Floquet Time Crystals

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Phys. Rev. Lett. 117, 090402 - Published 25 August 2016

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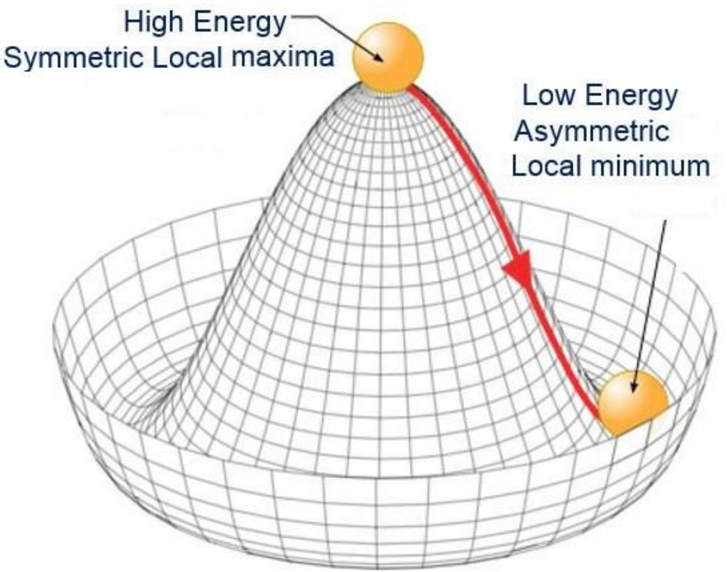
PHYS 596, Dec 1, 2016

Conventional (spatial) crystals exhibit spontaneous symmetry breaking

- The ground state of a system does not share the symmetry of the Hamiltonian
- Transition is conceptually described as a ball in a “Mexican hat” potential that breaks the symmetry around the axis by rolling down
- The Hamiltonian of conventional crystals has continuous translational symmetry, while the ground state has discrete translational symmetry

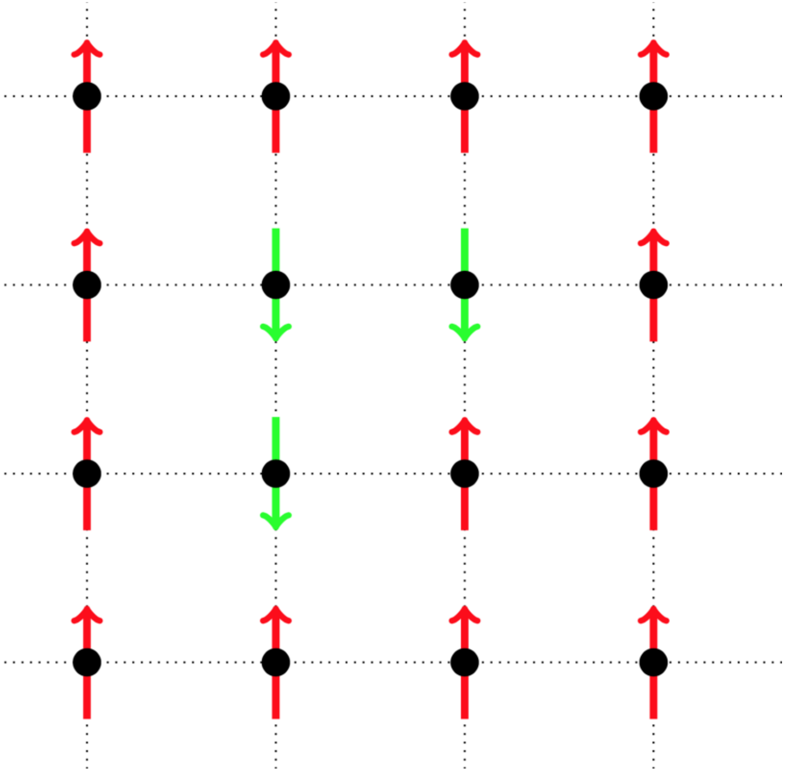


Exploration of time crystals requires a precise definition of time translation symmetry breaking

- In 2012, Frank Wilczek proposed the idea of crystals that break time translation symmetry, or “time crystals”
- In analogy with a space crystal, the ground state of a time crystal will spontaneously organize into periodic motion in time
- Wilczek considered a system that can spontaneously turn to periodic motion even in the lowest energy state (proven to be impossible by Watanabe and Oshikawa in 2015)
- In 2016, Else et al (this paper) proposed two definitions for time translation symmetry breaking in periodically driven systems and explicitly constructed a spin chain system satisfying these definitions.

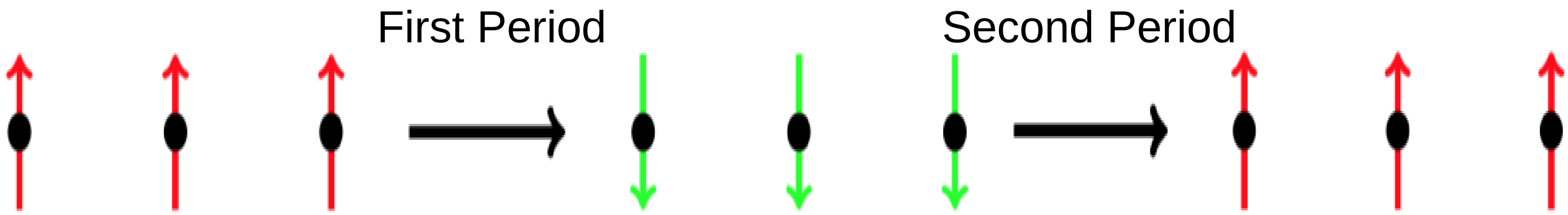
Example of Spontaneous Symmetry Breaking: The Ising Model

- Consists of a lattice with an Ising spin at each lattice site that is either up or down
- Energy comes from interactions between the spins: $H = -\sum_{i,j} J_i \sigma_i^{(z)} \sigma_j^{(z)}, \qquad J_i > 0$
- The physical ground states are spins all up or all down.
- A quantum state formed from a superposition of these is unstable to weak perturbations or interactions with the environment



Extending these ideas to Time Translation Symmetry Breaking

- The authors of this paper look at periodically driven systems with discrete time translation symmetry
- Time translation symmetry breaking occurs when the symmetry respecting “stationary states” are unstable to infinitesimal perturbations or interactions with the environment



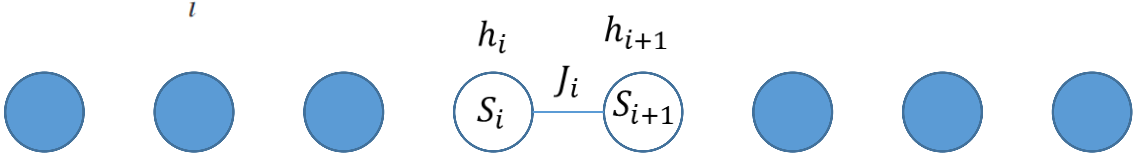
Analytical Example of TTSB proposed by the Author

Time evolution operator (Floquet operator)

$$U_f = \exp\left(-it_0 H_{\mathrm{MBL}}\right) \exp\left(it_1 \sum_i \sigma_i^x\right)$$

where

$$H_{\mathrm{MBL}} = \sum_i \left(J_i \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z + h_i^x \sigma_i^x\right)$$



- To simplify choose: 1. $t_1 \approx \pi/2 \Longrightarrow \exp(i(\pi/2)\sum_i \sigma_i^x) = \prod_i i\sigma_i^x$

  $h_i^x = 0 \Longrightarrow$ 2. $H_{\mathrm{MBL}} = \sum_i \left(J_i \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z\right)$

- The time period of H: $T = t_0 + \pi/2$

$$|\uparrow \cdots \uparrow\rangle \xrightarrow{t_1 = \pi/2} |\downarrow \cdots \downarrow\rangle \xrightarrow{t_0} |\downarrow \cdots \downarrow\rangle$$

Analytical Example of TTSB proposed by the Author

Analytical Solution of U_f and TTSB

- The eigenstates of U_f are analytic since $t_1 = \pi/2$ and $h_i^x = 0$

  $|\psi\{s_i\} >$ = $(\exp(it_0 E^-(\{s_i\})/2)|\{s_i\}\rangle \pm \exp(-it_0 E^-(\{s_i\})/2)|\{-s_i\}\rangle)/\sqrt{2}$

  with eigenvalue $\pm\exp(it_0 E^+(\{s_i\}))$

  where $E^-(\{s_i\}) = \sum_i (h_i^z s_i)$ and $E^+(\{s_i\}) = \sum_i (J_i s_i s_{i+1})$

- $\sigma_k^z|\{s_i\}\rangle = s_k|\{s_i\}\rangle$
- Period of Hamiltonian : T

  Period of the eigenstates of Hamiltonian $|\{s_i\}\rangle$: 2T

  Period of the eigenstates of Time evolution operator $|\psi\{s_i\} >$: T
- means that in this system TTSB occurs.

TTSB occurs in a region of parameter space (TTSB phase)

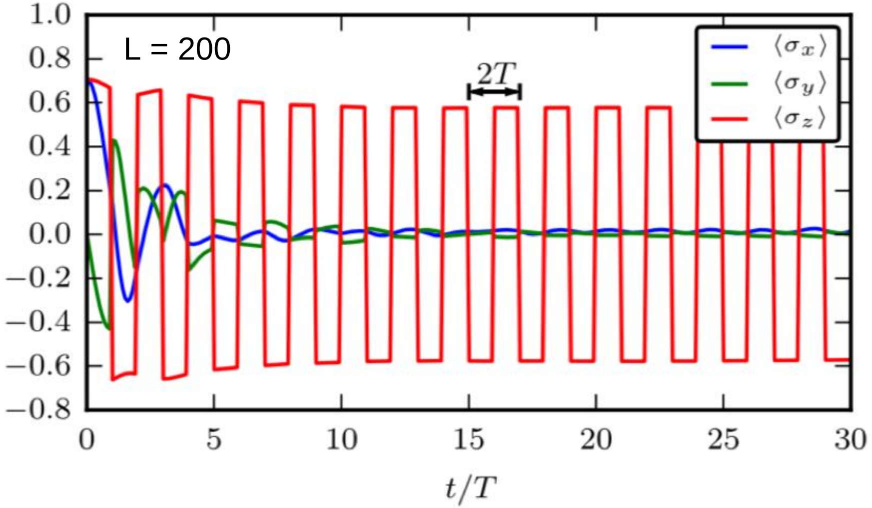
Two classes of perturbations:

1. Deviations of t_1 from pi/2
2. Non-zero h → h = 0.3

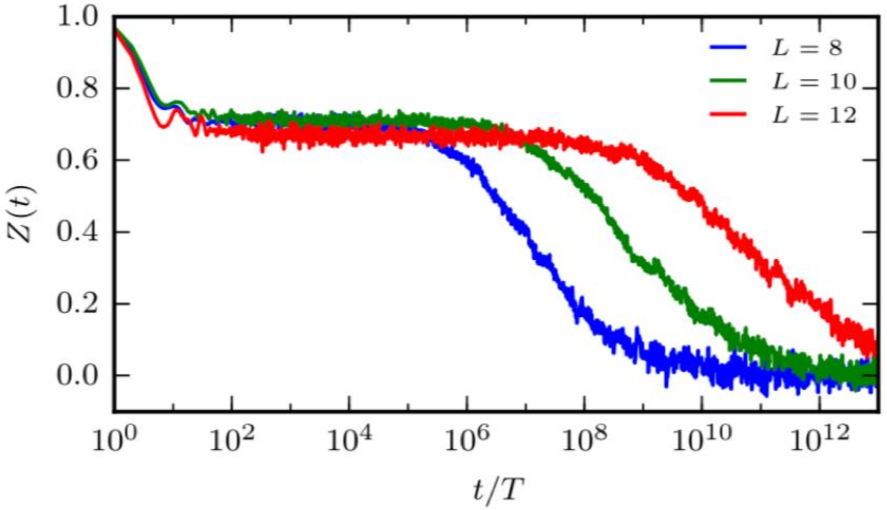
$$U_f = \exp\left(-it_0 H_{\mathrm{MBL}}\right) \exp\left(it_1 \sum_i \sigma_i^x\right)$$

$$H_{\mathrm{MBL}} = \sum_i \left(J_i \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z + h_i^x \sigma_i^x\right),$$

TTSB still occurs under small perturbation:



Simulation to much later times shows that the lifetime of TTSB diverges in system size:



Conclusion & Impact of the Paper

- First paper to put forward two consistent equivalent definitions of TTSB which can be used to test for TTSB in physical systems.
- First to show these TTSB states are robust with respect to small perturbations and show that these states are stable in the thermodynamic limit.
- The first experiment (Zhang et al., 2017) which physically demonstrated TTSB did so using the blueprint laid out in the paper.
- Implications for the definitions of many body localised systems.
- Cited 48 times (published) 97 times (published and pre-published)

Critique

The good

- Analytic and numerical analysis well supported the author's claims on TTSB.
- Offered a conceptually simple blueprint for an experiment to verify their conclusions.

The bad

- Paper is entirely focused on TTSB in MBL spin chain systems, fails to mention any possible generalisations or extensions.
- Authors determine that TTSB exists for a phase but do not provide a phase diagram.
- Their “Implication of TTSB” section feels underwhelming, only mentions a consequence for the the definition of many body localised systems.
- Published in PRL but presumed a large amount of domain specific knowledge from multiple fields.

Summary

The authors:

- Consistently formalise TTSB.
- Provide a model which explicitly shows TTSB.
- Give numerical and analytical evidence for the existence of a stable phase exhibiting TTSB.
- Provide a blueprint for an experiment to demonstrate TTSB, which has since been performed.

For further reading on the subject:

Sacha K, Zakrzewski J. 2017. *Time crystals: a review*. arXiv:1704.03735