## Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott, et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 061102 (2016), arXiv: 1602.03837.

Alexander Beach, Bora Basa, Carina Baker, Shraddha Agrawal

December 7, 2018

**ILLINOIS** 



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  - The (strong) principle of equivalence implies a correspondence between spacetime geometry and gravity: *Gravity*~ *Spacetime curvature due to matter*

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#### Einstein's Equation

Geometrically, spacetime is the data  $(M, g_{\mu\nu})$  with causal constraints (Lorentzian manifold). It is dynamical in the sense that  $g_{\mu\nu}$  obeys the field equation

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\mathsf{Tr}_{g}R_{\mu\nu}+g_{\mu\nu}\Lambda=8\pi\,T_{\mu\nu},$$

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• The solutions of Einstein's equations correspond to the possible configurations of the Universe at large length scales.

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  - It is generated by accelerating mass just as an electromagnetic vector potential is generated by accelerating charge.

[Tiec and Novak, 2017] **PHYS 596** 

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Group	1
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#### ILLINOIS

# • So a gravitational wave is just a wave in spacetime!

- So a gravitational wave is just a wave in spacetime!
  - Predicted by Einstein in 1916, **but** the amplitude is so small that "detecting them is like measuring the distance to a star ten light-years away with a precision equivalent to the diameter of a strand of hair"

-Royal Swedish Academy of Sciences

## • Gravitational Waves – First Experiments

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  - In the 1970's proposals for laser based interferometers were drafted
    - In the late 90's the first sets of interferometers were built, including TAMA 300, GEO 600, LIGO, and Virgo

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    - Ratio of observed to predicted orbital decay rate considering energy lost to gravitational waves is  $0.997 \pm 0.002$ [Weisberg et al., 2010]
    - $\bullet~$  In 2016 this was updated to  $0.9983\pm0.0016$  [Weisberg and Huang, 2016]

Background Experiment Analysis Future Conclusions

### • BICEP2 – Gravitational Waves?

• March 2014, BICEP 2 reported detection of B-mode primordial gravitational waves



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### BICEP2 – Gravitational Waves?

#### • It was just cosmic dust



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- Detect changes in length less of a ten-thousandth the charge diameter of a proton





#### Photodetector

Sensitivity of Detector





## L1 Livingston, LA

## H1 Hanford, WA



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- Collision Parameters
  - Redshift of 0.09
  - Primary black hole mass 36  $M_{\odot}$ 
    - Spin 0.32
  - Secondary black hole mass of 29  $M_{\odot}$ 
    - Spin 0.44
  - Final black hole mass of 62  $M_{\odot}$ 
    - Spin 0.67

- Collision Waveform
  - Model merger event using combination of analytic and numerical techniques



#### • Raw LIGO Data



#### Raw LIGO Data • How do they know this was 2 black holes? Hanford detector $5.\times 10^{-19}$ 0 -5.×10<sup>-19</sup> $h(t) = 1. \times 10^{-18}$ Livingston detecto $-1.5 \times 10^{-18}$ $-2.\times10^{-18}$ $-2.5 \times 10^{-18}$ 15 -15 -10 -.5 t(sec)10

# • Binary Coalescence Search



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  - For remaining signals, the background is re-estimated without the contribution of the signal

### • Generic Transient Search

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  - Devise a searching method, using a detection statistic

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- Determine background noise

#### • Statistical Analysis – Numerically Fitting Data

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  - Each site has seismometers, accelerometers, magnetometers, microphones, radio receivers, weather sensors, ac power line monitors, and cosmic ray detectors

## • Further Developments

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  - Improving the match filtering process
    - Better detection statistics
    - Expecting to see much higher frequency of events in the future


- Future Plans
  - In 2034 the European Space Agency plans to begin launching a Laser Interferometer Space Antenna (LISA)
    - Vacuum of space is better than vacuum achievable in LIGO
    - Almost no noise from Earth
    - The interferometer arms can be much larger than on Earth



- Future Plans
  - Fermilab E-990 'Holometer'
    - Most sensitive interferometer in the world
    - Meant to detect change in space-time due to quantum fluctuations
    - https://holometer.fnal.gov/faq.html#logo

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    - This correlation is maximised for both the signal and the noise, using the 6.9 ms time lag of the LIGO paper
- This paper is excellent in all other respects
  - Well written and unambiguous
  - Careful and thorough
  - Great example of international collaboration





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  - · Carefully isolated from environmental noise

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    - Binary coalescence search looks for gravitational waves from binary systems in a specific mass range. Found the signal with false alarm probability of 5.1  $\sigma$ .
  - Numerical models: mass and spin determined independently from the early stage and late stage agree.

- Why care?
  - Last piece of GR
  - New kind of astronomy

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▶ What it sound like though???

