Quantum-Field Tomography
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“Towards experimental quantum-field tomography with ultracold atoms”
Outline

- The Experiment - Quantum Field Tomography
- Theoretical Analysis
- Motivation and Additional Tomography Methods
- Future Work and Overall Considerations
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The recent experimental realization of continuous quantum AMO systems

• AMO systems can now be made larger than ever before (~1000s of atoms) while still maintaining precise control over individual constituents.

• Because such systems are essentially continuous, this motivates the use of quantum field theory.
What is Tomography?

• Tomography probes a system with an experimental technique, thereby reconstructing some aspects of the system. It does not assume any theory to begin with.

• Computer-Aided Tomography (CAT) scans are an illustrative example of tomography in practice.

• This sort of logic is common in the history of science. The only novelty is finally being able to apply it to these continuous, quantum systems.
The use of matter-wave interferometry in probing the Bose condensate

• After “kicking” the system out of equilibrium (through a process known as transversal quenching), two separate, but correlated, quantum systems are created. The correlation is picked up as a phase variation throughout the condensate.

• Matter-wave interferometry allows us to resolve the separation between the two halves in order to determine the relative phase.

1000-10000 Rb atoms
\[ T = 10-100 \text{ nK} \]
\[ \omega_R = 2\pi \times 2 - 3 \text{ kHz} \]
\[ \omega_L = 2\pi \times 5 - 10 \text{ Hz} \]
\[ \mu, k_B T \ll \hbar \omega_R \]
The notion of **sparsity** as taking simple data

• In principle, we could “kick” a system in any haphazard, arbitrary way we may like. But only a sufficiently simple “kick” will yield in easy to understand data. Hence, we probe the system in simple ways so as to excite only the basic structure.

• By probing the data in simple and symmetric ways, we also have the much needed luxury of making theoretically simplifying assumptions.
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Matrix Product State Representation

Every 1-dimensional quantum state $|\psi\rangle$ can be written as a product of matrices, called the matrix product state (MPS):

$$ |\psi\rangle = \sum_\sigma \text{Tr}(M^{[1]}_{\sigma_1} M^{[2]}_{\sigma_1} \ldots M^{[N]}_{\sigma_1}) |\sigma\rangle $$
Contracting the Matrix Product State Representation

• The expectation value of an observable $\hat{O} = \otimes_{k=1}^{k=N} \hat{O}_k$ can be computed: 
  $$\langle \psi | \otimes_{k=1}^{k=N} \hat{O}_k | \psi \rangle = \text{tr} \left[ \prod_{k=1}^{k=N} E_{O_k}^{[k]} \right], \text{where } E_{O_k}^{[k]} = \sum_{i,j=1}^{f} \langle i | \hat{O}_k | j \rangle \bar{M}_i^{[k]} \otimes M_j^{[k]}$$

For a state written in the MPS representation, the expectation value of an observable can be calculated and written graphically:
The quantum state described by a continuous MPS

- For a continuous 1-D translationally invariant bosonic system, a quantum many-body state can be written as

$$|\psi\rangle = \text{Tr} \left\{ P \exp \left[ \int_0^L dx (Q \otimes I + R \otimes \psi^+(x)) \right] \right\} |\Omega\rangle$$

- $Q, R \in \mathbb{C}^{d \times d}$ are matrices acting on the auxiliary d-dimensional virtual space and serves as variational parameters which can be determined from experimental data.
State Reconstruction from Phase Correlation Function

• The n-point correlation is defined as the phase correlation function:

\[ G^n(x_1, x_2, \ldots, x_n) = \text{Re} \left( e^{i(\theta_{x_1} - \theta_{x_2} + \theta_3 - \ldots)} \right) \]

• It can be computed in the thermodynamic limit:

\[ G^n(x_1, x_2, \ldots, x_n) = \text{Re} \left( \hat{n}(x_1)^{-1/2} \hat{\psi}(x_1)^+ \hat{\psi}(x_2) \hat{n}(x_2)^{-1/2} \ldots \right) \]

\[ = \text{Tr} \left[ e^{T(L-x_n)} \ldots e^{T(x_3-x_2)} \left( \frac{1}{R^2} \otimes R^{-1/2} \right) e^{T(x_2-x_1)} \left( \frac{1}{R^{-1/2}} \otimes R^2 \right) \right] \]

\[ T = \bar{Q} \otimes I + I \otimes Q + \bar{R} \otimes R \]
State Reconstruction from Phase Correlation Function

• By diagonalizing the matrix $T$ and taking the thermal dynamic limit, the correlation function takes the form

$$\sum_{\{k_j\}}^{d^2} \rho_{k_1, k_2, \ldots, k_{N-1}} e^{\lambda_{k_1} \tau_1} \ldots e^{\lambda_{k_{N-1}} \tau_{N-1}}.$$ 

• $\lambda_k$ are eigenvalues of $T$ and $\rho_{k_1, k_2, \ldots, k_{N-1}}$ is equal to $M_{1, k_{N-1}}^{-1} \ldots M_{k_2, k_1}^{-1} M_{k_{N-1}, 1}$ with $X^{-1}TX$ in diagonal form and

$$M = X^{-1} \left( \bar{R}^{-2} \otimes R^2 \right) X.$$ 

• In the Laplace transformation, $\lambda_k$ are poles and the prefactors $\rho_{k_1, k_2, \ldots, k_{N-1}}$ are referred to as residues.
Construction of Many-Body State

1. The poles are extracted and the dimension of the matrix product state is determined by using the experimental data of two-point correlation function.

2. The matrix $M$ is constructed from the data’s four-point and two-point correlation functions.

3. Having the poles and matrix $M$, we can predict all $n$-point functions.

4. Once the poles and the $M$ are determined, $R$ can be known. In this paper, $R$ is chosen to be diagonal and then $Q$ is expressed.

5. The quantum many-body state is obtained if $R$ and $Q$ are known.
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What can quantum field tomography be used for?

Creating new methods for state identification of continuous quantum many-body systems.

Can be used for precise, model-independent quantum-state identification: “quantum engineering”

Applications in quantum metrology, quantum information, and quantum simulation
Previous Quantum Tomography Efficient for small subsets of the full Hilbert Space

- 2010: “Quantum state tomography via compressed sensing”: Focus their measurements on pure states, states with symmetry, and ground states to limit experimental data acquisition. Use compressed sensing to recover sparse vector (containing only a few non-zero entries in specified basis) from small number of measurements. [2]

- 2010: “Efficient Quantum State Tomography”: two different methods, one requiring unitary operations on a constant number of subsystems, and the other requiring only local measurements with more elaborate post-processing. These methods have a linear number of experimental operations (always good!) and polynomially-scaled post-processing (not as good). [3]

- “Efficient and feasible state tomography of quantum many-body systems”: Focuses on discrete systems (atoms in a lattice). They show that measuring a tomographically complete set of observables is not necessary if a single observable is measured after allowing the state to evolve under appropriately-chosen quantum circuits. Results can then be generalized. [4]
Summary of efficient tomography for continuous quantum systems

• The continuous system (BEC) has infinitely many degrees of freedom → Quantum Field Tomography
  • Overcome this using quenching to form two separate quantum systems

• Sparsity - can use assumptions in theoretical analysis to simplify the system (low-order behavior)

• Continuous matrix-product states (cMPS) allows the creation of a theoretical model based on the data
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Data of Projected N-Point Function

First steps towards experimental quantum-field tomography!
What else can you do with this setup?

Experimental observation of a generalized Gibbs ensemble


• Using correlation functions to probe statistical properties
• Non-equilibrium system being described with multiple temperature-like parameters
Our critiques and final thoughts about this work

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<tr>
<th>Concerns</th>
<th>Conclusions</th>
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<tr>
<td>Accuracy?</td>
<td>• Probing quantum many-body systems using N-point correlation functions present the first steps towards efficient quantum-field tomography.</td>
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<td>Novelty?</td>
<td>• Reconstructing the quantum many-body states as a quantum field, without the need of a specific model (Hamiltonian) at hand is important for many applications in quantum information, as well as for general knowledge of strongly correlated states and far-from-equilibrium states.</td>
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<td>Influence?</td>
<td>• The authors could have better described the experimental setup, instead of focusing on the theory.</td>
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References


