Quantized electric multipole insulators


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Topological phases of matter give rise to quantized physical quantities

- Examples are
  - Charge polarization in crystals (1D) \( P_1 = -\frac{e}{2\pi} \int_{\text{BZ}} \text{Tr}[A] \)
  - Hall conductance (2D) \( \sigma_{xy} = -\frac{e^2}{2\pi \hbar} \int_{\text{BZ}} \text{Tr}[dA + iA \wedge A] \)
  - Magnetoelectric polarizability (3D) \( P_3 = -\frac{e^2}{4\pi \hbar} \int_{\text{BZ}} \text{Tr}[A \wedge dA + \frac{2i}{3} A \wedge A \wedge A] \)

- \( A \) is the Berry phase vector potential
- \( \sigma_{xy} \) and \( P_3 \) are natural mathematical extensions of the \( P_1 \) Berry phase expression
There is no generalization of the Berry phase expression for quantized polarization to higher electric multipole moments.

In the classical, continuous limit, multipole moments are:

- **Dipole**: \( \mathbf{p}_i = \int d^3 r \rho(r) r_i \)
- **Quadrupole**: \( q_{ij} = \int d^3 r \rho(r) r_i r_j \)
- **Octupole**: \( o_{ijk} = \int d^3 r \rho(r) r_i r_j r_k \)

Goal: construct crystalline insulator models exhibiting quantized quadrupole and octupole moments.

Bulk quadrupole (A) and octupole (B) moments and the induced moments: surface quadrupoles, edge polarization, corner charges.
The minimal components for a quadrupole insulator are 4 (2 occupied) bands and reflection symmetries $M_x, M_y$

- $\gamma, \lambda$ are hopping parameters
- Complex phases emulate flux quanta piercing each plaquette
- Topological: $|\gamma/\lambda| < 1$
  - Quantized edge polarization
    - $P = \pm e/2$
  - Quantized corner charge
    - $Q = \pm e/2$
- Trivial: $|\gamma/\lambda| > 1$
  - No $P$ or $Q$

Numerical simulations confirm quantized polarization and corner charges

- Corner states located at boundary of the boundary
- Exponential decay and sudden disappearance indicate topological origin
- Edge polarization also quantized, but there is no nice picture

Berry Phases in Quantum Mechanics

- Movement along curved paths can result in an acquired (geometric) phase

- Berry Phase $\theta$ : QM geometric phase
  - $e^{-i\theta} = \langle u_N | u_{N-1} \rangle \langle u_{N-1} | u_{N-2} \rangle \cdots \langle u_2 | u_1 \rangle \langle u_1 | u_0 \rangle$
  - $|u_N\rangle$ is the orbital wavefunction

- Crystal momentum space is a torus, allowing nontrivial loops

- Berry phase is equivalent to location of electrons in the unit cell (polarization) Zak (1989)

- How to generalize to multiple bands (quadrupole/octupole moments)?
Wilson Loops are a generalization of the Berry phase integral in multiple band systems

• Wilson loops over 2D energy bands give 1D bands of Wannier centers (electron positions)

• Wilson loops on 1D Wannier bands give polarizations of each Wannier center

• Each electron contributes opposite polarizations

• Quantized as 0 or $\pm e/2$

Benalcazar, Bernevig, Hughes, Science, 357(6346), (2017).
Cold atoms in optical lattices could realize a quantized quadrupole moment

- A 2D superlattice is created using orthogonal standing optical waves
- X-hopping inhibited with a magnetic gradient
- X-hopping is restored with a complex phase via laser beams
- This phase mimics a $\pi$ flux per plaquette

Benalcazar, Bernevig, Hughes, Science, 357(6346), (2017).
Bragg transitions between plane-wave BEC states can also model the quadrupole.

- Local atomic orbitals -> BEC planewaves
- Hopping -> 2-photon transitions
- Acousto-optic modulators control hopping amplitude and phase
  - Allows effective flux per plaquette
- Has only been achieved in 1D so far

Recent advancements in photonics allows this model to be realized with laser etched waveguides

- Model can be replicated with arrays of parallel waveguides
- Orbitals -> Waveguides
- Hopping -> Evanescent Tunneling
- New negative couplings allow complex hopping
- Topology can be confirmed by illuminating a corner of the lattice
This paper is of extremely high quality overall

• Good:
  • The paper is reasonably accessible
  • The figures are very illustrative and aid in understanding
  • The work represents a significant advancement in understanding of topology and provides a new framework for calculating invariants (nested Wilson loops)
  • The predictions have been verified in multiple experiments
    • arXiv:1708.03647 (topoelectrical circuit)
    • arXiv:1710.03231 (microwave circuit)

• Bad
  • The supplement is enormous compared to the core paper, but that is nearly unavoidable
Citation Analysis
Summary

• Authors wanted to extend the quantum theory of polarization to higher multiple moments

• Designed Hamiltonians demonstrating quantized quadrupole and octupole moments

• Discovered new topological paradigm (nested Wilson loops)

• Provided experimental proposals for physical realizations of quantized quadrupole insulators