conglomerate into a finite number of energy states.

• The number of Landau levels filled is given by:

\[ \nu = \frac{N}{d} = \frac{Nh}{eB} \]

Where N is the number of free valence electrons. Note that \( \nu \) is a small for high B.
Quantum Hall Effect

• Illustration of Landau levels filled as the magnetic field is increased.

The graph on the right shows the Hall resistivity and the diagonal resistivity with increasing B field.
Currents in the Quantum Hall Effect

- Geometrical picture of electron orbits in the 2-d gas.

Edge currents of opposite direction form at the edges along the y-direction.

No net current exists unless an E-field is applied along the y-direction.
Quantum Spin Hall Effect

- A special class of Topological Insulators that contain edge states that are spin filtered.
- That is one edge state made up of spin up carriers and another edge state made up of spin down carriers.
- Both carriers go in opposite directions.
- No external B field is needed. The spin currents and due to the internal electronic structure.
Constructing the Quantum Spin Hall Effect

• The QSH can be explained intuitively by using the QH.
• The “acting” magnetic field is due to the nucleus B field from the strong spin-orbit coupling.

Currents in the Quantum Spin Hall Effect

- A spin up carrier will see an effective B-field going into the page due to the spin-orbit coupling.
- A spin down carrier will see an effective B-field going out of the page due to the spin-orbit coupling.
- This generates two opposing edge currents that are spin filtered.
- The currents are suffer no dissipation.
Resistance in Quantum Spin Hall Effect

• The Spin up and Spin down edge currents give each a Hall resistance of:

\[ R_H = \frac{h}{e^2}, \nu = 1 \]

• Hence, the total Spin Hall Resistivity is:

\[ R_H = \frac{2h}{e^2} \]

• And the total Spin Hall Conductivity is:

\[ \rho_H = \frac{2e^2}{h} \]

• Since the edge currents have quantized Hall resistance, the Spin Hall Conductivity is quantized.
Why This Paper is Important

- Importance of Quantum Spin Hall effect
  - Provides new physics and new devices
- Previously published work
  - What lead to the theoretical prediction
  - What the previously proposed real world examples are
  - Why those proposed real world examples are unrealistic
- Importance of this paper
  - This paper fills the missing piece -> giving a real world example
Quantum spin hall effect has zero mass charge carriers

- Linear relationship between momentum and energy

\[ E^2 = p^2 c^2 + m^2 c^4 \]
\[ E = pc \]

- Therefore the charge carriers behave like massless particles
- Velocity determined by slope
- This effect may have applications in high frequency electronics
Coupling in electric and magnetic images

• Similar to having an induced image charge on the surface of a conductor, the quantum spin hall effect will induce a magnetic monopole image when an electric point charge is outside.

Red lines are electric fields, blue lines are magnetic field. This shows a point charge create an image magnetic monopole.
Electric and Magnetic coupling

- The electric charge combined with the image magnetic monopole form a dyon; a particle with both magnetic and electric charge
- because of the Aharonov–Bohm effect, and the magnetic field of the monopole image, exchanging two charges will give a phase change

\[ |\psi_1\psi_2\rangle = e^{i\theta} |\psi_2\psi_1\rangle \]

this gives an entirely new set of statistics compared to bosons and fermions

\[ |\psi_1\psi_2\rangle = \pm |\psi_2\psi_1\rangle \]
Previous work

Haldane, proposed that the Quantum Hall Effect could occur without an external B field

B Bernevig & S-C Zhang propose that QSH effect may occur in specially strained GaAs

S Murakami predict QSH effect to be found in 2-D bismuth

C Kane and E Mele predict a quantized spin hall effect in graphene

C Kane and E Mele establish QSH effect as having $Z_2$ topological order

H Min shows that the gap in graphene is too small to support QSH effect

1988 | 2005 | 2006 | 2007

This Paper

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Summary and Results of this Paper

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Proposal of Experiment:

- HgTe between two pieces of CdTe.
- Thickness of Hg less or more than a critical point ($d_c$).

Voltage drop is measured to see whether the conductance becomes zero or not.

1. If conductance becomes zero ➔ Normal Insulator.
2. If conductance is not zero ➔ Topological Insulator.

There are six basic atomic states per unit cell in HgTe and CdTe.

\[ \Psi = (|\Gamma_6, \frac{1}{2}\rangle, |\Gamma_6, -\frac{1}{2}\rangle, |\Gamma_8, \frac{3}{2}\rangle, |\Gamma_8, \frac{1}{2}\rangle, |\Gamma_8, -\frac{1}{2}\rangle, |\Gamma_8, -\frac{3}{2}\rangle) \]

\( \Gamma_6 \) is a s-type band \( \Gamma_8 \) is a p-type band.
CdTe has a normal band progression (s-type $\Gamma_6$ band lying above the p-type $\Gamma_8$ band).

HgTe has an inverted band progression ($\Gamma_6$ band lies below the $\Gamma_8$ band).
| E₁, mJ \rangle state is formed from the combinations of |Γ₆, mJ = ±1/2 \rangle & |Γ₈, mJ = ±1/2 \rangle states.

| H₁, mJ \rangle state is formed from the |Γ₈, mJ = ±3/2 \rangle states.
$E_1$ and $H_1$ states flip when thickness of HgTe is more than $d_c$. 
In a **normal insulator**, conductance vanishes while Fermi level is between valence and conductance band.

In a **topological insulator**, conductance is not zero between conductance and valence bands.

In a **six-terminal measurement**, the QSH state would exhibit electric voltage drop between the terminals ($\mu_1$ and $\mu_2$ and between $\mu_3$ and $\mu_4$), in the zero temperature.
Experimental verification

- König et al. confirmed the prediction an year later (2007)

Critical Analysis

- Overall we found this to be a very good paper with very important consequences in condensed matter experiment and theory.
- However, effects of inversion symmetry breaking is not considered in the paper which the materials HgTe and CdTe actually have.
- In that case $S_z$ Conservation is also broken, and with no conservation of spin, the material does not show true Quantum Spin Hall effect.
Extended to 3D

- In 2007 Kane and Fu predicted that this idea can be extended to 3D materials as well.
- $\text{Bi}_{1-x}\text{Sb}_x$ alloy can act as topological insulator.
- In 2008 by Zahid Hasan at Princeton University, observed topological surface states in that system.
Citations

- Scopus - 773
- Web of Knowledge - 748
2012 Dirac Medal & 2010 Europhysics prize

S Zhang
Summary

• The quantum spin hall effect can be constructed as two separate quantum hall effect currents, one for spin up, one for spin down
• Materials with this behavior have quantized resistance, can carry spin currents. With applications in spintronics and quantum computing
• Bernevig, Hughes and Zhang were the first to propose a realistic method of observing this effect
• As a result of this discovery there is a huge amount of research in condensed matter theory and experiment

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# References