Topological Quantization in Units of the Fine Structure Constant

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Fundamental topological phenomena in condensed matter physics are associated with a quantized electromagnetic response in units of fundamental constants. Recently, it has been predicted theoretically that the time-reversal invariant topological insulator in three dimensions exhibits a topological magneto-electric effect quantized in units of the fine structure constant $\alpha = e^2/hc$. In this Letter, we propose an optical experiment to directly measure this topological quantization phenomenon, independent of material details. Our proposal also provides a way to measure the half-quantized Hall conductances on the two surfaces of the topological insulator independently of each other.
Outline

- Introduction and Background
- Proposed Experiment
- Critique
- Citation Analysis
Topological Quantization in Condensed Matter Systems

- **Superconductor (SC):**
  Magnetic flux is quantized in the units of flux quantum \( \phi_0 = \frac{\hbar}{2e} \)

- **Quantum Hall effect (QHE):**
  Hall conductance is quantized in the units of conductance quantum \( G_0 = \frac{e^2}{h} \)

- **Topological magnetoelectric effect (TME):**
  A quantized coefficient in the units of fine structure constant \( \alpha = \frac{e^2}{hc} \)

Provide the most precise measurement of fundamental physical constants \( e, h \) and \( c \)
Topological Magnetoelectric Effect (TME)

For the time-reversal ($T$) invariant topological insulator (TI), the effective Lagrangian is

$$\mathcal{L} = \frac{1}{8\pi} \left( \varepsilon E^2 - \frac{1}{\mu} B^2 \right) + \frac{\theta}{2\pi} \frac{\alpha}{2\pi} E \cdot B,$$

Under $T$:

- $B \rightarrow -B$
- $E \rightarrow E$

Described the topological magnetoelectric effect

The system is invariant under shifts of $\theta$ by any multiple of $2\pi$

The only allowed value of $\theta$ by time reversal symmetry is 0 or $\pi$

For topological insulators, $\theta=\pi$; trivial insulators, $\theta=0$. 
Faraday Effect vs. Kerr Effect

The polarization of light is rotated when light is transmitted through (Faraday) or reflected from (Kerr) magnetized materials.
Purpose of the Paper

- Provide an optical experiment to measure the topological magnetoelectric effect

**How?**

- Exploit the Faraday & Kerr effects
- Measure Kerr and Faraday Angles (easy)
- Deduce the quantization of the parameter $\theta$ from the measured angles

\[
\mathcal{L} = \frac{1}{8\pi} \left( \varepsilon E^2 - \frac{1}{\mu} B^2 \right) + \frac{\theta}{2\pi} \frac{\alpha}{2\pi} E \cdot B,
\]

\[\alpha = e^2 / \hbar c.\]
Experimental Setup

1. A film of TI (topological insulator) of thickness $\ell$ and parameter $\theta$

2. A topologically trivial substrate with parameter $\theta_{\text{subs}} = 2p\pi$ with $p \in \mathbb{Z}$

3. A magnetic field is applied in the $z$ direction in order to have the Faraday-Kerr effects
Kerr and Faraday Angles

- **Incident Monochromatic Light**:
  \[ E_{in} = E_{in} \hat{x}, \]

- **Reflected Light**:
  \[ E_r = E_r^x (-\hat{x}) + E_r^y \hat{y} \]
  - Kerr Angle:
    \[ \tan \theta_K = E_r^y / E_r^x \]

- **Transmitted Light**:
  \[ E_t = E_t^x \hat{x} + E_t^y \hat{y} \]
  - Faraday Angle:
    \[ \tan \theta_F = E_t^y / E_t^x \]
What parameters determine the rotation of the light polarization?

- Generally the Kerr and Faraday angles will depend on many parameters (dielectric constants of materials, length, frequency of light, multiple reflections, $\theta$, $\theta_{\text{subs}}$)

- It seems dubious that we could extract the exact quantization of the TME from just the Kerr/Faraday angles

- But the paper suggests a trick that simplifies the expressions - Measure the angles at reflectivity minima and maxima (simplifies the expressions significantly)
Reflectivity minima

- Find specific frequency so that we have minimum reflectivity $R \equiv |E_r|^2/|E_{in}|^2$

- Equation between Faraday and Kerr angles:

$$\frac{\cot \theta'_F + \cot \theta'_K}{1 + \cot^2 \theta'_F} = \alpha p, \quad p \in \mathbb{Z}. \quad \alpha = e^2/\hbar c.$$ (Independent on other properties of the materials)
Reflectivity maxima

- $\omega$ is now tuned to a reflectivity maxima
- Kerr and Faraday angles - more complicated formulas

\[
\begin{align*}
\tan \theta_F'' & = \frac{2\alpha \left( p - \frac{\theta}{2\pi} + Y_3 \frac{\theta}{2\pi} \right)}{Y_3 + Y_2^2 - 4\alpha^2 \frac{\theta}{2\pi} \left( p - \frac{\theta}{2\pi} \right)}, \\
\tan \theta_K'' & = \frac{4\alpha \left[ Y_2^2 \left( p - \frac{\theta}{2\pi} \right) - \tilde{Y}_3^2 \frac{\theta}{2\pi} \right]}{\tilde{Y}_3^2 - Y_2^4 + 4\alpha^2 \left[ 2Y_2^2 \frac{\theta}{2\pi} \left( p - \frac{\theta}{2\pi} \right) - \tilde{Y}_3^2 \left( \frac{\theta}{2\pi} \right)^2 \right]}, \\
\tilde{Y}_3^2 & = Y_3^2 + 4\alpha^2 \left( p - \frac{\theta}{2\pi} \right)^2
\end{align*}
\]

$Y_i \equiv \sqrt{\varepsilon_i/\mu_i}$
Universal Function

- Use previous formulas to find an equation of the form:

\[ f(\theta_K', \theta_F', \theta_K'', \theta_F''; p, \theta) = 0 \]

- \( f \) does not depend explicitly on any material parameter \( \varepsilon_i, \mu_i \)

- We can find the zero crossing \( f(\theta) = 0 \)

- Hence find the parameter \( \theta \)

- Demonstrate the quantization of the TME effect in the TI bulk by verifying it is always 0 or \( \pi \)!
Summary

- Authors propose an optical experiment to directly measure the topological quantization phenomenon.

- By considering both the Kerr and Faraday effects, the authors derive the universal function $f$, which is independent of material parameters and can be used to determine $\theta$. 
Critique

● Is the experiment feasible?
  ○ Theory only applies when $w << E_g / h$.
  ○ To achieve maximum or minimum reflectivity $\omega$ or the thickness of the TI film must be changed continuously.
  ○ Samples do not yet have sufficient quality to yield strongly developed Quantum Hall effects.
  ○ This paper neglects reflection from the substrate-vacuum boundary.
Impact of the paper

- This paper has been cited 88 times.
- The TME effect has yet to be observed.
  - Considers light at oblique incidence
  - Considers both thin and thick films
  - Examines possible experimental difficulties
Thank you!