Transport in Out-of-Equilibrium XXZ Chains: Exact Profiles of Charges and Currents

Bertini et al., PRL 117, 207201 (2016)

Julian May-Mann, Yutao Ma, Rong Nie, Kevin Ly
Solving Many-Body Systems

- For systems with small parameters perturbation theory can be used (QED)
- However there are many systems that do not even have a perturbative solution (QCD, Hubbard Model, XXZ model)
- However in 1 spatial dimension there are actually a large class of solvable models called integrable models (XXZ model, Heisenberg spin $\frac{1}{2}$ chain)

$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}^\dagger_{i,\sigma} \hat{c}_{j,\sigma} + \hat{c}^\dagger_{j,\sigma} \hat{c}_{i,\sigma}) + U \sum_{i=1}^{N} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$
Integrable Models

● Actual definition is pretty complicated, but here are some key properties of the scattering processes of these systems:
  ○ Particle number conservation
  ○ Momentum conservation conserved during scattering
  ○ Scattering can be factored into products of 2 particle scattering

● We can solve integrable systems using the Bethe Ansatz
Bethe Ansatz

- Basic process:
- A given theory has an S matrix which characterizes all scattering processes
- The Integrability of the theory places constraints on the S matrix
- Using these constraints you can construct equations “Bethe Equations”
- The solutions to the “Bethe Equations” give the eigenvalues of the theory

\[
S_{fi} \equiv \lim_{t \to +\infty} \langle \Phi_f | \Psi(t) \rangle \equiv \langle \Phi_f | S | \Phi_i \rangle, \\
\left( \frac{\sinh (\lambda_j + i\frac{\gamma}{2})}{\sinh (\lambda_j - i\frac{\gamma}{2})} \right)^L = \prod_{l \neq j}^N \left( \frac{\sinh (\lambda_j - \lambda_l + i\gamma)}{\sinh (\lambda_j - \lambda_l - i\gamma)} \right)
\]
Problems with Bethe Ansatz

- Because they are exactly solvable Integrable Models have high theoretical value, and can be used to understand more realistic systems.
- However, a large number of experimental systems we would like to study are incompatible with the Bethe Ansatz.
- For example, spatially inhomogeneous systems in 1 dimension:
  - These systems can be realized in quasi 1d cold atom/AMO systems.
- As a result, to study 1d inhomogeneous systems, we need other tools.
Model: XXZ spin-$\frac{1}{2}$ chain

Features:

- Diagonalizable by the Bethe ansatz
- In the thermodynamic limit the states can be characterized by “root densities,” which are distributions of quasiparticles
- Local observables in long times can be described by a locally quasistationary state (LQSS), via dephasing

Goal: Put together two globally different chains and try and get analytical* result for charges and currents

\[
H = J \sum_{\ell=1}^{L} (s_{\ell}^x s_{\ell+1}^x + s_{\ell}^y s_{\ell+1}^y + \Delta s_{\ell}^z s_{\ell+1}^z),
\]
Kinetic theory

\[
\langle q \rangle_{x,t+\delta t} - \langle q \rangle_{x,t} = \int d\tilde{x}\left( \Delta^q_{\tilde{x}\to x,t} - \Delta^q_{x\to \tilde{x},t} \right),
\]

Steps:

- Observe how charge density changes at a given point
- Use what is known about excitations and LQSS to rewrite RHS
- Relate the motion of charges to root densities
Equations of continuity

Final result:

$$\partial_t \rho_{\xi,k}(\lambda) + \partial_x (v_{\xi,k}(\lambda) \rho_{\xi,k}(\lambda)) = 0.$$  

$$\langle \rho | i e[Q] | \rho \rangle \sim \sum_k \int d\lambda q_k(\lambda) v_k(\lambda) \rho_k(\lambda),$$

To include inhomogeneity:

$$\vartheta_{\xi,k}(\lambda) = \Theta_H(v_{\xi,k}(\lambda) - \xi)(\vartheta_k^L(\lambda) - \vartheta_k^R(\lambda)) + \vartheta_k^R(\lambda).$$
Charge currents defined by
\[ j_{\ell+1}(Q) - j_\ell(Q) = i[q_\ell, H] \]

Symbols are predictions

Full lines are numerical data from matrix product density operator (MPDO)

Unquestionably perfect agreement: the discrepancies are smaller than the MPDO
Two chains prepared at different temperatures and then joined together.

- Full lines are predictions.
- Symbols are MPDO data.

Numerical data are in excellent agreement with the analytical predictions.

Strongly suggests that the solution characterizes the state of the system at late times.
Joining together two globally different pure states

Lines are predictions

Symbols are time-evolving block decimation (TEBD) data

The vertical dotted-dashed lines denote the light-cone edges

The agreement is fairly good
Domain-wall initial state
\[|\uparrow\ldots\uparrow\rangle \otimes |\downarrow\ldots\downarrow\rangle\]

Lines are predictions

Symbols are numerical data

Predictions are fairly good
Critiques

Pros:

- From the continuity equation, the paper obtains analytic result characterizing the dynamics of integrable XXZ chain
- The solution agrees well with numerical simulations within numerical accuracy
- The approach can in principle be applied to other integrable models

Cons:

- The paper does not describe the steps of their construction in detail.
- There is no new physics; the paper just proposes a new perspective on existing model
Citation Analysis

Total number of times cited : 43
Citation Analysis

An example of papers that cite the current paper

- It has been Cited 11 times since it was published
- It uses hydrodynamic approach to efficiently compute Drude weights
Summary

- A continuity equation describing the dynamics of integrable XXZ chain is developed.
- The validity of the equation is confirmed by comparing the analytic solutions with numerical results.
- The construction can be applied to other integrable models.