High Dimensional Quantum Teleportation

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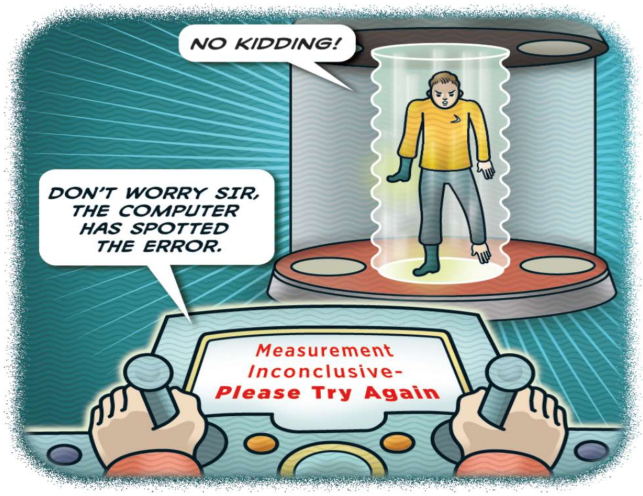
Phys. Rev. Lett. ***123***, *070505 (2019)*



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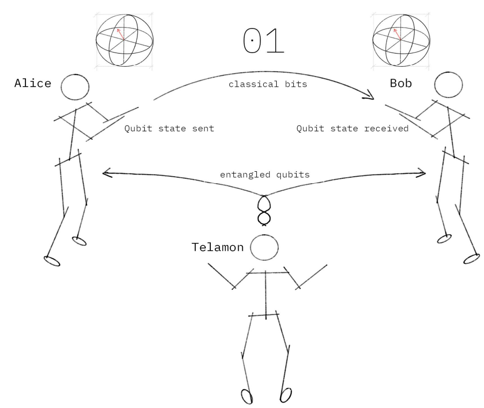


Source: https://qiskit.org/

Outline

- The benefits of HD quantum teleportation
- 2D Quantum teleportation
- 3D teleportation methodology
- Experiment setup using paths of photons
- Measurement results interpretation
- Citation analysis

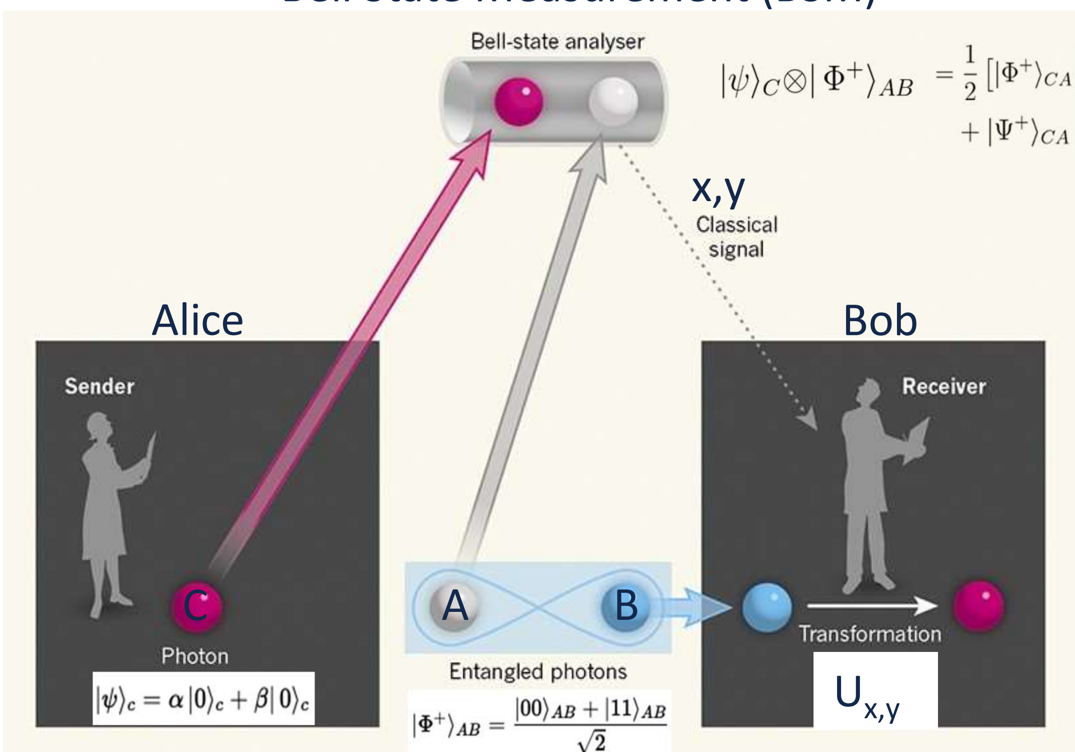
The Benefits of High-Dimensional Quantum Teleportation

- **Quantum teleportation:** transferring unknown quantum states without physical transmission.
- **High-dimensional (HD) entanglement:** has advantages such as higher capacity, noise resilience and more efficient quantum simulations.
- Previous work limited to two-dimensional subspaces (qubits) and coherent control of a single-particle HD state.

This paper proposed a scheme for HD photonic quantum teleportation and demonstrated an example in 3D.

Quantum Teleportation in 2D

Bell State Measurement (BSM)



$$|\psi\rangle_C \otimes |\Phi^+\rangle_{AB} = \frac{1}{2}\left[|\Phi^+\rangle_{CA} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + |\Phi^-\rangle_{CA} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B) + |\Psi^+\rangle_{CA} \otimes (\alpha|1\rangle_B + \beta|0\rangle_B) + |\Psi^-\rangle_{CA} \otimes (\alpha|1\rangle_B - \beta|0\rangle_B)\right]$$

Source: Fig1, Quantum-teleportation experiments turn 20, Nature 552, 42-43 (2017)

HD Quantum Teleportation is challenging

- N^2 Bell states for an N-level bipartite system
- Hard to implement the BSM by using linear optics in practice.[1][2]
- This paper utilized N-2 ancilla and N-N multiport to achieve HDBSM.

[1] H. Weinfurter, Europhys. Lett. 25, 559 (1994).
[2] J. Calsamiglia, Phys. Rev. A 65, 030301(R) (2002).

Methodology for N-Dimensional Quantum Teleportation

Alice wants to teleport a quNit to Bob: $|\phi\rangle_{\mathrm{a}} = \sum_{j=0}^{N-1} \alpha_j |j\rangle_{\mathrm{a}}$

- A shared initial entangled state: $|\psi\rangle_{\mathrm{bc}} = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} |j\rangle_{\mathrm{b}} |j\rangle_{\mathrm{c}}$

- Alice performs a N-dimensional Bell-State Measurement to a and b:

$$|\psi_{nm}\rangle_{\mathrm{ab}} = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{i2\pi jn/N} |j\rangle \otimes |(j+m) \bmod N\rangle,$$

- After receiving results {m, n} from Alice, Bob applies a unitary transformation to c:

$$U_{nm} = \sum_{k=0}^{N-1} e^{i2\pi kn/N} |k\rangle \otimes \langle (k+m) \bmod N|$$

Steps for Three Dimensional Teleportation

1. Decompose quTrits a, b, c into 3-d Bell state basis:

$$|\phi\rangle_a |\boldsymbol{\psi}_{00}\rangle_{bc} = |\boldsymbol{\psi}_{00}\rangle_{ab} (\alpha_0|0\rangle_c + \alpha_1\omega|1\rangle_c + \alpha_2|2\rangle_c)/3 +$$

$$|\boldsymbol{\psi}_{10}\rangle_{ab} (\alpha_0|0\rangle_c + \alpha_1\omega^2|1\rangle_c + \alpha_2\omega|2\rangle_c)/3 +$$

$$|\boldsymbol{\psi}_{11}\rangle_{ab} (\alpha_0|0\rangle_c + \alpha_1\omega|1\rangle_c + \alpha_2\omega^2|2\rangle_c)/3 +$$

…

2. Alice performs a projective measurement on a, b:

$$|\boldsymbol{\psi}_{10}\rangle_{ab} (\alpha_0|0\rangle_c + \alpha_1\omega^2|1\rangle_c + \alpha_2\omega|2\rangle_c)$$

3. Bob applies a corresponding unitary transformation to c:

$$U_{10}(\alpha_0|0\rangle_c + \alpha_1\omega^2|1\rangle_c + \alpha_2\omega|2\rangle_c) = \alpha_0|0\rangle_c + \alpha_1|1\rangle_c + \alpha_2|2\rangle_c$$

$$= |\boldsymbol{\psi}\rangle_c$$

4. Compared to what Alice has initially:

$$|\phi\rangle_a = \alpha_0|0\rangle_a + \alpha_1|1\rangle_a + \alpha_2|2\rangle_a$$

$$|\phi\rangle_a = |\boldsymbol{\psi}\rangle_c$$

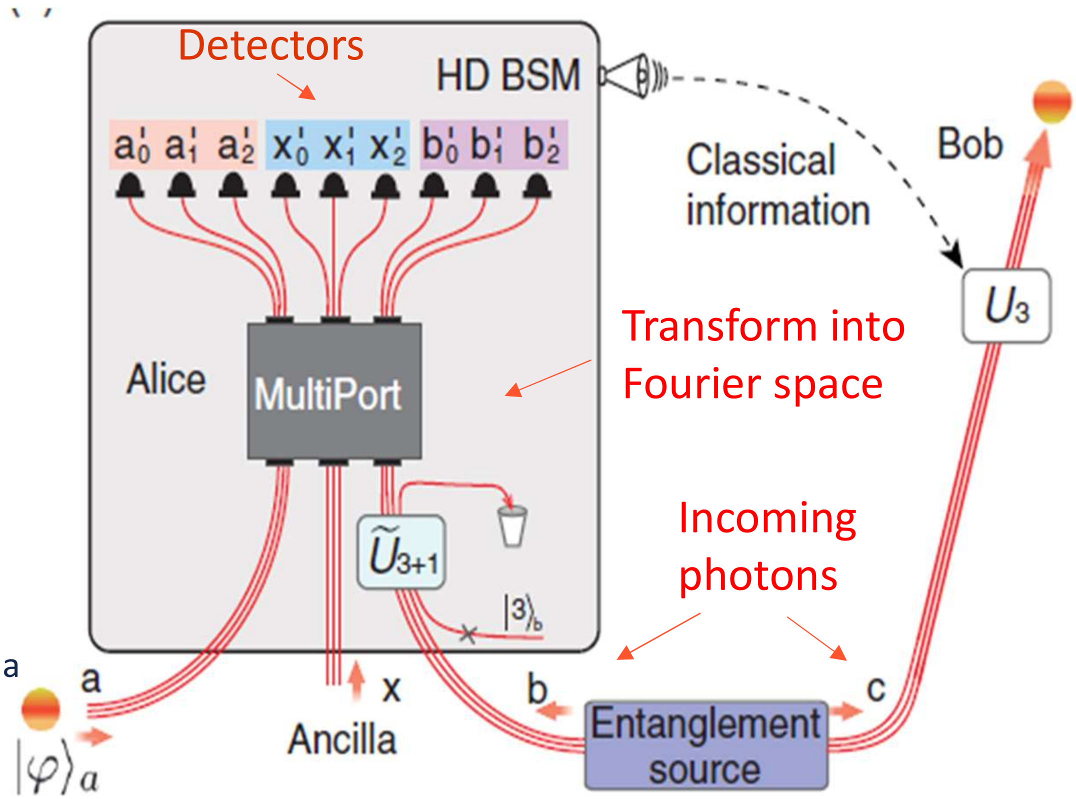
Teleportation is achieved!

Realization of the Scheme Using Paths of Photons

- $|0\rangle, |1\rangle, |2\rangle$: three paths of a photon

- Multiport beam splitter:
    - Allows quantum interference
    - Plays the role of a quantum Fourier transform

- Specific click patterns of different detectors represents one of the Bells States:
    - example: A click at a_0' indicates $|\psi\rangle_a$ has a $|0\rangle$ state dependence

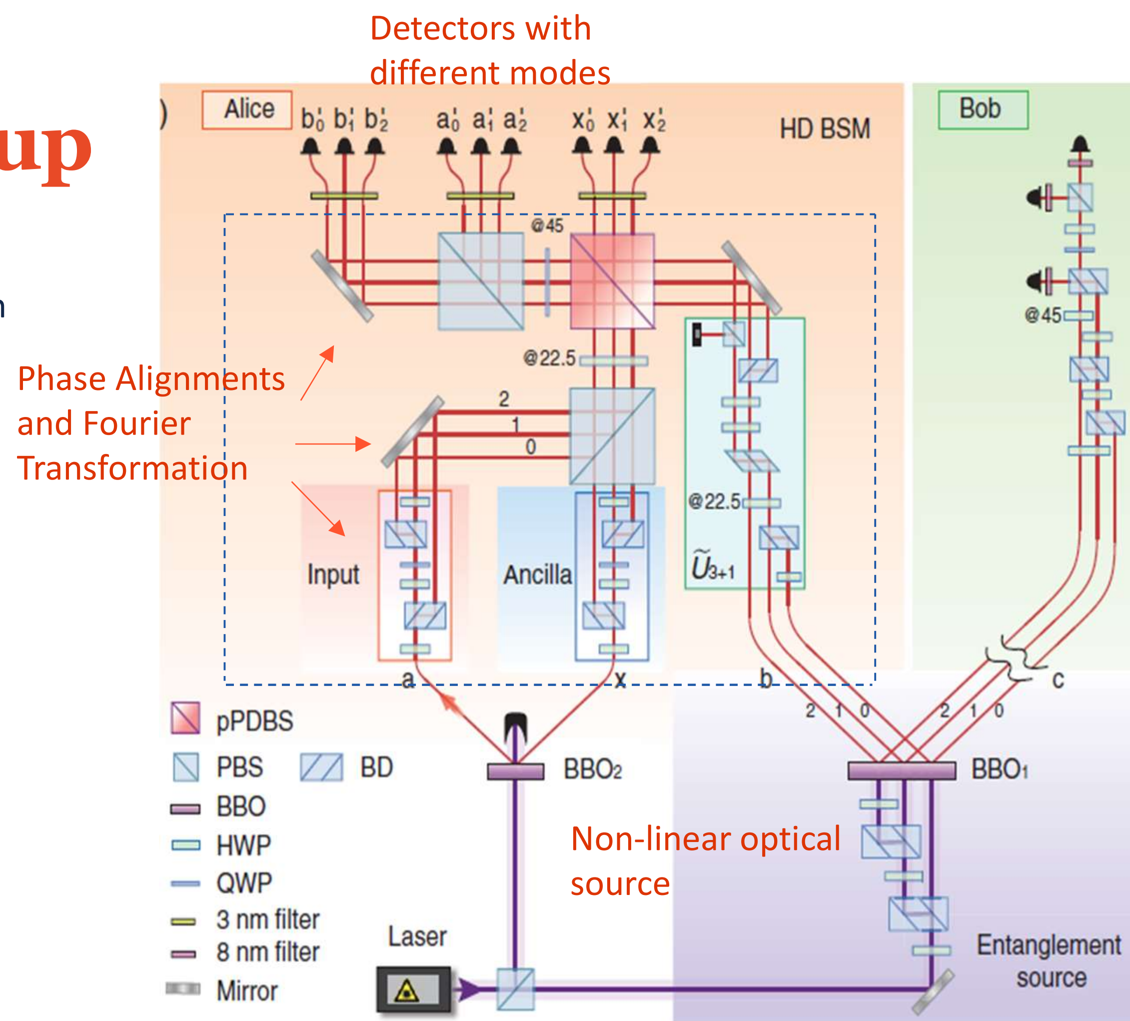


Experiment Setup

- Ultraviolet laser creates entangled photon pairs in BBO crystal

- Beam displacers (BDs), controlled by half and quarter-wave plates (HWP, QWP), prepare teleported quTrits

- A simultaneous click of three detectors in the ports $\{a'_0\ a'_1\ a'_2\}$, $\{b'_0\ b'_1\ b'_2\}$, $\{x'_0\ x'_1\ x'_2\}$ implies photon a and b goes to $|\psi_{00}\rangle$



Properties of Mutually Unbiased Bases (MUB)

Defination: Two orthornormal bases A and B of dimension d are **mutually unbiased** iff $|\langle a|b\rangle| = 1/\sqrt{d}$ for all $a \in A, b \in B$.

- Example: MUB for a qubit system

$$M_0 = \{|0\rangle, |1\rangle\}$$
$$M_1 = \left\{\frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right\}$$
$$M_2 = \left\{\frac{|0\rangle + i|1\rangle}{\sqrt{2}}, \frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right\}$$

For a qutrit system, measurements done in MUB allow **reconstruction** of the **initial density matrix**.

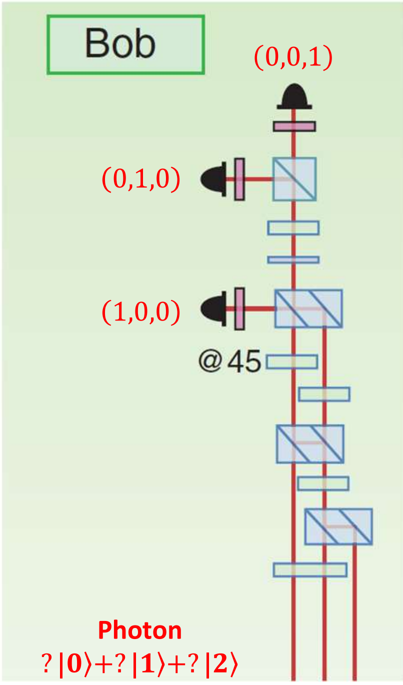
MUB define operators whose measurements yield partial (sometimes full) information of the initial state.

Measuring Qutrit State with MUB

For a qutrit system:

- 4 groups of MUB: $B^{(1)}, B^{(2)}, B^{(3)}, B^{(4)}$
- Each group has 3 bases, 12 bases total

- $B^{(1)}_{\{1,2,3\}}$: (1,0,0), (0,1,0), (0,0,1)
  - $1|0\rangle + 0|1\rangle + 0|2\rangle = |0\rangle$



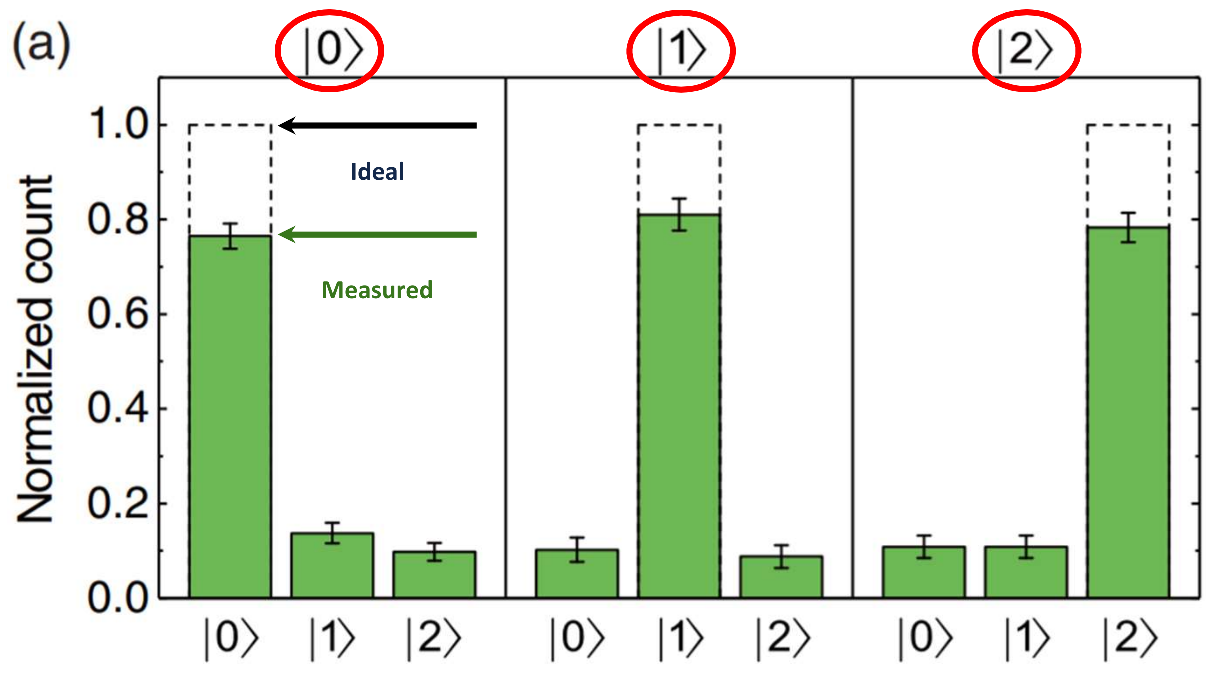
In this experiment, need to analyze the teleported photon.

- Photon in unknown state in $B^{(1)}$.
- Configure each detector to represent 1 basis.
- Gather counts – normalized probabilities – fill out density matrix.
- Photon = $\alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$

Measuring Qutrit State with MUB – Results

Wish to teleport Alice's photon $|\psi\rangle_a$ to Bob's photon $|\phi\rangle_c$

- $|\psi\rangle_a$ = $|0\rangle$, (1,0,0).
- Measure counts in $B^{(1)}$.
- Compare ideal and measured result – fidelity.
- Repeat with $|\psi\rangle_a$ = $|1\rangle$ and $|\psi\rangle_a$ = $|2\rangle$.
- Repeat for $B^{(2)}$, $B^{(3)}$, $B^{(4)}$



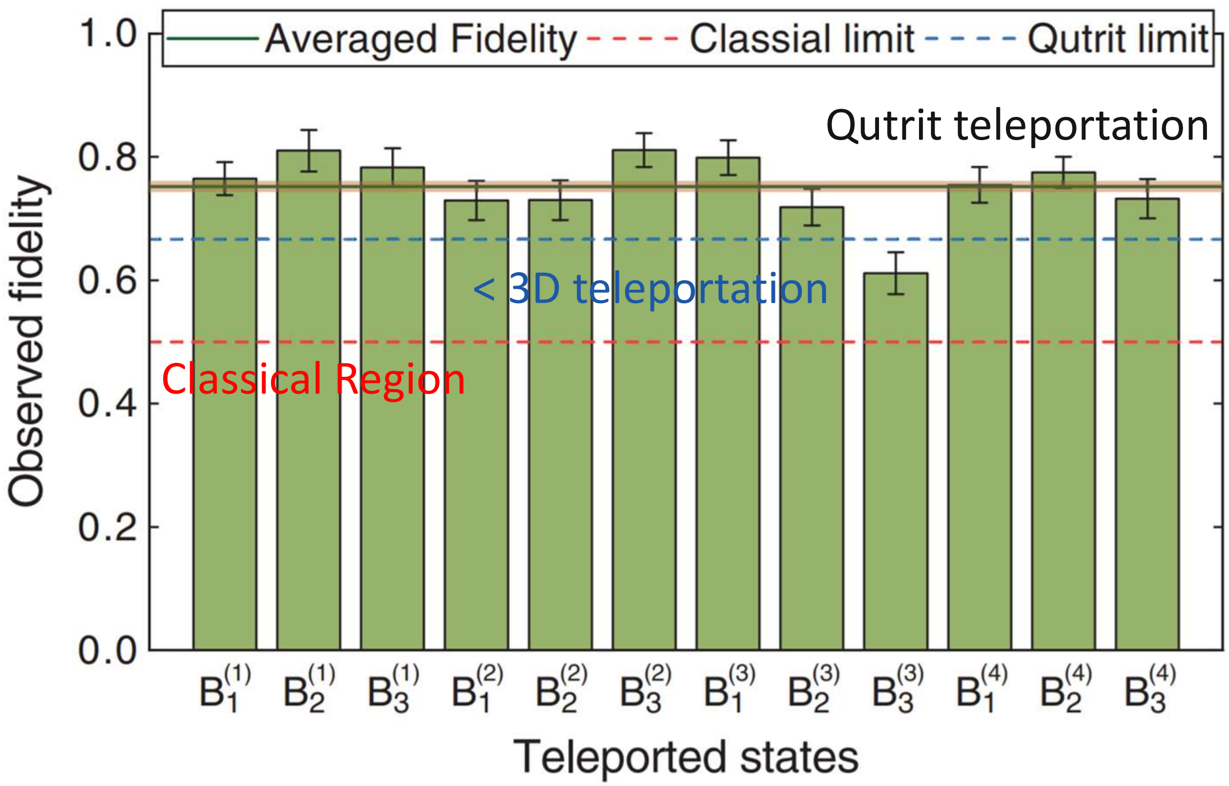
Quantum Teleportation Quality Analysis

Fidelity calculation:

- Input ideal state: $|\psi\rangle_a$.
- Measured teleported state: $\rho = (|\phi\rangle\langle\phi|)_c$.
- Fidelity $= \mathrm{Tr}(|\psi\rangle\langle\psi|\rho)$.

Fidelity measures the similarity between input state and teleported state – quality of teleportation.

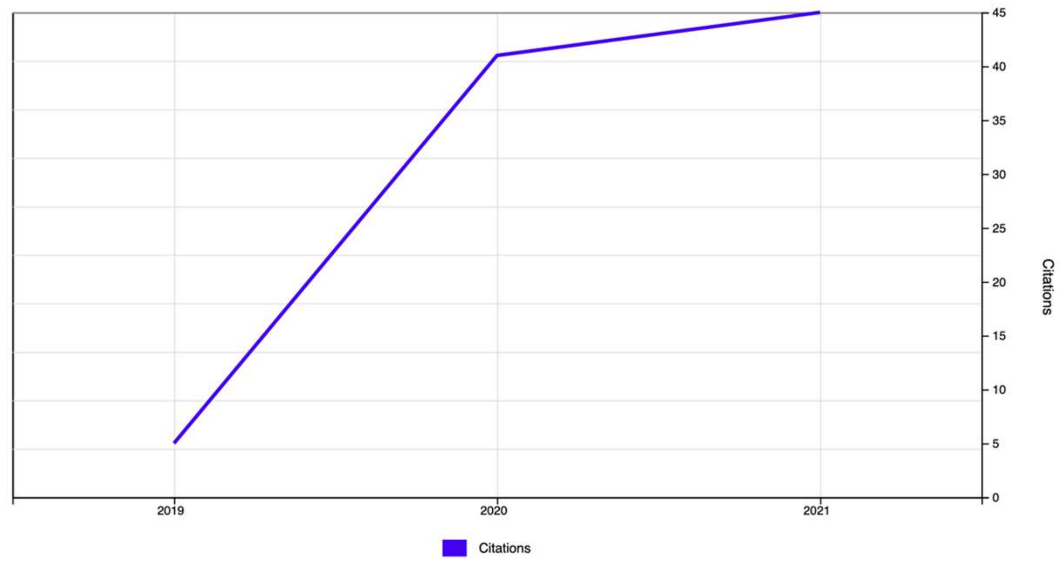
Measured fidelity exceeds qutrit limit by 9 standard deviations.



Impact of the Paper

Citations:

2019	2020	2021	Average per year	Total
5	41	45	33.33	91



Impact of the Paper

- We can see that the impact continues to grow every year. This represents the importance of this work as the field of quantum information continues to grow at a fast rate.

- Around 80% of the citations are in AMO and quantum information papers.

- The importance of teleportation is very well known since cloning quantum states is impossible. We can expect the impact of the paper to continue to grow because of its importance to the field.

Impact of the Paper

- The field has grown into many directions since the publication of this paper:

1. Reproducing high dimensional teleportation (*Experimental High-Dimensional Quantum Teleportation*, Physical Review Letters)

2. More research on the nature of entanglement (*Identifying genuine quantum teleportation,* Physical Review A)

3. Proposing real world applications based on such results (*Proposal for practical multidimensional quantum networks*, Physical Review A)

Feelings of the paper

- Incredible experimental setup with the realization of HD BSM (three-photon nine-path Hong-Ou-Mandel Interferometer).
- It's very theoretically-driven, but with typos and confusing notations in the Supplemental section
- The roles of ancillary photons performing Fourier transformation are not clear
- Correspondence between the click patterns of detectors and the Bell-State measurement lacks explanation