Self-organized Criticality: An Explanation of 1/f noise

Bak, P., Tang, C., Wiesenfeld, K.,
Self-organized criticality: An explanation of the 1/f noise
(1987) *Physical Review Letters*, 59 (4), pp. 381-384.

Presented by A. Luu, Y. Lv, M. Lynch, G. Mattson
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Phys 596, Dec 2, 2016

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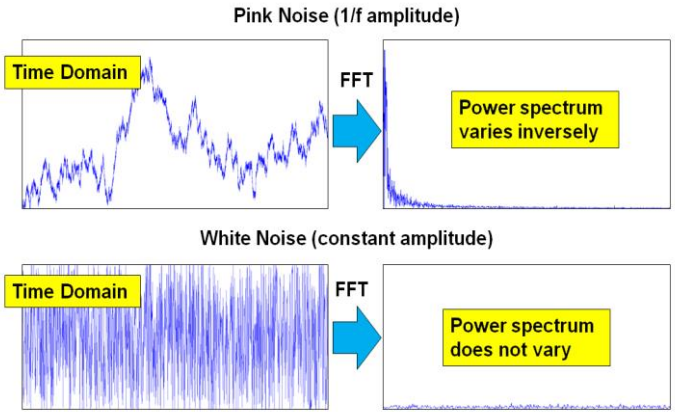

Source:http://math.stackexchange.com/questions/216006/1-f-pink-noise-for-the-math-disabled

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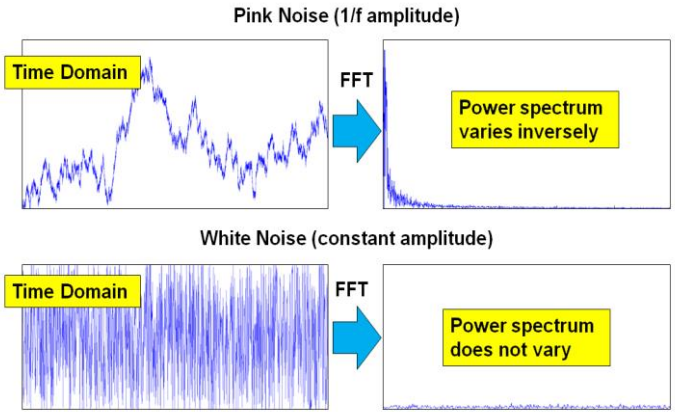

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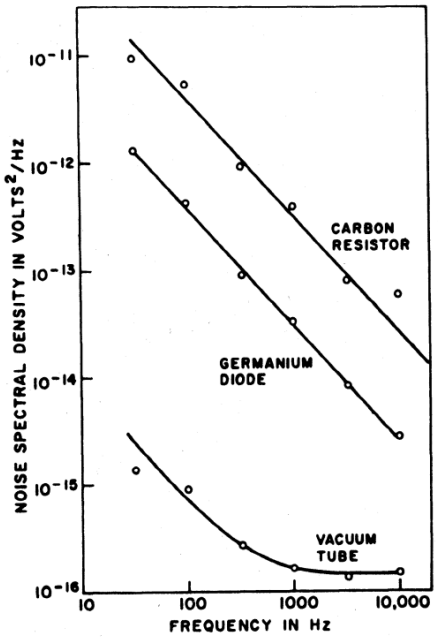

J.J. Brophy, J. Appl. Phys. **40,** 3551 (1969)

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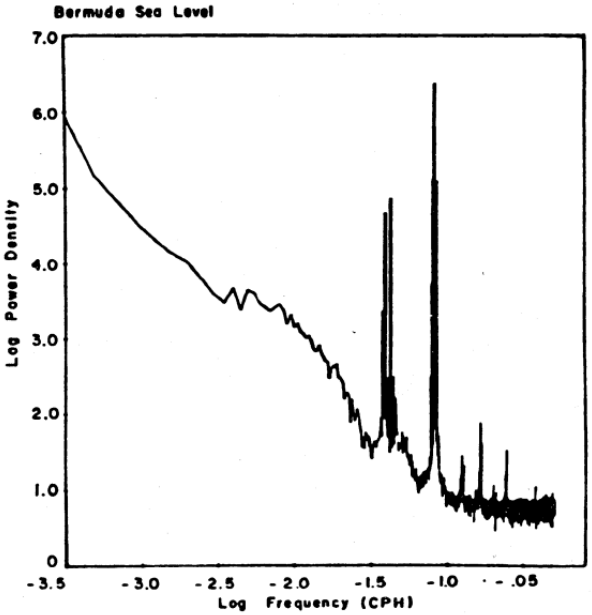

C. Wunsch, Rev. Geophys. **10,** 1 (1972)

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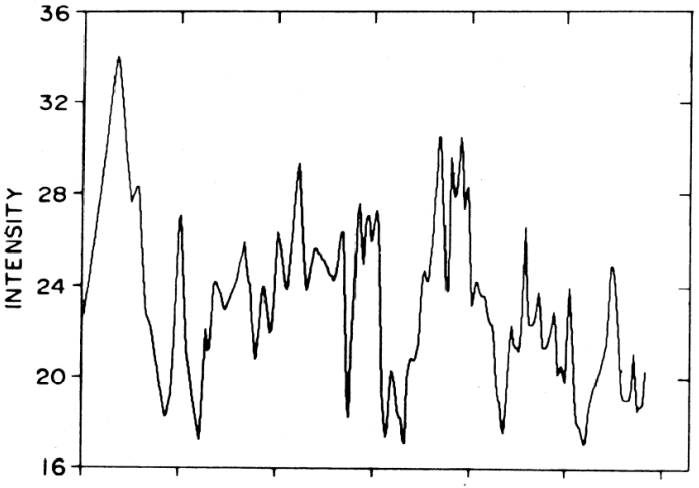

Intensity spectrum of the quasar 3C273 from 1887 to 1967

Fahlman and Ulrych, Ap. J. **201,** 277 (1975)

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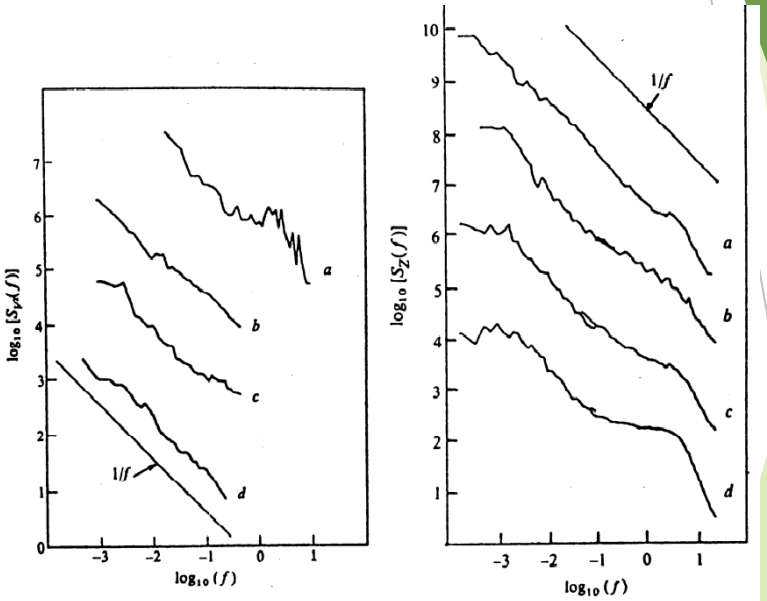

Loudness (left) and pitch (right) of radio broadcasts: (a) piano rags; (b) classical music; (c) rock music; (d) news and talk

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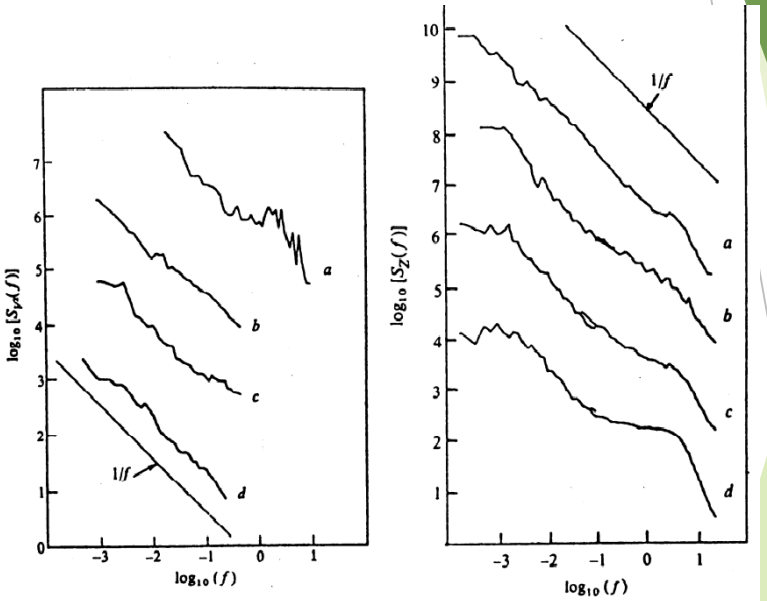
- Ubiquitous, but origin unknown!



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- Prior work (e.g. Richardson 1950, Hooge 1969, Voss & Clark 1976) focused on conductors and other solids
- Proposed 1/f dependence as a consequence of diffusion and thermal fluctuations in electrical resistance
- Still no general explanation for 1/f noise, but theories increasingly point to relationship with non-equilibrium phenomena and scaling invariance

First model accounting for 1/f noise without specific physical details

- In the past, people have used diffusion theory to explain the 1/f noise in vacuum tubes, resistors and amplifiers.
- Those theories require specific physical details, and rely on fine tuning of parameters in the diffusion equation.
- The parameters needed in this model are very few.

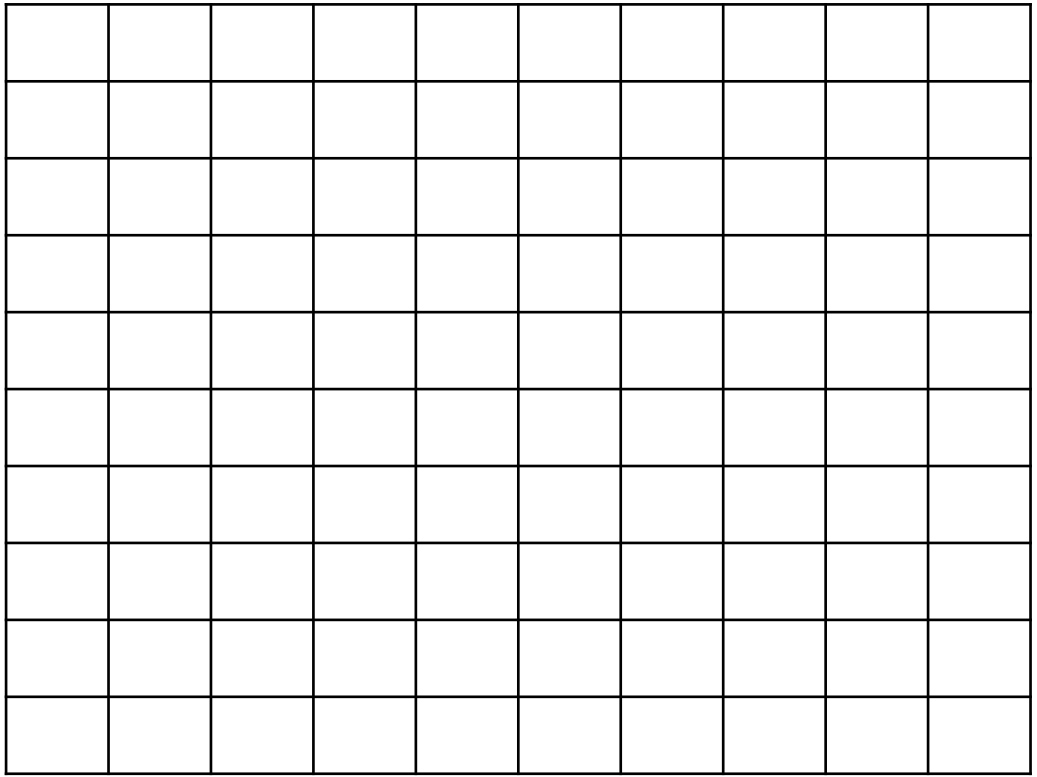
First computational simulation based on our theory

- Based on their model, the authors produced simulations demonstrating the f^(-1.1) power law for noise in three dimensions.

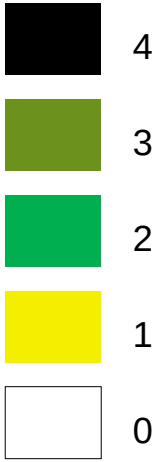
Simulation's Implications

- This model suggests the two seemingly unrelated phenomena- the fractal structure of self organizing systems and f^-β noise- might have the same underlying mechanism
- Due to few parameters are required, this model has great universality
- Later works have used the same model to interpret more phenomena

Simulation Rules



Cellular automata model on square lattice



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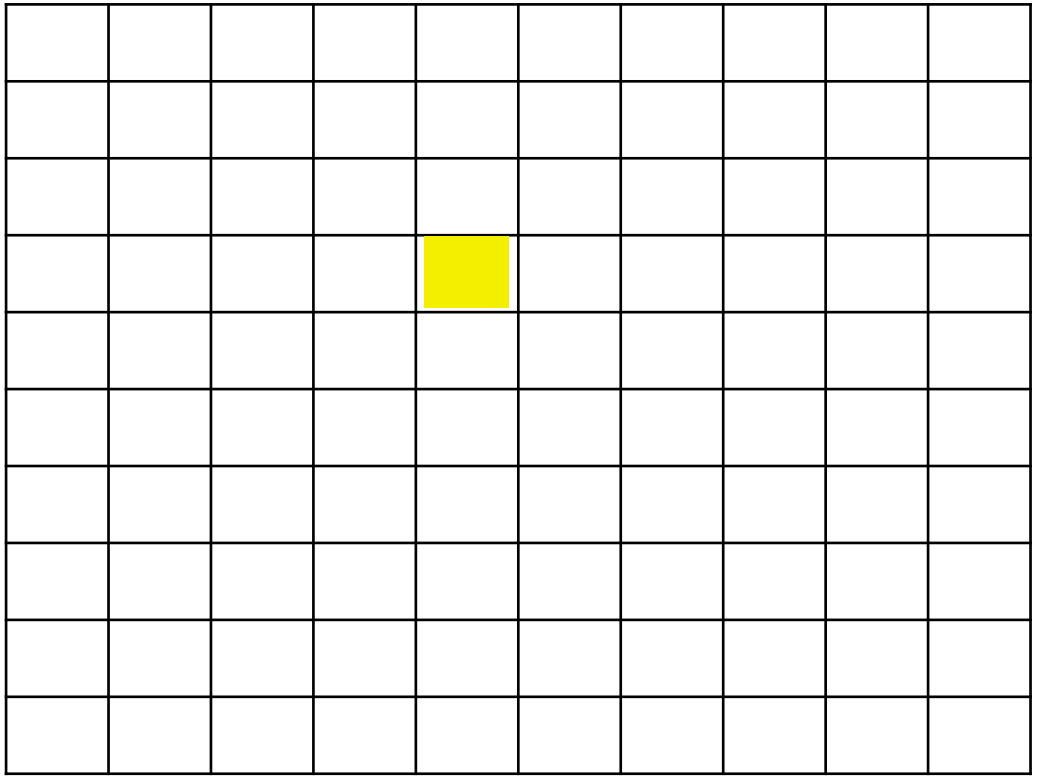
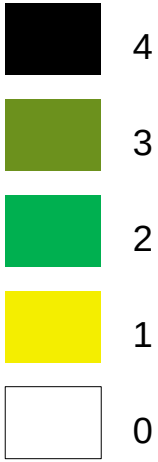


Each time step, increment random lattice value by 1



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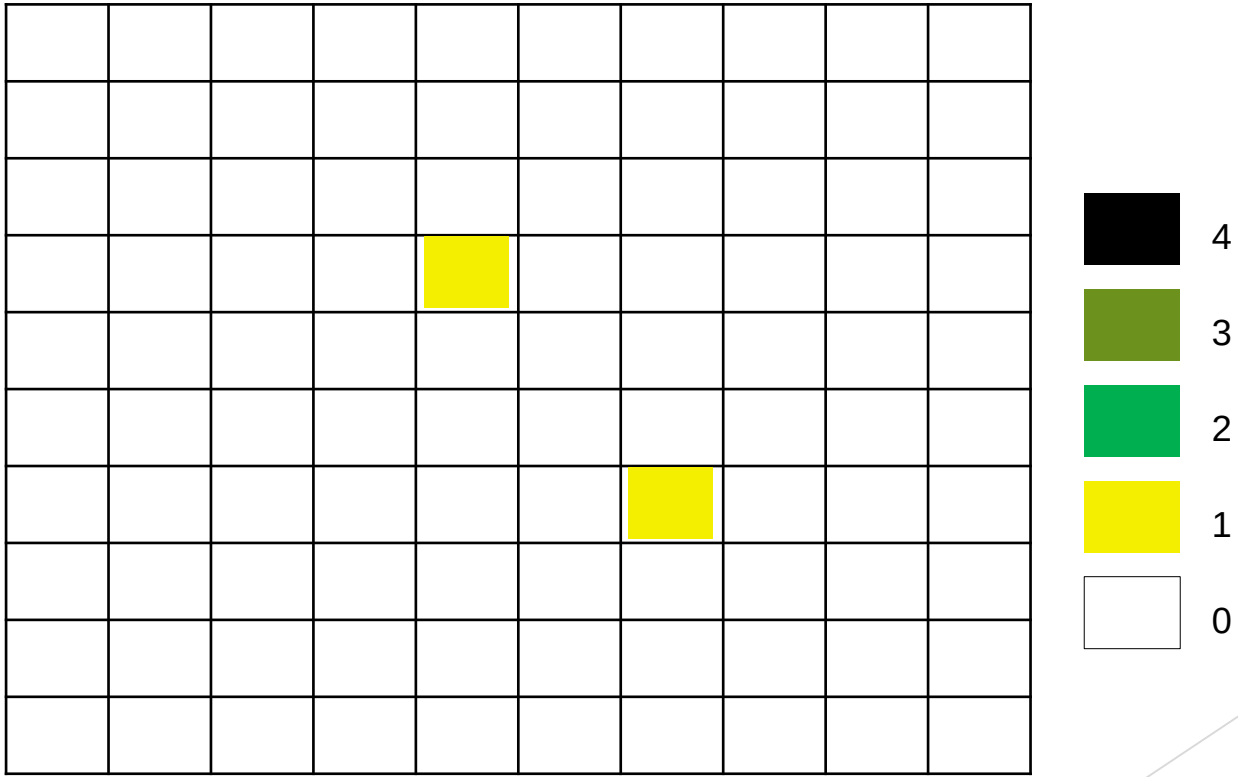

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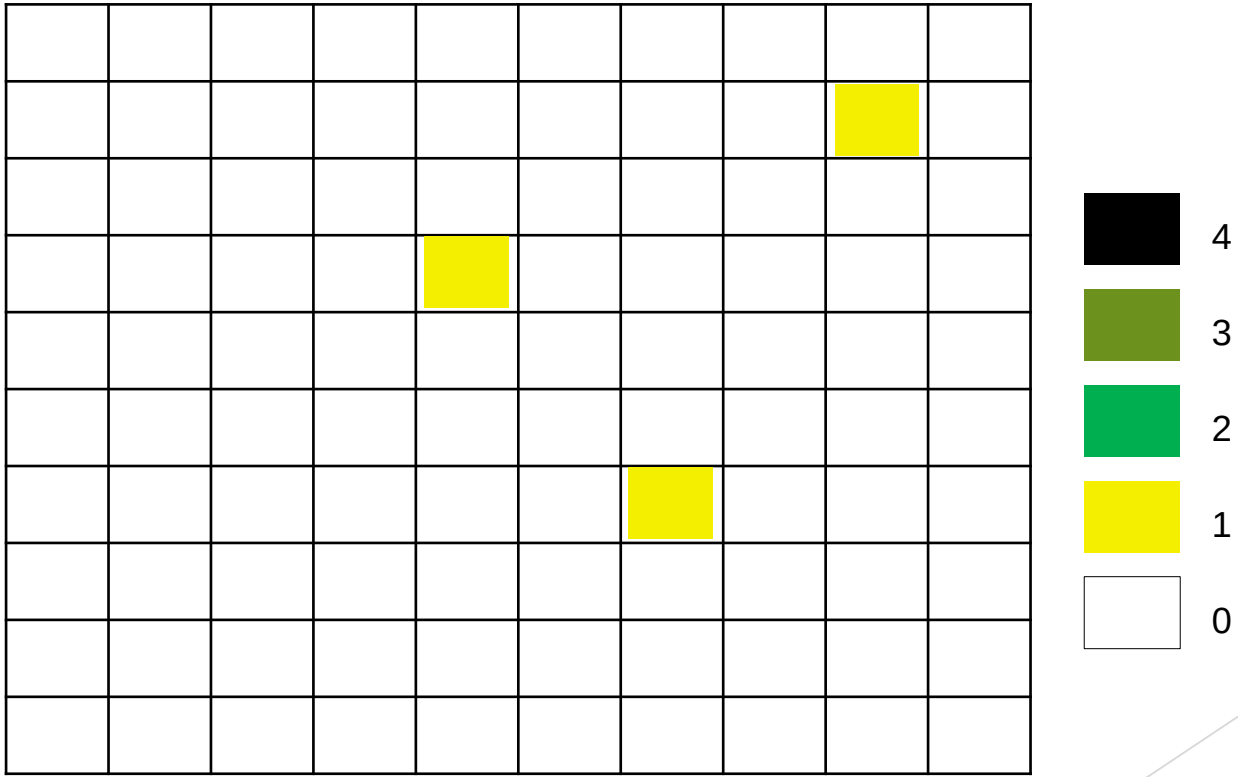

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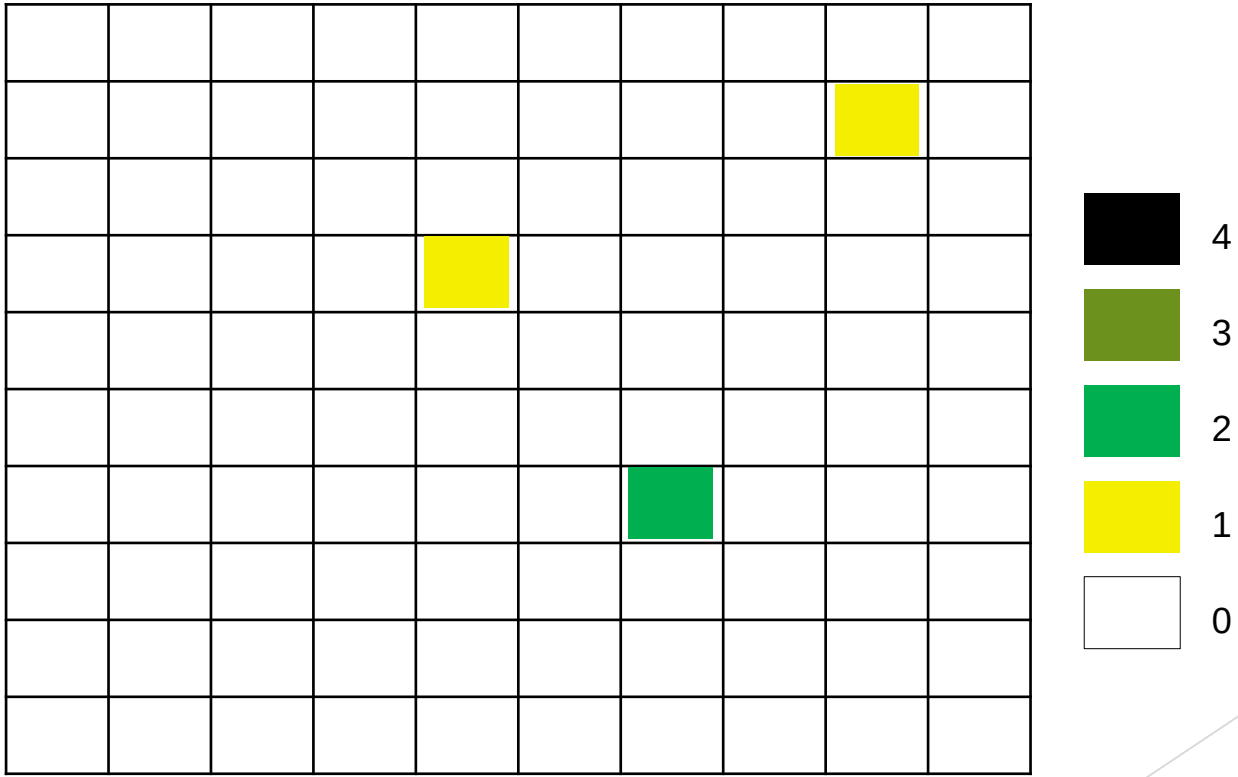

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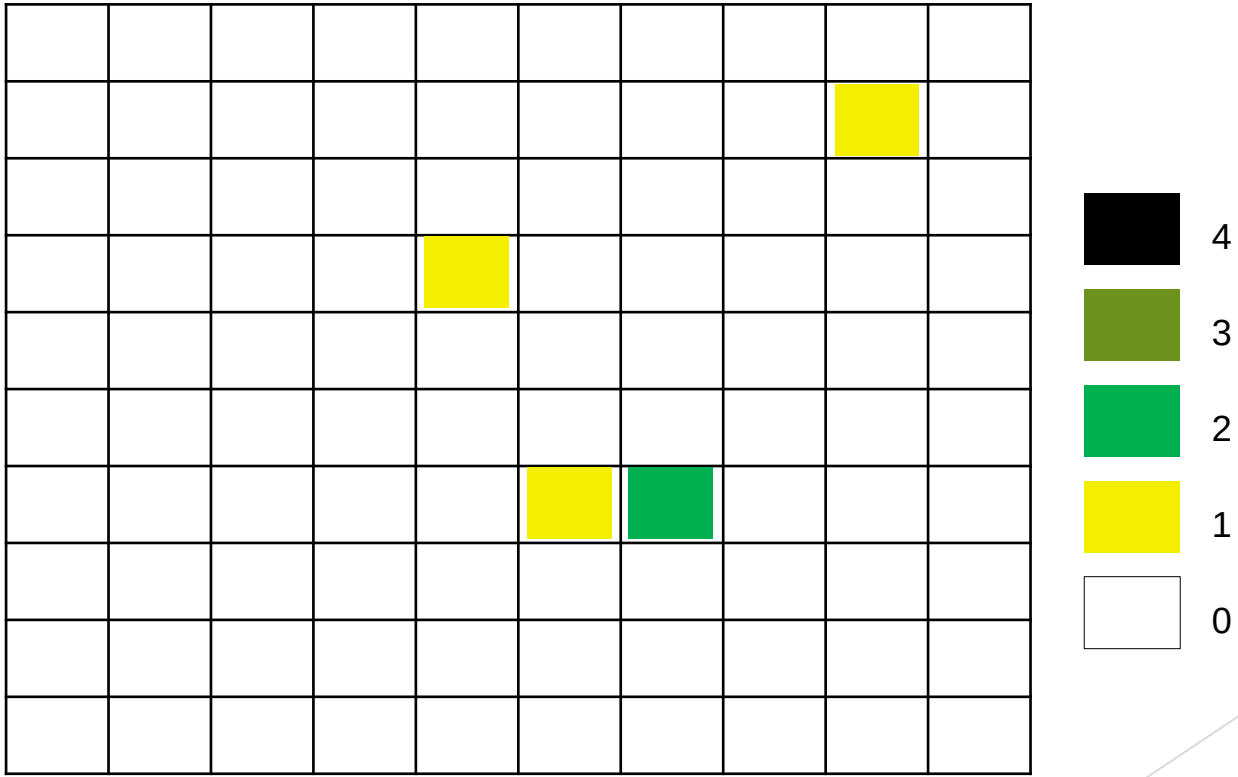

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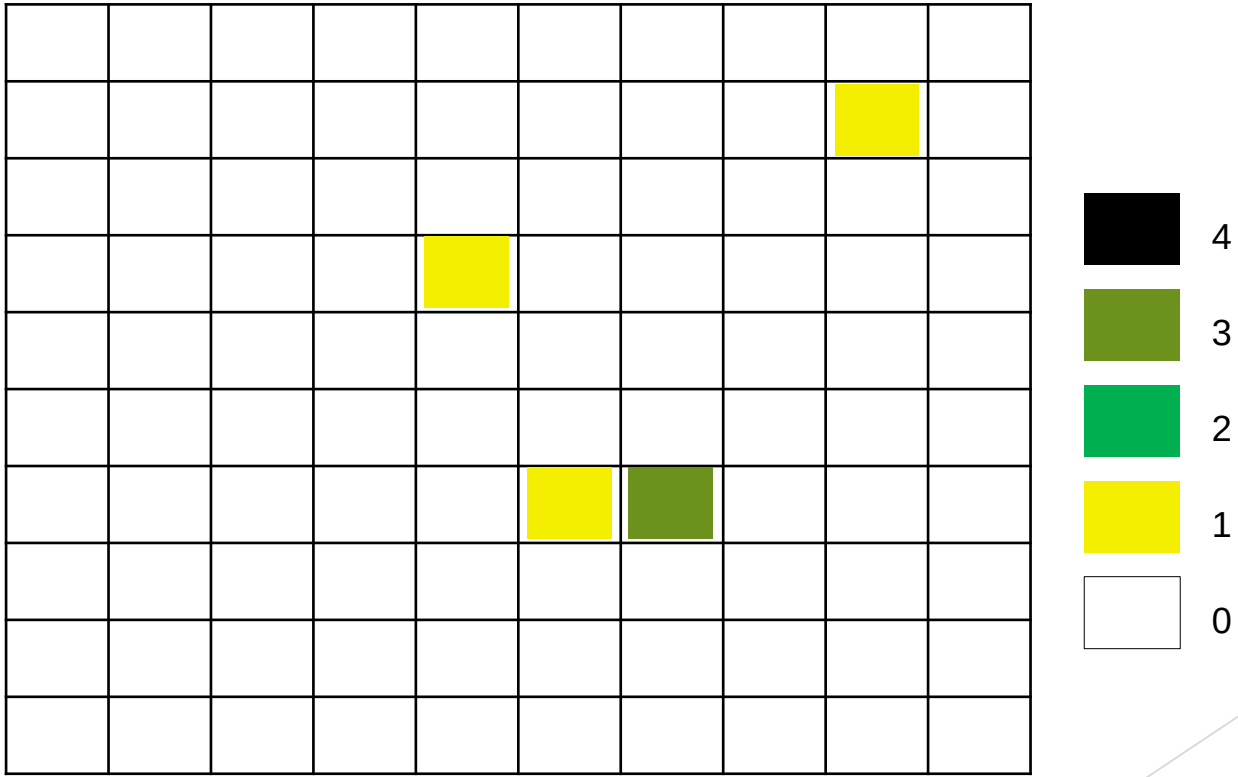

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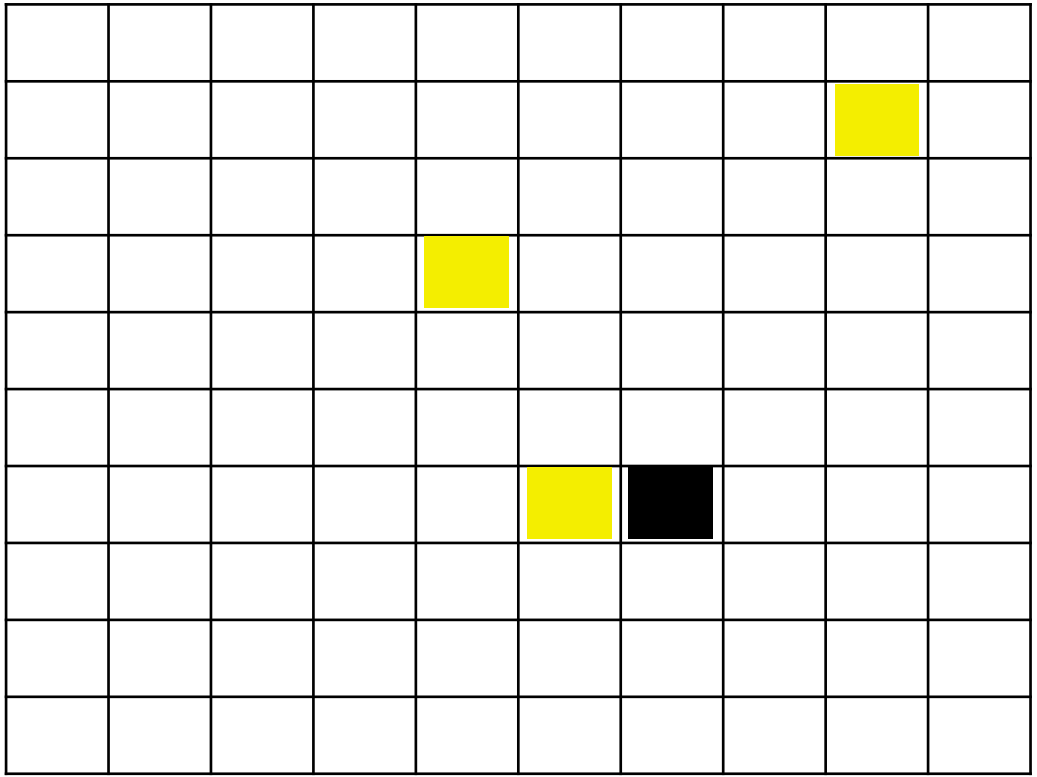
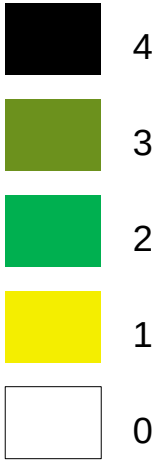


When lattice point reaches threshold, decrease its value by 4 and increment nearest cardinal neighbors



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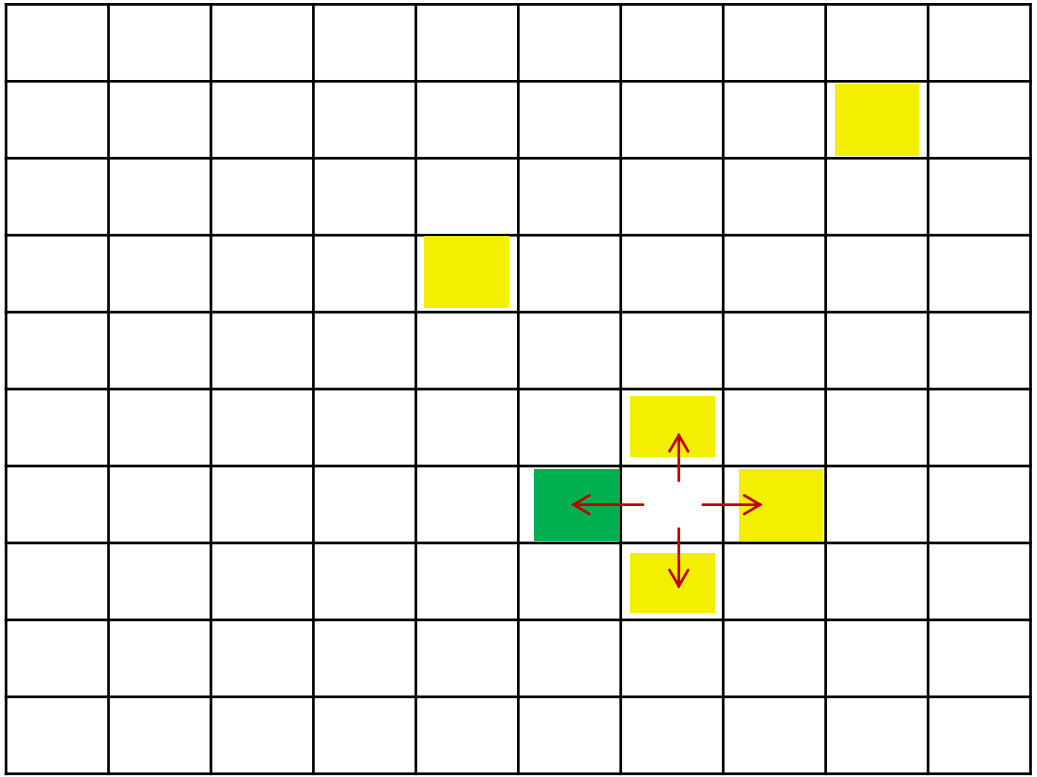
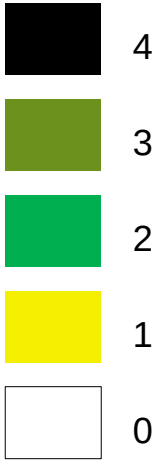


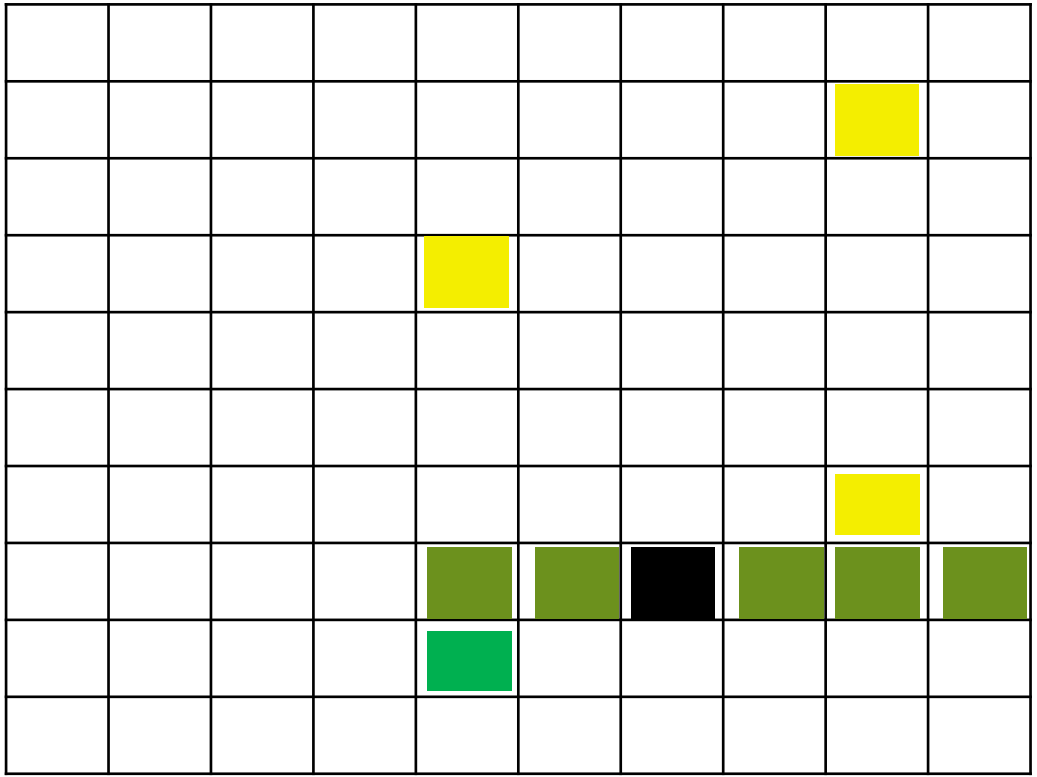
$$z(x,y) \rightarrow z(x,y) - 4,$$

$$z(x \pm 1, y) \rightarrow z(x \pm 1, y) + 1,$$

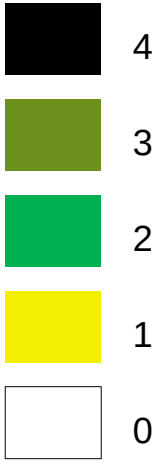
$$z(x, y \pm 1) \rightarrow z(x, y \pm 1) + 1,$$



Simulation Rules: Cascades



"Cascades" of unstable clusters can occur



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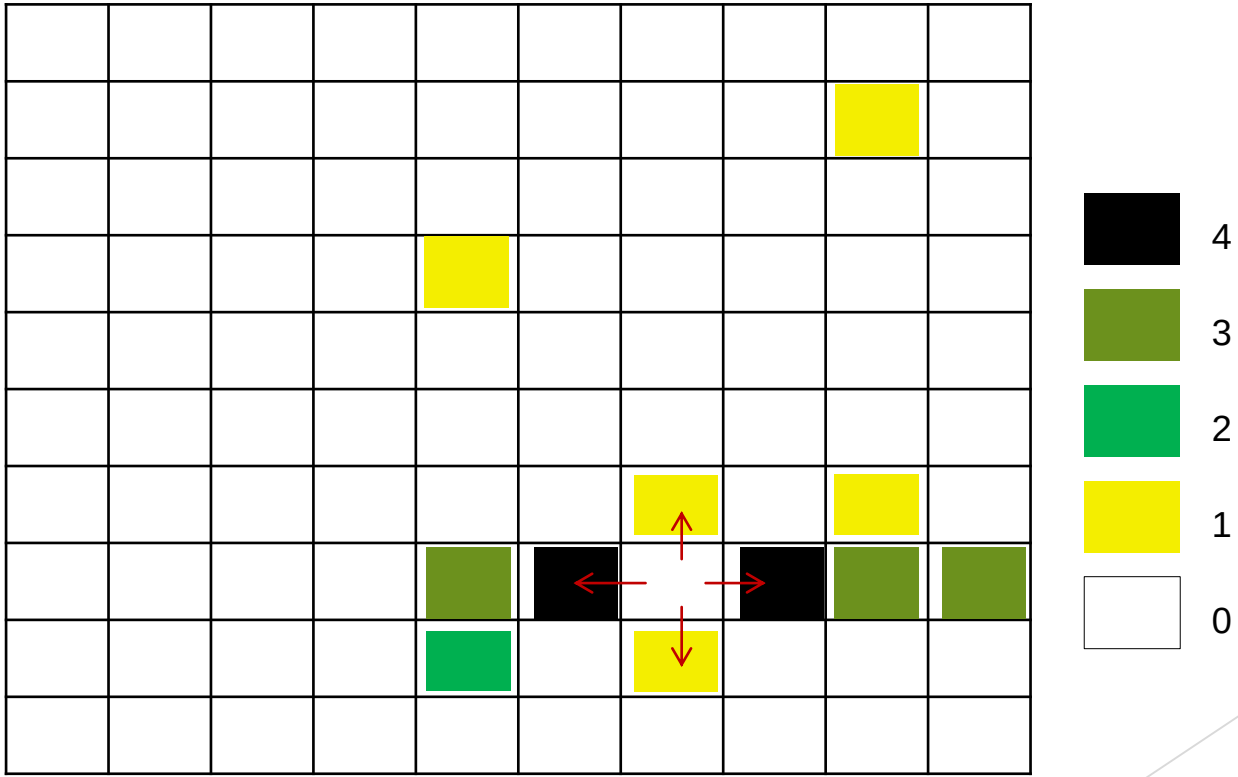

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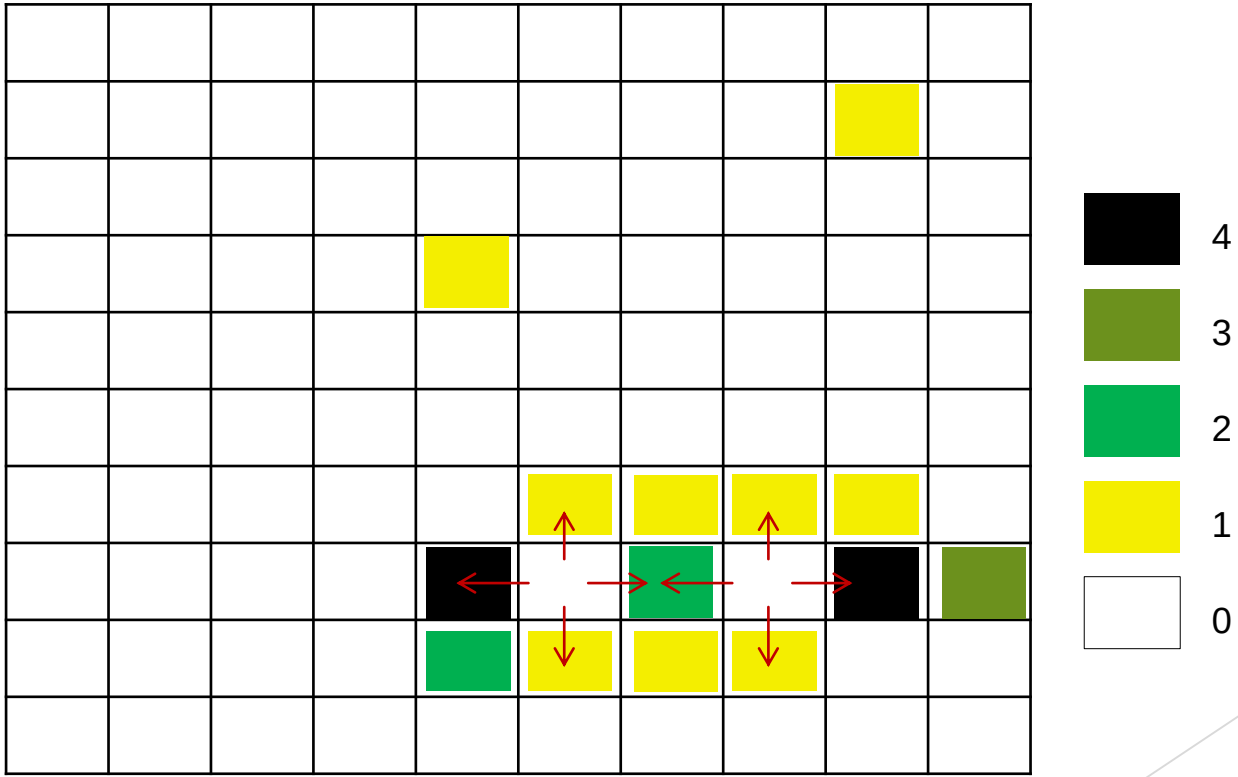

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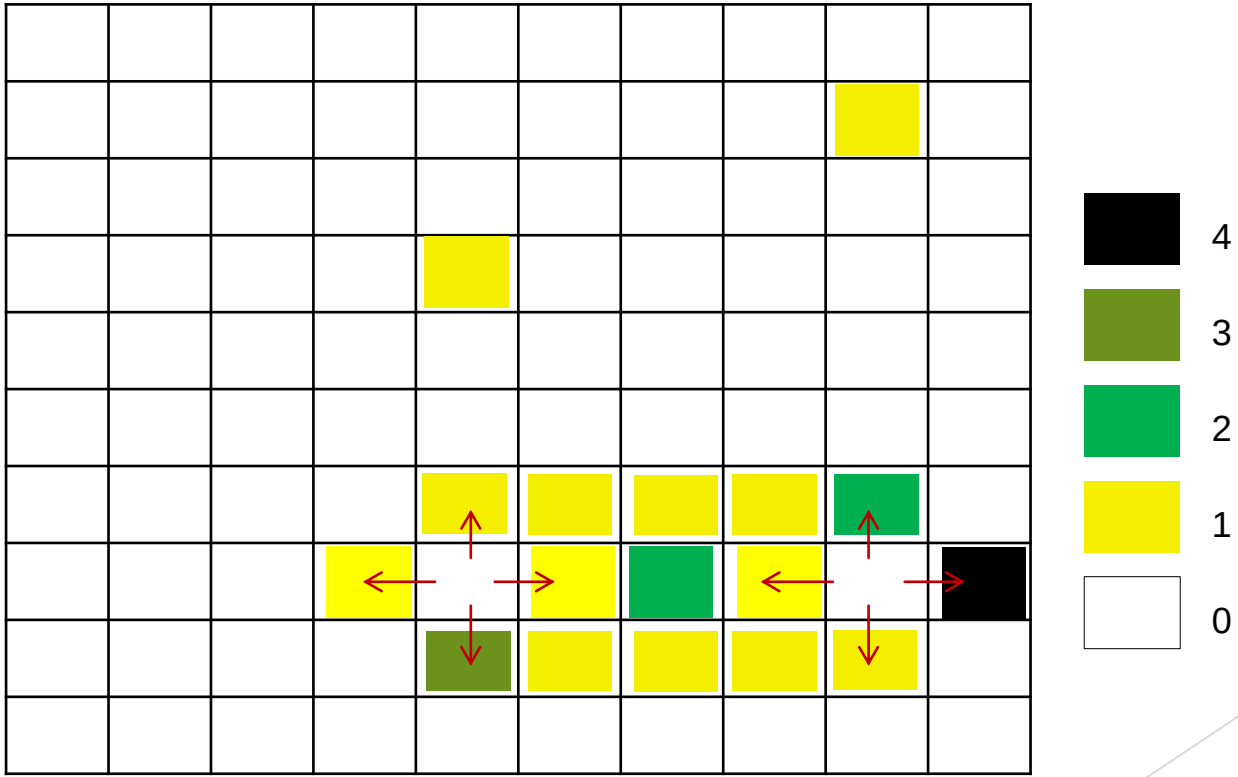

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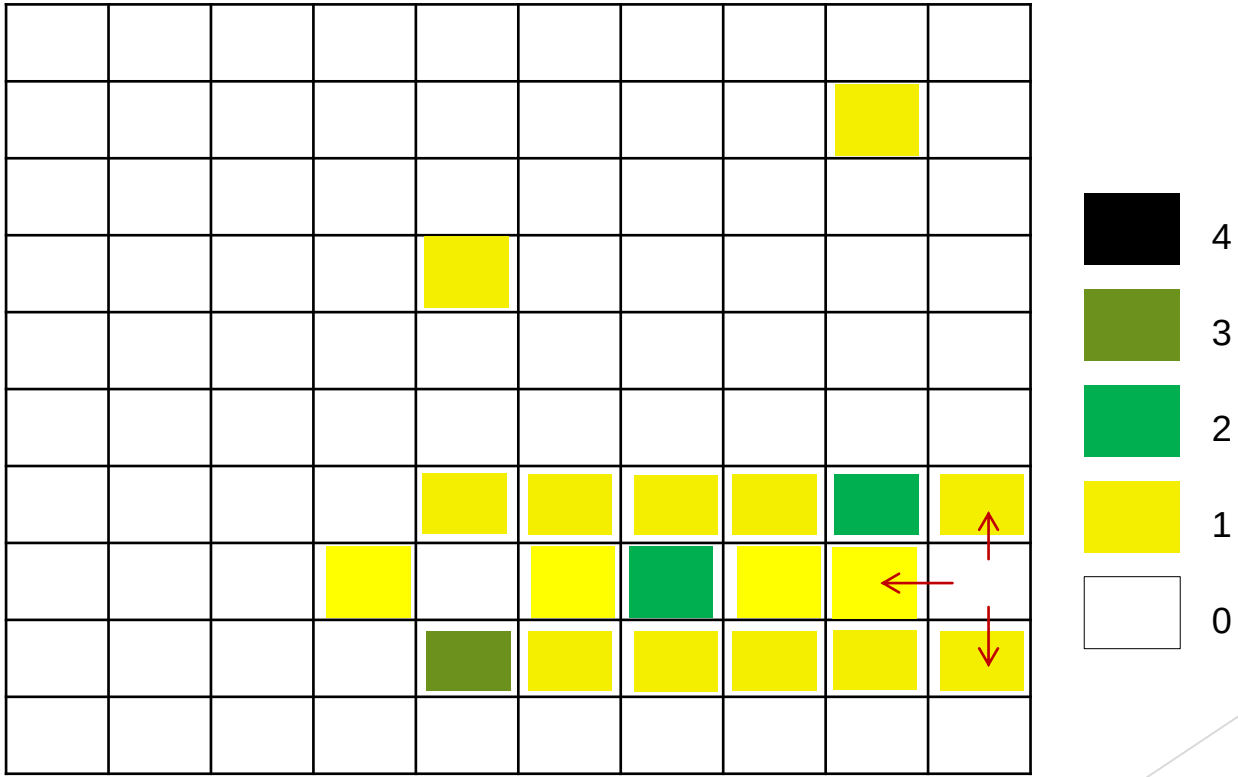
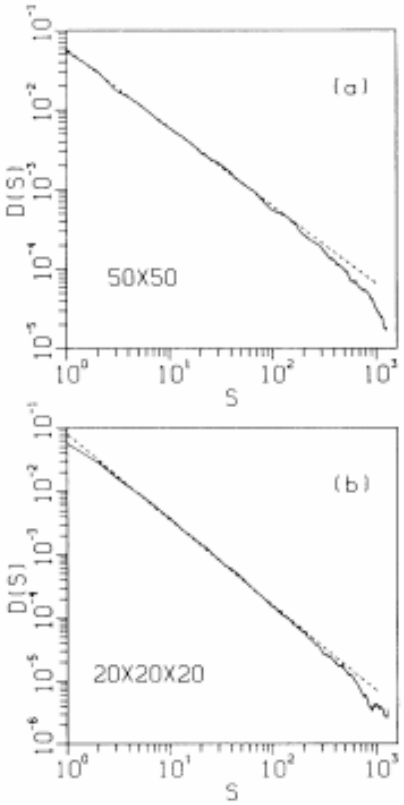


Simulation Setup

- Initialization
  - Randomly assign z[x,y] >> threshold
  - Evolve system until it stops (all z[x,y] less than threshold) at locally minimally stable state
- Simulation
  - Each time-step, randomly increment lattice point
  - Count number and “size” of cascades
  - Size defined as number of lattice points affected

Distribution of Cluster Lifetimes Gives 1/f Noise

- $D(s) \sim s^{(-\tau)}$
  - S = size of cascade
  - D(s) = how frequently cascade of size s occurs
  - τ = parameter
- Distribution of cluster lifetimes D(t) calculated from D(S)
  - $D(t) = (s/t)D(s(t))(ds/dt) \equiv t^{(-\alpha)}$
- Noise spectrum $S(\omega)$ calculated from D(t)
  - $S(\omega) = \int dt\, [t*D(t)]/[1+(\omega t)^2]$
  - $S(\omega) \approx \omega^{(-2 + \alpha)}$
- 2D: $S(\omega) \approx \omega^{(-1.58)}$
- 3D: $S(\omega) \approx \omega^{(-1.1)}$



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  - Simulation produces $S(\omega) \approx \omega^{(-1.1)}$
- Generality of model's applicability is yet unknown
- Could become the canonical model for temporal and spatial scaling in a wide variety of dissipative systems

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- Brevity of $\beta \approx 1.1$ claim undermines significance of results

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Thank You!