Review of *Signatures of Majorana Fermions in Hybrid Semiconductor-Superconductor Nanowire Devices* paper by V. Mourik et al.

Group 2
J. W. Bowers, F. A. Ceyhan, Y. Chang, G. Cao
University of Illinois at Urbana-Champaign
12/09/2016

Overview

Theory

1. What Majorana Fermions are and their presence in the context of Condensed Matter Physics. (Fikret Ceyhan)

2. The use of Majorana Fermions in Quantum Computation. (John Bowers)

Experiment

3. Setup and results of the experiment presented in the paper. (Yueqing Chang)

Others

4. Summary, impact and critique. (Goten Cao)

Mathematical Formalism of Majorana fermions

- Discovered out of mathematical curiosity: Real solutions to Dirac Equation (normal fermions have complex solutions) with a small twist on gamma matrices

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

4 x 4 Matrices

Complex (particle/hole) solutions (Dirac spinors, which has 4 components) with ±E eigenenergies

$$\gamma_0 = I \otimes \sigma_3$$
$$\gamma_1 = i\sigma_1 \otimes \sigma_2$$
$$\gamma_2 = i\sigma_2 \otimes \sigma_2$$
$$\gamma_3 = i\sigma_3 \otimes \sigma_2$$

Transformation to Majorana Equation

$$\tilde{\gamma}_0 = \sigma_2 \otimes \sigma_1$$
$$\tilde{\gamma}_1 = i\sigma_1 \otimes 1$$
$$\tilde{\gamma}_2 = i\sigma_3 \otimes 1$$
$$\tilde{\gamma}_3 = i\sigma_2 \otimes \sigma_2$$

Mathematical Formalism of Majorana fermions

c : electron
γ : majorana

- Majorana Fermions are their own antiparticles.

$$\gamma_i = \gamma_i^*$$

- A Majorana Operator can be described as a linear combination of two Fermion Operators.

Creation Operator

$$\gamma_{i,1} = c_i^\dagger + c_i \qquad \gamma_{i,2} = i(c_i^\dagger - c_i)$$

Annihilation Operator

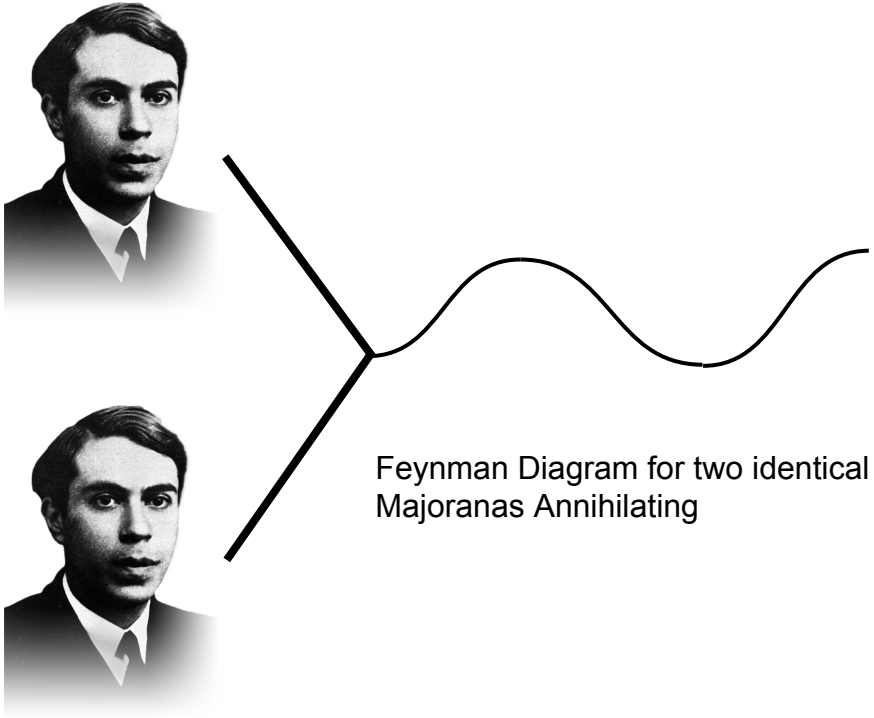
1st Majorana of ith fermion

- The algebra they satisfy is different from conventional fermions.

$$(c_i^\dagger)^2 = 0 \qquad {c_i}^2 = 0 \qquad \gamma_i^2 = 1$$

Finding Majorana Fermions

- Elementary Majorana fermions have never been observed directly by particle physicists.
- Neutrinos are suspected by some to be Majorana, but neutrinoless double beta decay (the product neutrinos annihilate with each other) has not been observed.
- Certain quasiparticles (excitation states) in condensed matter physics are also predicted to be Majorana.



Feynman Diagram for two identical Majoranas Annihilating

Majorana Fermions in Condensed Matter

c : electron
γ : majorana

Kitaev Model (in Electron Basis):

Cooper pairs

$$H = \sum_j [t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \Delta(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger) - \mu(c_j^\dagger c_j - \frac{1}{2})]$$

Hopping amplitude Superconducting gap Chemical Potential

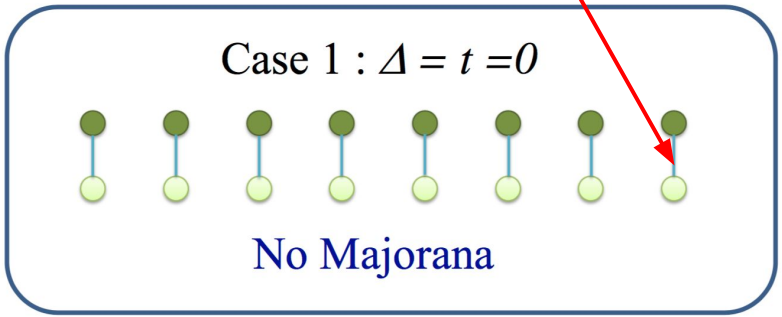
Expressed in the Majorana Basis, this is

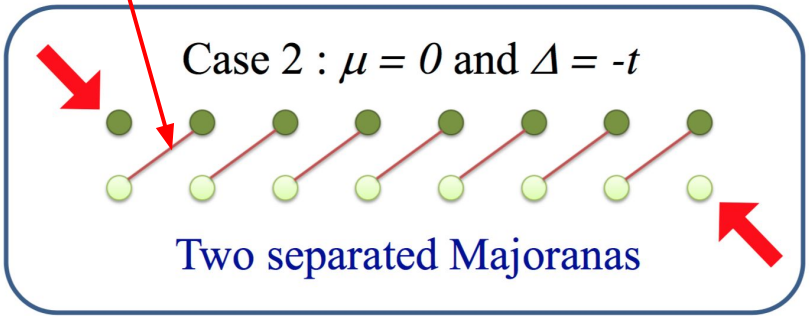
$$H = \sum_j \left[i\mu \gamma_j^A \gamma_j^B + i(t-\Delta)\gamma_j^B \gamma_{j+1}^A - i(t+\Delta)\gamma_j^A \gamma_{j+1}^B \right]$$

Majorana Fermions in Condensed Matter

Two Special Cases for Zero-Energy States

$$H = \sum_j \left[i\mu \underline{\gamma_j^A \gamma_j^B} + i(t-\Delta)\underline{\gamma_j^B \gamma_{j+1}^A} - i(t+\Delta)\gamma_j^A \gamma_{j+1}^B \right]$$





Geneviève Felury, Jian Li, Markus Büttiker. "Introduction to Majorana Fermions in Condensed Matter Physics."

Majorana Fermions in Condensed Matter

- Physical Example: Superconductor placed in proximity to a nanowire with strong Spin-Orbit Coupling (Fu and Kane 2008)

$$H_0(x) = \frac{k_x^2}{2m} - \mu + \tilde{\alpha} k_x \sigma_y + \frac{1}{2}\tilde{B}\sigma_z$$

Kinetic Term

Spin-Orbit Coupling

Zeeman Splitting

$$\mathcal{H}_S = \int d^D r \, d^D r' \, \Psi_\downarrow(\mathbf{r}) \Delta(\mathbf{r}, \mathbf{r}') \Psi_\uparrow(\mathbf{r}') + h.c.$$

Tunneling potential from superconductor to nanowire

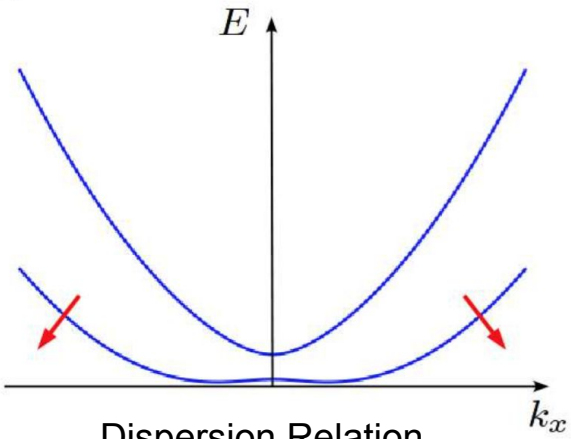
Condition for Majorana States

- Energy Spectrum for Different Spin Polarizations

$$E_{\pm}(k_x) = \frac{k_x^2}{2m} - \mu \pm \sqrt{(\tilde{\alpha} k_x)^2 + \tilde{B}^2}$$

- Condition for observing a Majorana Bound State

$$|\tilde{B}| > \sqrt{\Delta^2 + \mu^2}$$



Dispersion Relation

Leijnse, Martin, and Karsten Flensberg. "Introduction to topological superconductivity and Majorana fermions." *Semiconductor Science and Technology* 27.12 (2012): 124003.

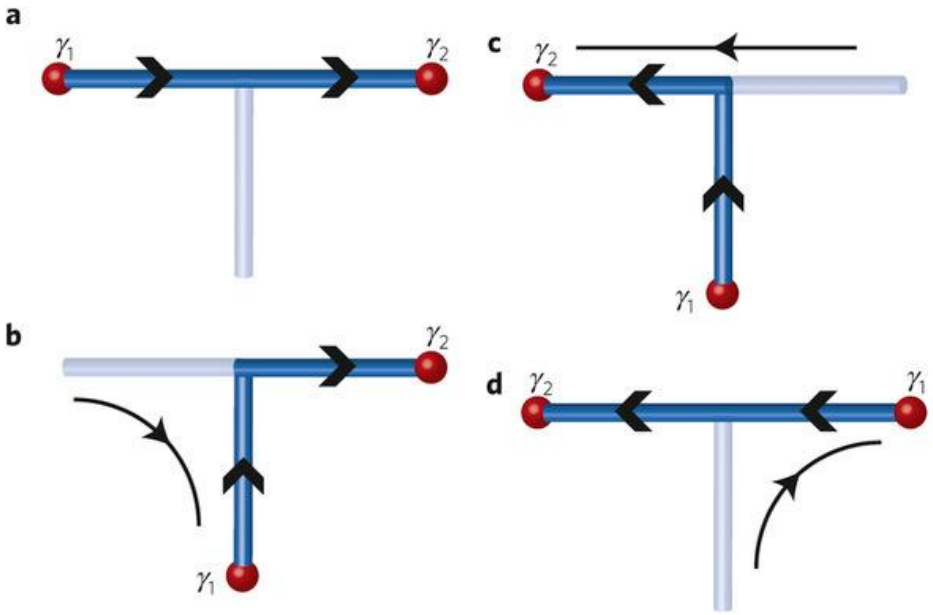
- Why is this important? Zero-Energy States, Highly non-localized nature of Majorana Bound States protect them from local erasures.

Majorana Edge States as Anyons

- Topological protection makes Majorana edge states ideal qubits.
- In two dimensions and lower, they behave as ***anyons***, which obey a symmetry rule

$$|\psi_1\psi_2\rangle = e^{i\theta}\,|\psi_2\psi_1\rangle$$

- We can therefore encode information in the system by moving anyons around.
- The information can differ for an exchange of the same two particles.
- We can model such behavior using braid theory.



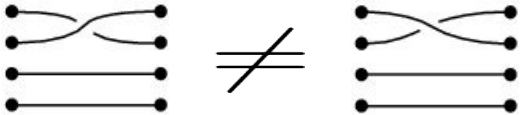
Two Majoranas on a T-shaped wire can be exchanged in three steps by first moving one to another wire, moving the other to the previous position of the first, and then moving the first particle to the previous position of the second.

Alicea, Jason, et al. "Non-Abelian statistics and topological quantum information processing in 1D wire networks." *Nature Physics* 7.5 (2011): 412-417.

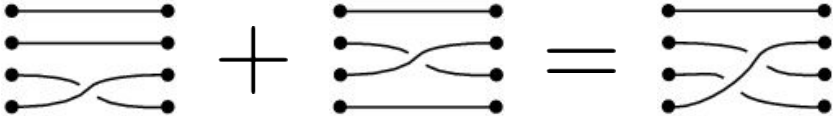
Computation via Braiding

- An n-braid is a mathematical object made from performing a permutation on n-points, and connecting each point to its image via a ‘string’.

- They differ from permutations in that any two points can be permuted with each other in 2 ways, rather than one.



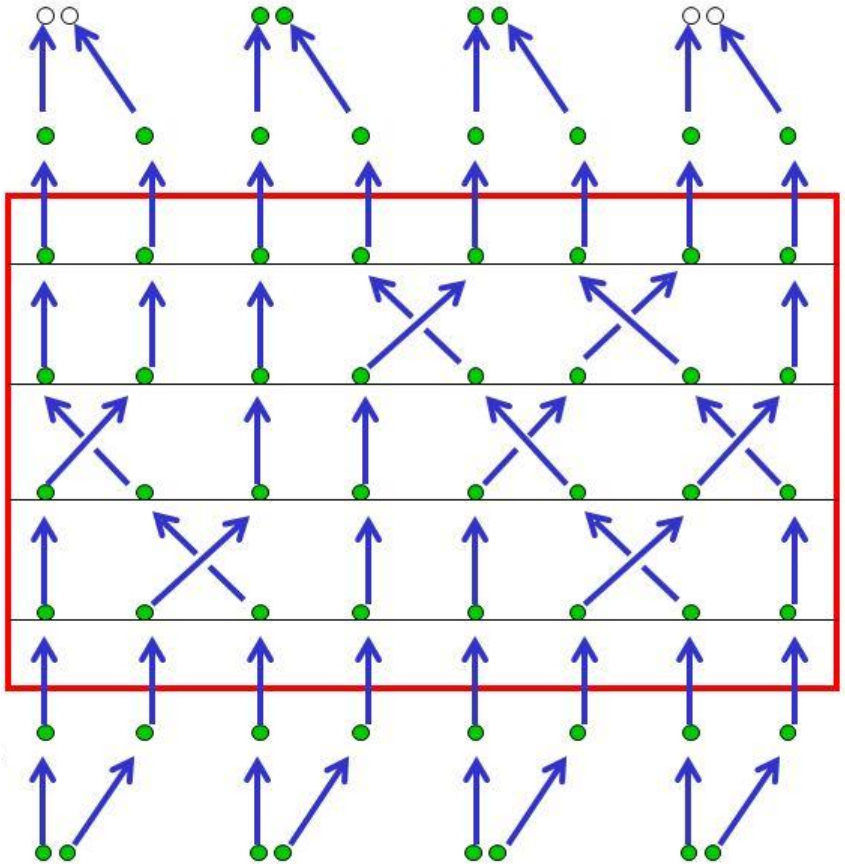
- Two n-braids can be put together to form another n-braid. This composition is a nonabelian group binary operation.



“Braid Group.” *Wikipedia, the Free Encyclopedia*. 12 Dec. 2016, https://en.wikipedia.org/wiki/Braid_group

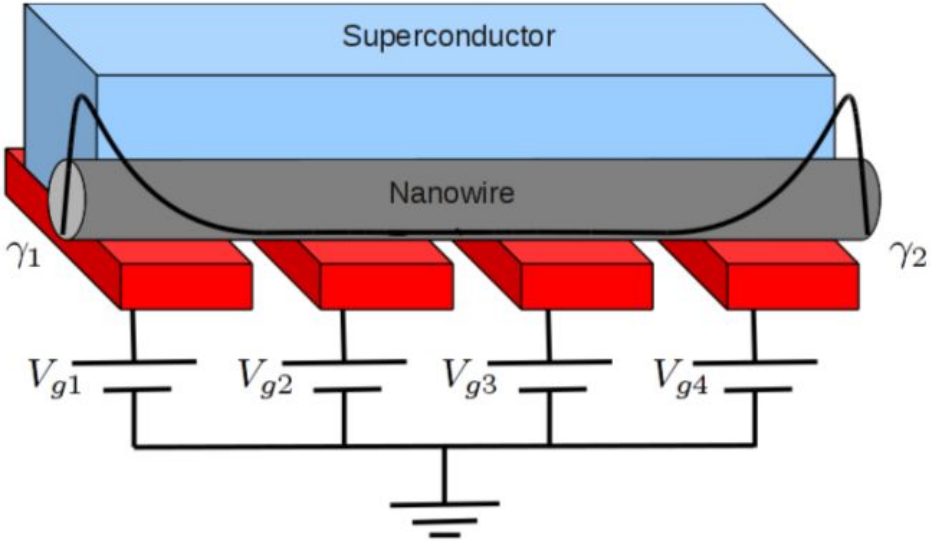
Computation via Braiding

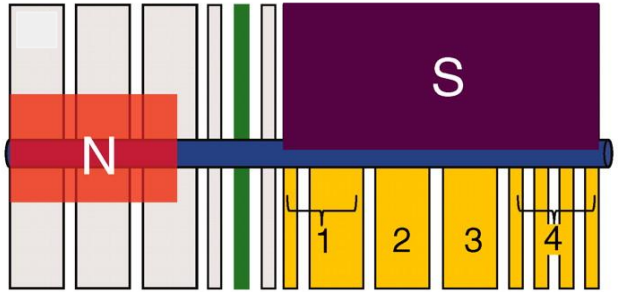
- Using an array of anyon pairs, we encode information by switching their positions.
- The worldlines of n-particles form an n-braid, as shown in the figure for n=8.
- Readout is done by bringing particles together and seeing which ones annihilate.



Preskill, J. “Quantum Information Science Atomic-Molecular Optical Physics Condensed Matter Physics Exotic Quantum State of Matter.” 3 Dec. 2008, California Institute of Technology, Pasadena, CA. Lecture.

Experimental Setup: InSb semiconductor nanowire on s-wave superconductor, with external Zeeman field



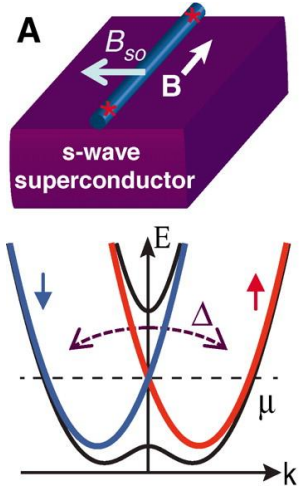


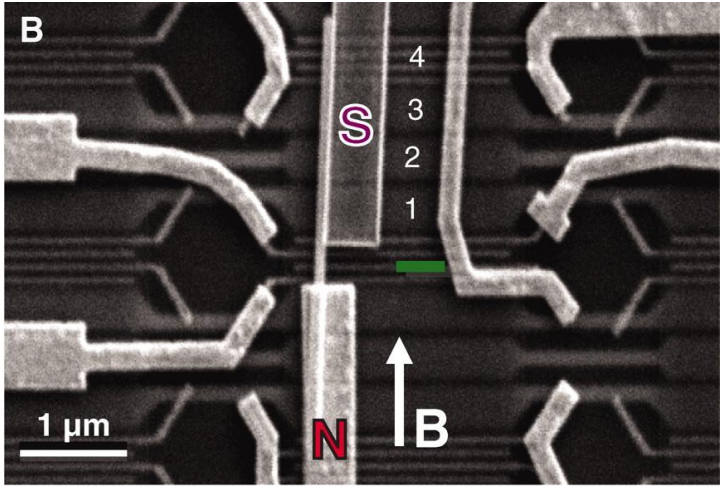
N: normal semiconductor
S: superconductor
1,2,3,4: gates

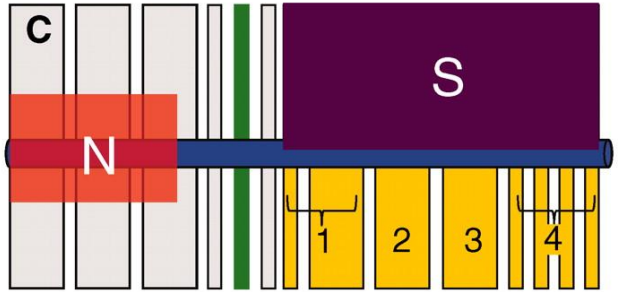
Leijnse, Martin, and Karsten Flensberg. "Introduction to topological superconductivity and Majorana fermions." *Semiconductor Science and Technology* 27.12 (2012): 124003.

Ç

Experimental Setup: InSb semiconductor nanowire on s-wave superconductor, with external Zeeman field





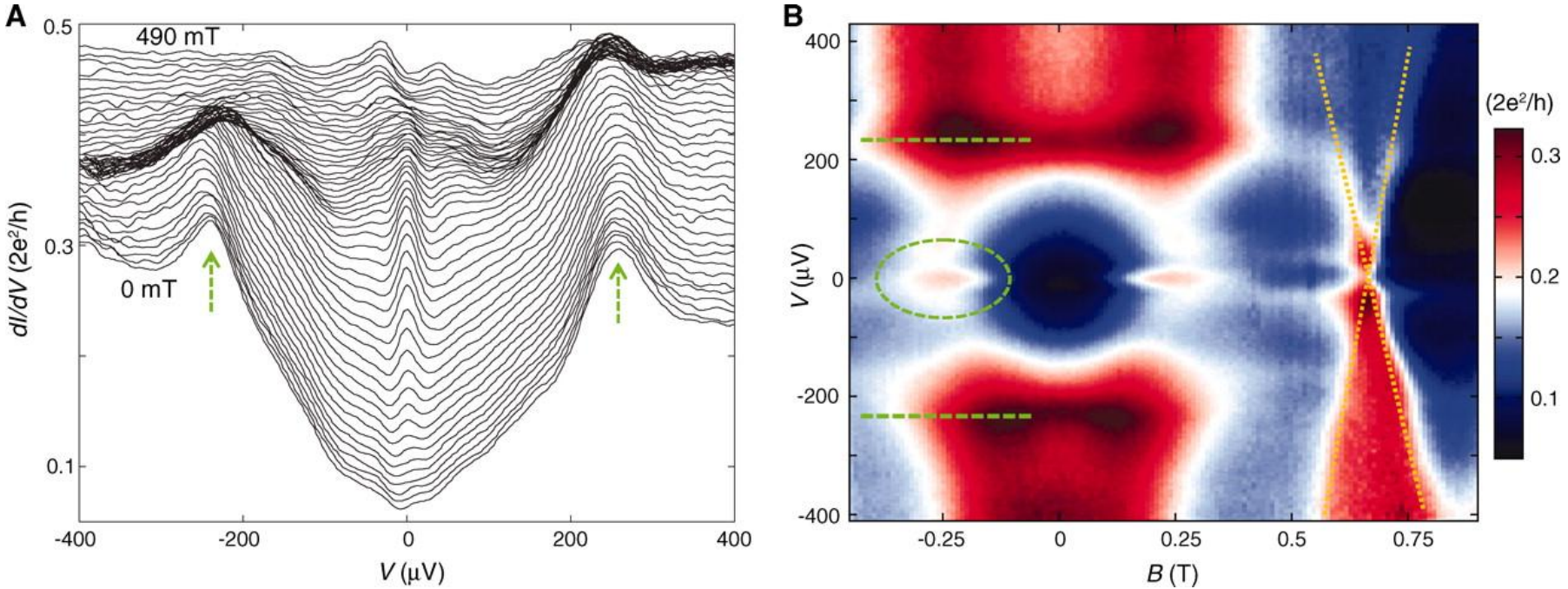


$$E_{\pm}(k_x) = \frac{k_x^2}{2m} - \mu \pm \sqrt{(\tilde{\alpha} k_x)^2 + B^2}$$

B: External Zeeman field:
Bso: Effective spin-orbit-coupling field
N: normal semiconductor
S: superconductor
Green rectangle: tunneling barrier

Mourik, Vincent, et al. "Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices." *Science* 336.6084 (2012): 1003-1007.

Results: zero-bias-peaks (ZBPs) remains stuck to zero energy over considerable changes in *B*

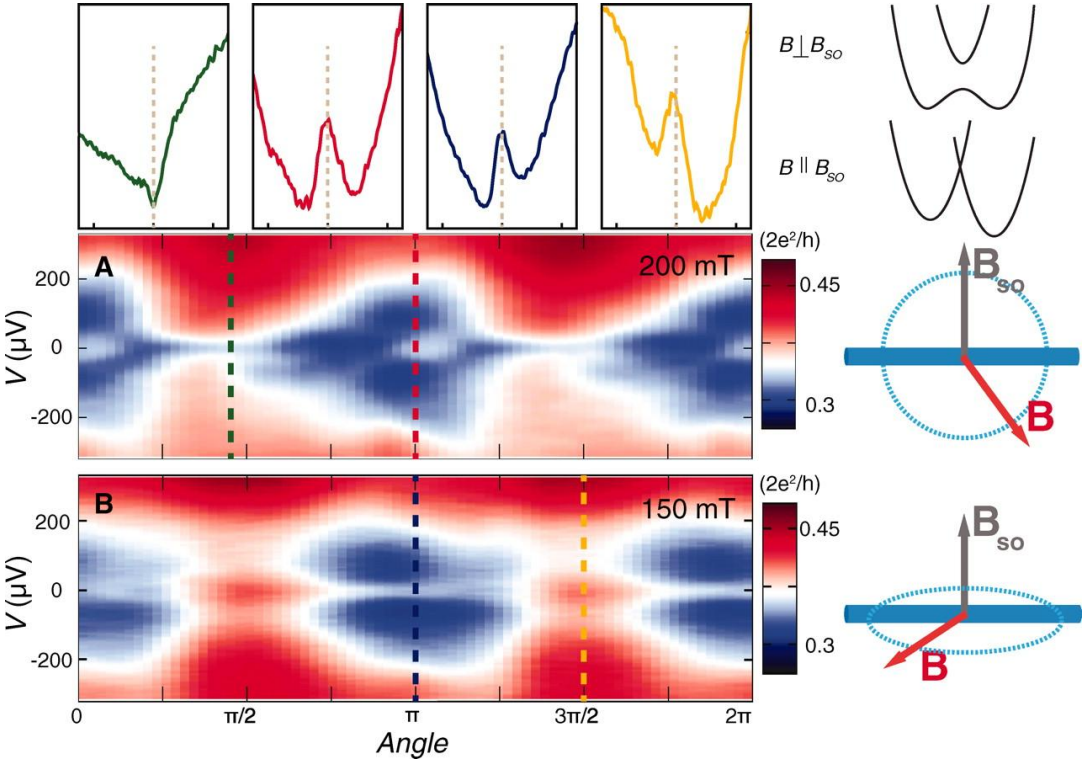


Possible origins of ZBPs: Kondo effect, Andreev bound states, weak antilocalization, reflectionless tunneling and **Majorana bound states**.

Mourik, Vincent, et al. "Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices." *Science* 336.6084 (2012): 1003-1007.

Verification: Signatures of Majorana Fermions

- **Superconductivity:** no ZBP in N-NW-N setup
- **Nonzero *B* field:** no ZBP when *B = 0*
- **External *B* has a component perpendicular to B_{so}:** sweep *B* in all directions and ZBP is absent when ***B*** is parallel to B_{so}



Mourik, Vincent, et al. "Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices." *Science* 336.6084 (2012): 1003-1007.

Summary

- Majorana Fermions are a special type of fermions which are the antiparticles of themselves and they can be created by “splitting” electrons (fermions).
- Majorana Fermions can be used as qubits in the form of anyons under a braid algorithm.
- Zero-bias peak was observed, which strongly suggests the existence of Majorana Fermions.
- Known phenomena with similar spectra were eliminated so that this observation was verified.

Impact

- First possible observation of Majorana edge state in a condensed matter system.
- This paper has been cited 1,113 times since it was published in 2011.
- Web of Science shows there were 1,092 publications with the keywords “Majorana” and “Topological” from 2012-2016. By contrast, there were only 152 such publications from the years 2008-2011.
- Similar experiments are being done using thin films, which provide a 2D environment, vs the 1D nanowires.

Critique

- The paper concisely summarizes a novel result, and presents a convincing argument of the existence of Majorana edge states.
- Despite otherwise convincing evidence, statistical uncertainties of this finding are not mentioned.
- Some aspects of the experimental setup are not well-explained. For example, the spin-orbit magnetic field, and why some gates are finer than the others.

Anyon, Anyon

- Anyon, anyon, where do you roam?
  Braid for a while, before you go home.

- Though you're condemned just to slide on a table,
  A life in 2D also means that you're able
  To be of a type neither Fermi nor Bose
  And to know left from right - that's a kick, I suppose.

- You and your buddy were made in a pair
  Then wandered around, braiding here, braiding there.
  You'll fuse back together when braiding is through
  We'll bid you adieu as you vanish from view

- Alexei exhibits a knack for persuading
  That someday we'll crunch quantum data by braiding,
  With quantum states hidden where no one can see,
  Protected from damage through topology

- Anyon, anyon, where do you roam?
  Braid for a while, before you go home.

-John Preskill

References

- Alicea, Jason, et al. "Non-Abelian statistics and topological quantum information processing in 1D wire networks." *Nature Physics* 7.5 (2011): 412-417.
- “Braid Group.” *Wikipedia, the Free Encyclopedia*. 12 Dec. 2016, https://en.wikipedia.org/wiki/Braid_group
- Mourik, Vincent, et al. "Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices." *Science* 336.6084 (2012): 1003-1007.
- Wilczek, Frank. "Majorana returns." *Nature Physics* 5.9 (2009): 614-618.
- Leijnse, Martin, and Karsten Flensberg. "Introduction to topological superconductivity and Majorana fermions." *Semiconductor Science and Technology* 27.12 (2012): 124003.
- Kitaev, A. Yu. "Unpaired Majorana fermions in quantum wires." *Physics-Uspekhi* 44.10S (2001): 131.
- Fu, Liang, and Charles L. Kane. "Superconducting proximity effect and Majorana fermions at the surface of a topological insulator." *Physical review letters* 100.9 (2008): 096407.
- Preskill, J. “Quantum Information Science Atomic-Molecular Optical Physics Condensed Matter Physics Exotic Quantum State of Matter.” 3 Dec. 2008, California Institute of Technology, Pasadena, CA. Lecture.