Is the proton as “big” as we thought?

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“Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen”

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The proton-size puzzle

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- One attempt to do so runs into trouble - it seems inconsistent with everything else!
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- "Radius" of the proton can be defined and determined in various ways.
- One attempt to do so runs into trouble - it seems inconsistent with everything else!
- Is this a window for new physics, or another experimental goof-up?
Preliminaries: how things got dirty!

**Diagram:**
- **Lyman series**
- **Balmer series**
- **Paschen series**

**Labels:**
- \( n = 1 \)
- \( n = 2 \)
- \( n = 3 \)
- \( n = 4 \)
- \( n = 5 \)
- \( n = 6 \)

**Wavelengths:**
- 122 nm
- 103 nm
- 97 nm
- 95 nm
- 94 nm
- 656 nm
- 486 nm
- 434 nm
- 410 nm
- 1875 nm
- 1282 nm
- 1094 nm
Preliminaries: how things got dirty!

- Spin-orbit coupling
- Lamb\(^\dagger\) shift
- Hyperfine splitting
Preliminaries: how things got dirty!

† lamb = a small sheep ;)

✓ Spin-orbit coupling
✓ Lamb† shift
✓ Hyperfine splitting
Standard way to write down the angular momentum quantum numbers of a state.

\[ \mathcal{N}^{2S+1}L_J \]

\( L \) : Orbital angular momentum (S, P, D, F ... for \( \ell = 0, 1, 2, 3, \ldots \))

\( J \) : Total angular momentum
Digression: the spectroscopic notation

✓ Standard way to write down the angular momentum quantum numbers of a state.

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\( L \): Orbital angular momentum (S, P, D, F \ldots for \( \ell = 0, 1, 2, 3, \ldots \))

\( J \): Total angular momentum

e.g.: single electron states (spin-\( \frac{1}{2} \) system!), like in Hydrogen

\[ \begin{align*}
1^2S_{\frac{1}{2}} & \quad 2^2S_{\frac{1}{2}} & \quad 2^2P_{\frac{3}{2}} & \quad 2^2P_{\frac{1}{2}} & \quad 3^2S_{\frac{1}{2}} & \quad 3^2P_{\frac{3}{2}} & \quad 3^2P_{\frac{1}{2}} & \quad 3^2D_{\frac{5}{2}}
\end{align*} \]
Fig: A subtle structure of the n=2 level in hydrogen according to Bohr’s, Dirac’s and QED with Lamb Shift.
Thou art not solid, proton!

✓ Finite probability of electron to be found inside the nucleus.

\[
P(r \leq R) = \frac{1}{\pi a^3} \int_0^{2\pi} \int_0^\pi \int_0^R e^{-2r/a} r^2 \sin \theta \, dr \, d\theta \, d\phi \approx \frac{4}{3} \left( \frac{R}{a} \right)^3
\]

V inside charge distribution is smaller than the corresponding field produced by a point charge.

\[V = \frac{kQ}{2R} \left( 3 - \frac{r^2}{R^2} \right)\]

\[V = \frac{kQ}{R}\]

\[V = \frac{kQ}{r}\]

\[R \quad 2R \quad 3R \quad 4R \quad 5R\]

⇒ measured transition frequencies depend on proton size!
Two ways of defining the “radius” of the proton

✓ Neither atoms nor their nuclei have definite boundaries - we define a
  
  - **Charge radius** \((r_E)\), based on the distribution of charge) and a
  
  - **Zemach radius** \((r_Z)\), reflects the spatial distribution of \(\vec{\mu}\) smeared out by \(\rho(\vec{r})\).
Working with exotic atoms

✓ Historically, $r_E$ and $r_Z$ were determined using measurements of the \textbf{differential cross section} in elastic e-p scattering.

✓ A more accurate measurement is expected from \textbf{laser spectroscopy} of “Muonic Hydrogen”. (Why?)

Muon (e$^-$’s heavier twin) orbiting the proton instead of electron.

\[ m_\mu = 207m_e \]
\[ r_\mu = \frac{1}{186}r_e \]
The experiment

(We’ll try not to make it boring!)
The experiment

We’re measuring $r_E$ from spectroscopy of the $2S_{1/2} - 2P_{3/2}$ transition.
Step 1/3: prepare muonic hydrogen in 2S state

✓ Highly energetic $\mu^-$ stopped in H$_2$ gas
✓ Highly excited $\mu$-p atoms form ($n \approx 14$)
✓ $\sim 1\%$ populate long-lived 2S state

Diagram:
- $n=14$
- 1% transition to 2P
- 99% transition to 2S
- 2 keV x-rays ($K_\alpha$, $K_\beta$, $K_\gamma$)
- 1S state
Step 2/3: induce $2S \rightarrow 2P$ transitions

- Laser pulse induces $2S \rightarrow 2P$ transitions
- Immediately follows $2P \rightarrow 1S$ de-excitation
Step 3/3: measure the transition frequencies, what else!

Two $2S \rightarrow 2P$ transitions measured

$\checkmark \quad \nu_s \equiv \nu(2S^{F=0}_{1/2} \rightarrow 2P^{F=1}_{3/2})$

$\checkmark \quad \nu_t \equiv \nu(2S^{F=1}_{1/2} \rightarrow 2P^{F=2}_{3/2})$

$^\dagger \quad N^{2S+1}L_J \quad F = J + I$
Do they conform with known data?

✓ Finite proton size significantly affects Lamb shift and 2S HFS

✓ Some linear combinations of $\hbar \nu_s$, $\hbar \nu_t$, $\Delta E^{2P}_{FS}$, $\Delta E^{2P}_{HFS}$ yield Lamb shift and 2S HFS
Do they conform with known data?

- Finite proton size significantly affects Lamb shift and 2S HFS

- Some linear combinations of $\hbar\nu_s$, $\hbar\nu_t$, $\Delta E_{FS}^{2P}$, $\Delta E_{HFS}^{2P}$ yield Lamb shift and 2S HFS

- From 2S hyperfine splitting, we get $r_Z$, the Zemach radius.

\[
\Delta E_{HFS}^{th} = \ldots + (\ldots) r_Z + (\ldots)
\]

\[r_Z = 1.082(37) \text{ fm}\]
Do they conform with known data?

✓ Finite proton size significantly affects Lamb shift and 2S HFS

✓ Some linear combinations of $\hbar \nu_s, \hbar \nu_t, \Delta E_{FS}^2, \Delta E_{HFS}^2$ yield Lamb shift and 2S HFS

✓ From 2S hyperfine splitting, we get $r_Z$, the Zemach radius.

\[
\Delta E_{HFS}^{th} = \ldots + (\ldots) r_Z + (\ldots)
\]

$r_Z = 1.082(37) \text{ fm}, \quad r_Z^{\text{Friar}} = 1.086(12) \text{ fm}, \quad r_Z^{\text{Distler}} = 1.045(40) \text{ fm}$

Experimental limit for measuring $r_Z \sim 3.4 \%$

✓ So far, so good!
Do they conform with known data?

✓ From Lamb shift, we get the charge radius.

\[ \Delta E_L^{\text{th}} = \ldots + (\ldots) r_E^2 + (\ldots) \]

\[ r_E = 0.84087(39) \text{ fm}, \quad r_E^{\text{CODATA}} = 0.8775(51) \text{ fm} \]
Do they conform with known data?

✓ From Lamb shift, we get the charge radius.

$$\Delta E_L^{th} = \ldots + (\ldots) r_E^2 + (\ldots)$$

$$r_E = 0.84087(39) \text{ fm}, \quad r_E^{\text{CODATA}} = 0.8775(51) \text{ fm}$$

At 7 $\sigma$ variance with CODATA!
Did we miss something?

\[ r_E = 0.84087(39) \text{ fm}, \quad r_E^{\text{CODATA}} = 0.8775(51) \text{ fm} \]

! Maybe we should try a different \( \rho(r) \)? \( \Longrightarrow \) changes \( r_E \) by less than the quoted uncertainty! (more on a later slide.)
Did we miss something?

\[ r_E = 0.84087(39) \text{ fm}, \quad r_E^{\text{CODATA}} = 0.8775(51) \text{ fm} \]

\[ \Rightarrow \]

Maybe we should try a different \( \rho(r) \)? changes \( r_E \) by less than the quoted uncertainty! (more on a later slide.)

Other possibilities (spectroscopy of \( p\mu \) or \( \mu pe^- \) instead of \( \mu - p \)) can be excluded.
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Maybe we should try a different \( \rho(r) \)? \( \implies \) changes \( r_E \) by less than the quoted uncertainty! (more on a later slide.)

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Yet other recent e-p scattering measurements support the CODATA value.
For the thinking layman . . .

? Is the \( \mu \)-p interaction different (in what way?) from the e-p interaction?

? Is this a window (albeit possibly small) of new physics?

? Do new force carriers (MeV-mass) exist? (In conformity with other results)
More serious attempts at resolution

× Gorchtein: uses finite-energy sum rule to find the correction to the proton-polarizability of -(40 +/- 5) $\mu$eV. Not enough to explain the 300 $\mu$eV difference.

× Griffith: uses bound-state field theory on proton structure in $\mu$-p. No positive results yet.

? Moumni: Corrects for the noncommutativity in space-space and space-time versions. Says discrepancy is solved by the corrections depending on $m^3$ giving shifts in the spectrum. New paper though, hasn’t been reviewed yet.
Citation Evaluation

No. of citations : 24

- **13** new proposals, theories, or experiments : Trying to explain the “proton radius puzzle”.
- **2** were self-citations (by someone among the authors)
- **9** were papers using the determined values or similar techniques as this experiment.
From the reviewer’s POV

What we thought was good:

- Attention to detail - sources of experimental errors; possible sources of discrepancy
- Clear flow of reasoning

What we thought wasn’t:

- Talk about changing $\rho(r)$ out of the blue
- No obvious reason for de-excitation into 1S and 2S only
... and apart from that, the world is still as beautiful.
Definitions of the radii

The charge radius is defined as

$$ r_E^2 = \int_0^\infty r^2 \rho(r) \, dr $$

In general, of course, we could define it in any way, for e.g.

$$ r_E = \left( \int_0^\infty r^n \rho(r) \, dr \right)^{1/n} $$
What we should retain!

- We aim to measure the **charge** and **Zemach** radii of the proton.
- Useful as inputs for tests of bound-state QED
- Earlier experiments with e-p scattering in H-atom - this time, laser spectroscopy of muonic Hydrogen
- From spectroscopy, we deduce **Lamb shift** and **2S hyperfine splitting** - and from them the **charge** and **Zemach** radii.
- Turn out to be significantly **smaller** than accepted proton radius
- Consequences? Window for new physics?