Significance of Electromagnetic Potentials in the Quantum Theory

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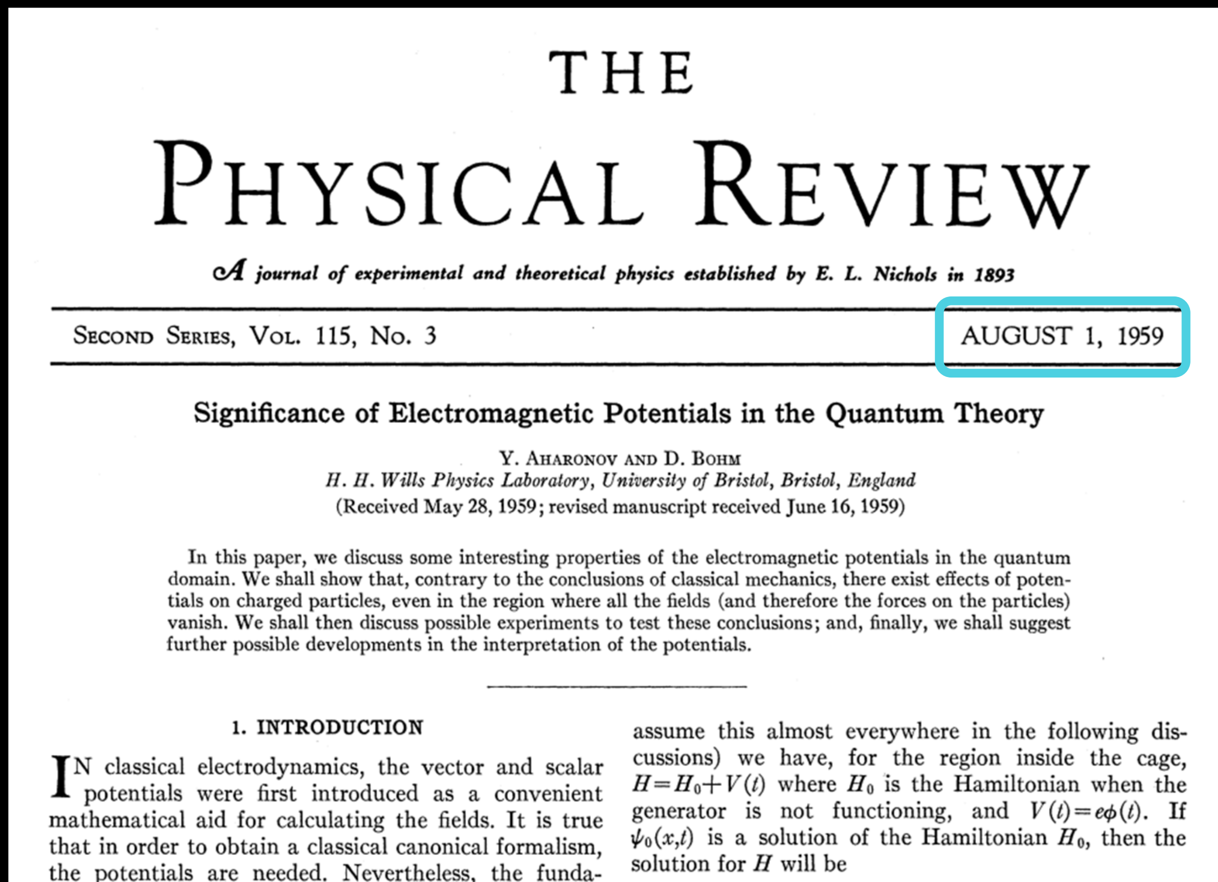
Article: Significance of Electromagnetic Potentials in the Quantum Theory by Aharonov–Bohm

Background: science in 1950s



THE

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Significance of Electromagnetic Potentials in the Quantum Theory

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In this paper, we discuss some interesting properties of the electromagnetic potentials in the quantum domain. We shall show that, contrary to the conclusions of classical mechanics, there exist effects of potentials on charged particles, even in the region where all the fields (and therefore the forces on the particles) vanish. We shall then discuss possible experiments to test these conclusions; and, finally, we shall suggest further possible developments in the interpretation of the potentials.

1. INTRODUCTION

IN classical electrodynamics, the vector and scalar potentials were first introduced as a convenient mathematical aid for calculating the fields. It is true that in order to obtain a classical canonical formalism, the potentials are needed. Nevertheless, the funda-

assume this almost everywhere in the following discussions) we have, for the region inside the cage, $H=H_0+V(t)$ where H_0 is the Hamiltonian when the generator is not functioning, and $V(t)=e\phi(t)$. If $\psi_0(x,t)$ is a solution of the Hamiltonian H_0, then the solution for H will be

Key points: Electromagnetism

- **Fields** → Physical quantities.
- The **potential** → mathematical tool to obtain the field.
- **Solenoid** → electromagnet formed by a helical coil of wire whose length is substantially greater than its diameter

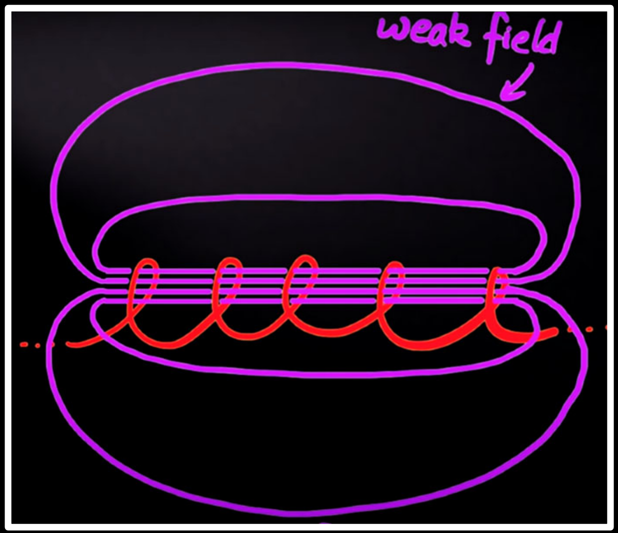
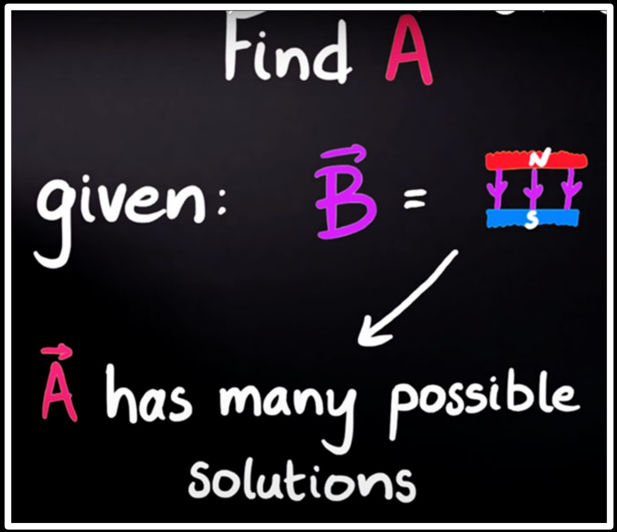


Fig. 1 - Solenoid



Key points: Locality in Physics

- An object is influenced directly only by its immediate surroundings.

- An action at one point to have an influence at another point, something in the space between those points must mediate the action.

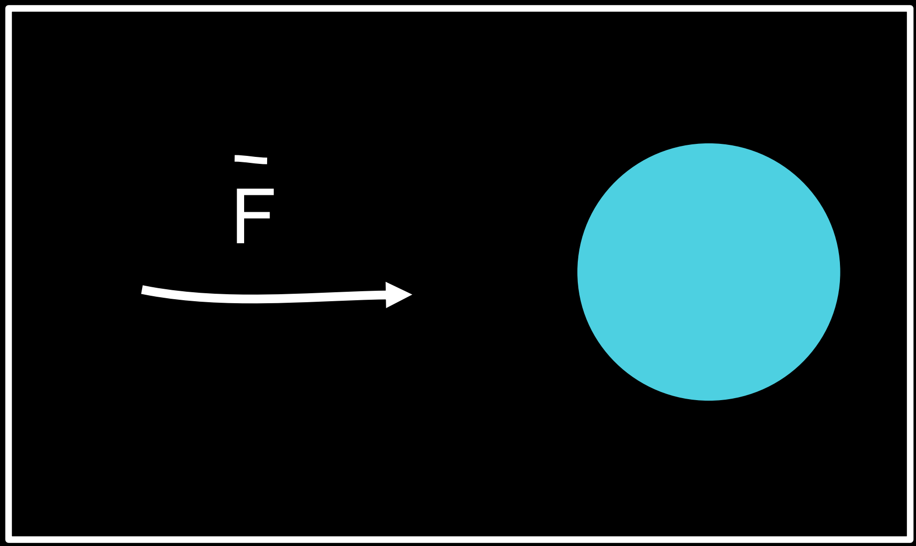
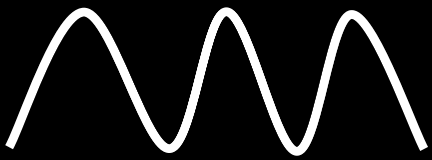


Fig. 2 - Force on object-Classical Mechanics

Key points: Quantum Mechanics

Fig. 3 - Schrödinger



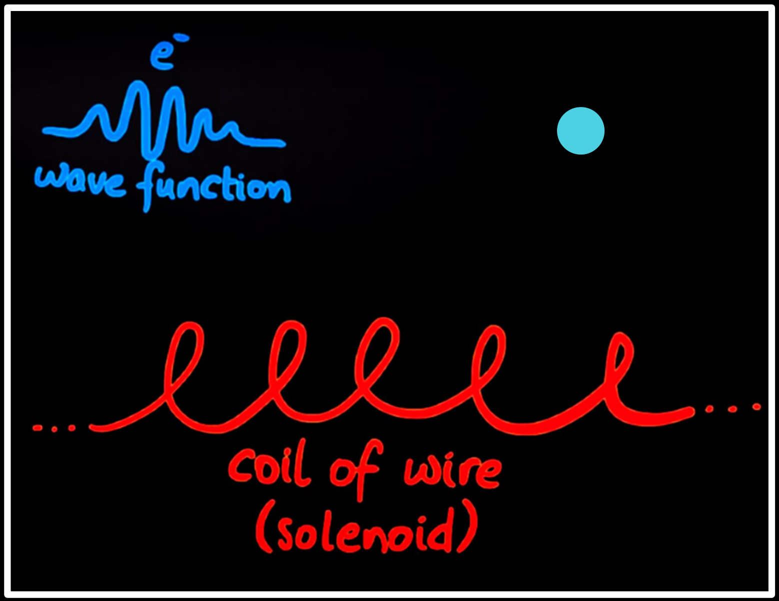
A wave function: mathematical description of the quantum state

- The canonical formalism is necessary → potentials cannot be eliminated from the basic equations
- These equations, as well as the physical quantities, are all gauge invariant; so that it may seem → the potentials have no independent significance

What is the next step?

- Study the Electromagnetic Potentials in the Quantum Mechanics domain: further interpretation of the potentials is needed in the quantum mechanics.

- Are there effects of the potentials on charged particles?



What is the article about?

- Classical canonical formalism - Potential needed but EOM has fields.

  $\partial L / \partial \Phi - \partial_\beta(\partial L / \partial(\partial_\beta \Phi)) = 0$ (E-L equation)

- QM Canonical formalism necessary - Potential needed but gauge invariant

  ($A \rightarrow A + \nabla\psi$, $\emptyset \rightarrow \emptyset - \partial\psi/\partial t$)

- No significance of Potential?!
- Does local E and B field contain full information?!
- $\psi = \psi_0 e^{(-iS/\hbar)}$ - Is phase really irrelevant?!

Methods - Hypothetical experiment 1

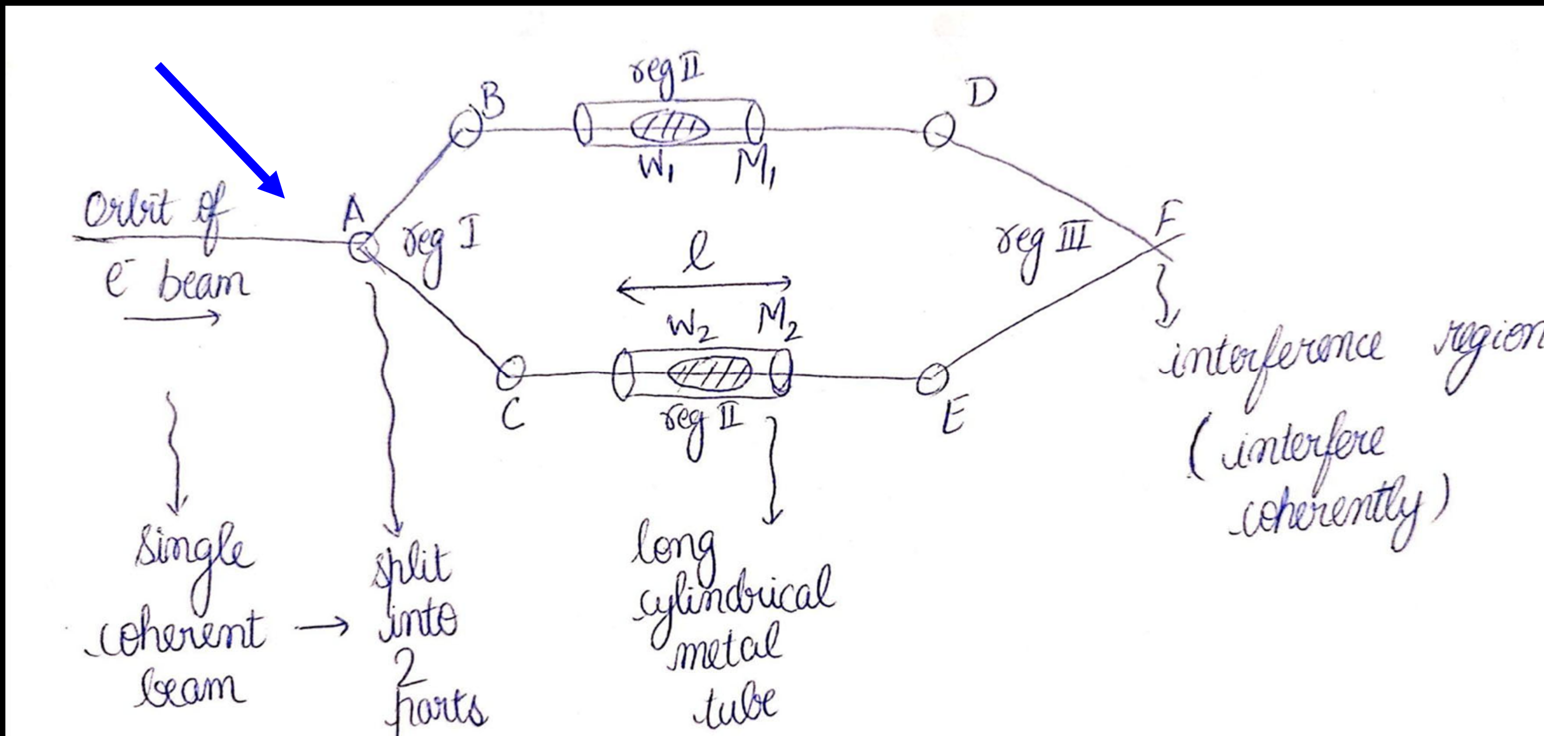
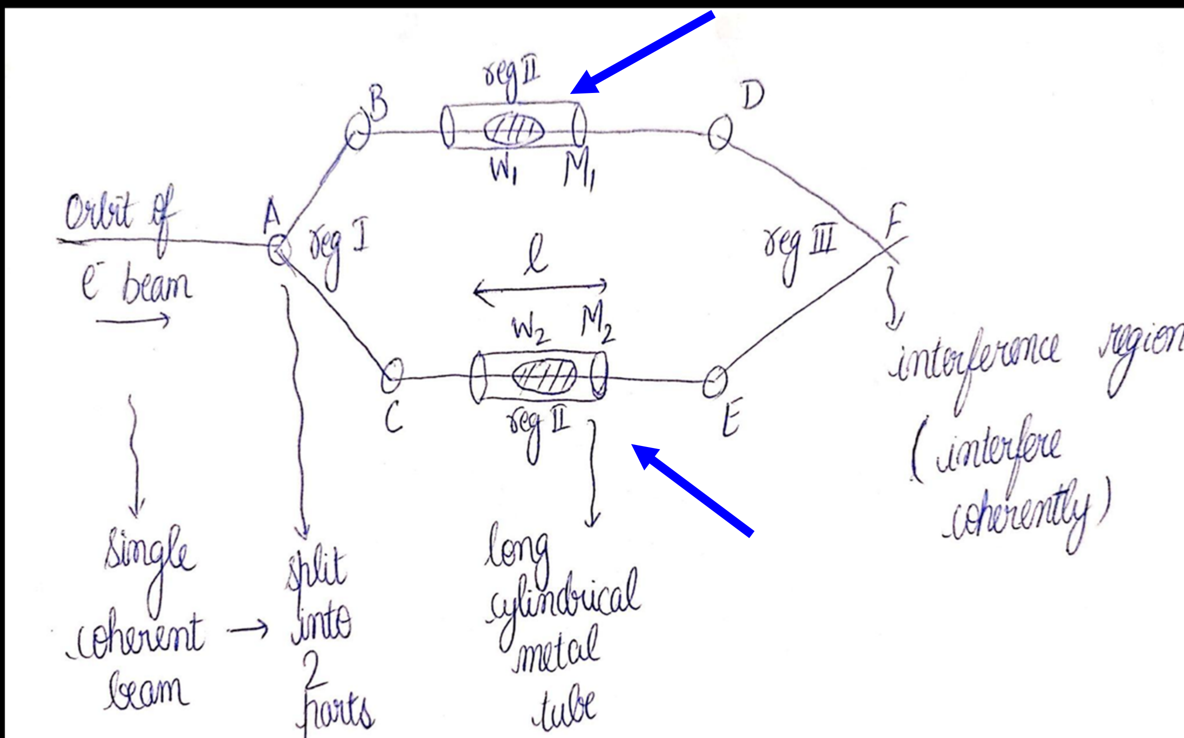


Fig.-Schematic experiment to demonstrate interference with time-dependent scalar potential. (https://link.aps.org/doi/10.1103/PhysRev.115.485)

Methods - Hypothetical experiment 1



$\psi_1 = \psi_1^o e^{(-iS_1/\hbar)}$

$S_1 = e \int \phi_1 \, dt$

$\psi_2 = \psi_2^o e^{(-iS_2/\hbar)}$

$S_2 = e \int \phi_2 \, dt$

Fig.-Schematic experiment to demonstrate interference with time-dependent scalar potential. (https://link.aps.org/doi/10.1103/PhysRev.115.485)

Methods - Hypothetical experiment 1

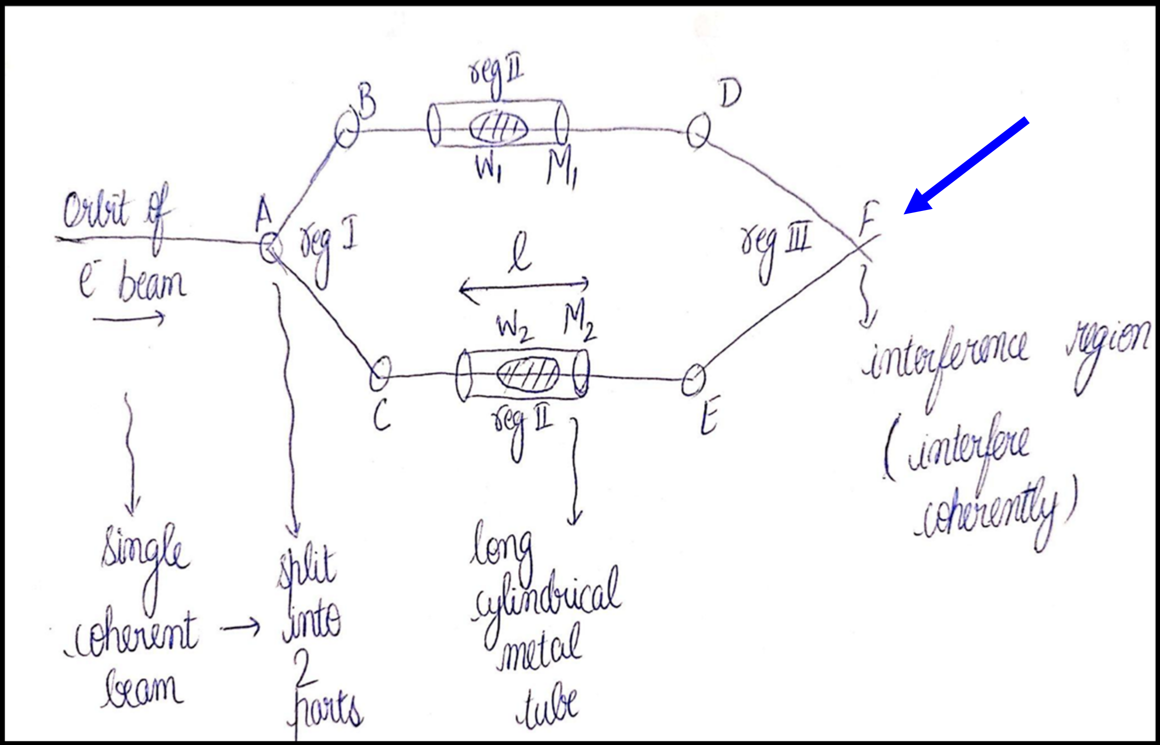


Fig.-Schematic experiment to demonstrate interference with time-dependent scalar potential. (https://link.aps.org/doi/10.1103/PhysRev.115.485)

The interference at F depends on the phase diff. $(S_1 - S_2)/\hbar$

No force has been exerted on e- but potential still had a physical effect.

Interference - Quantum mechanical in nature hence effect not seen in CM.

Relativistic Consideration

Vector 4 - potential: $A^i = (\phi, \bar{A})$

Relativistic consideration: Covariance of above demands similar results from $\bar{A}$.

Phase diff: $(e/\hbar) \oint \phi dt - (\bar{A}/c) \cdot dx$

where ϕ is evaluated at center of wave packet.

Methods - Hypothetical experiment 2

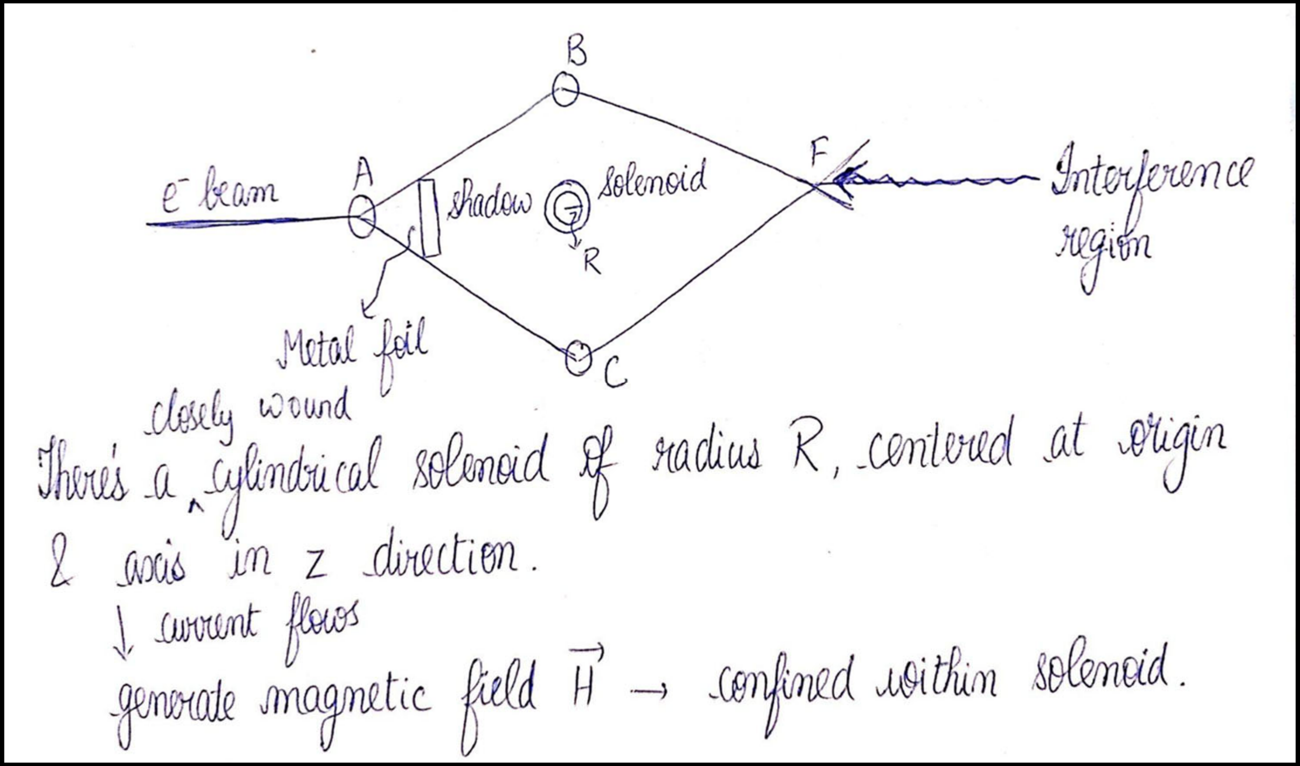


Fig - Schematic experiment to demonstrate interference with time-independent vector potential. (https://link.aps.org/doi/10.1103/PhysRev.115.485)

Methods - Hypothetical experiment 2

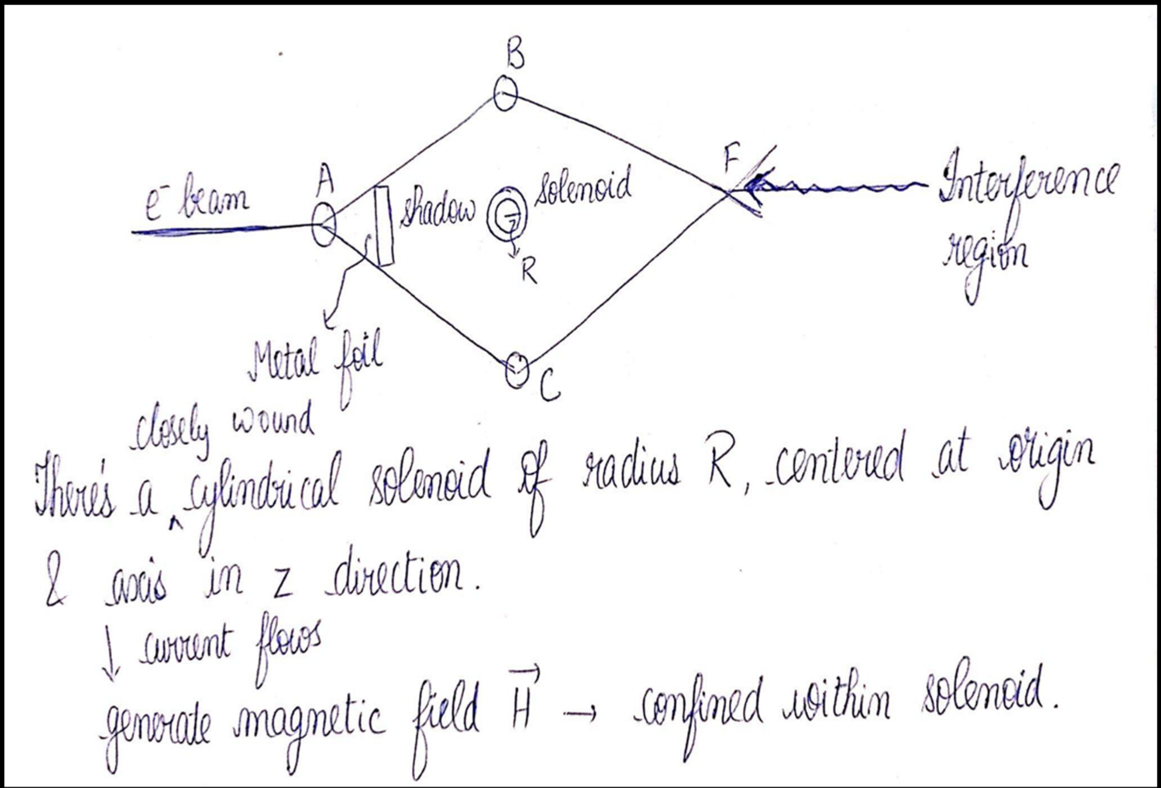


Fig - Schematic experiment to demonstrate interference with time-independent vector potential. (https://link.aps.org/doi/10.1103/PhysRev.115.485)

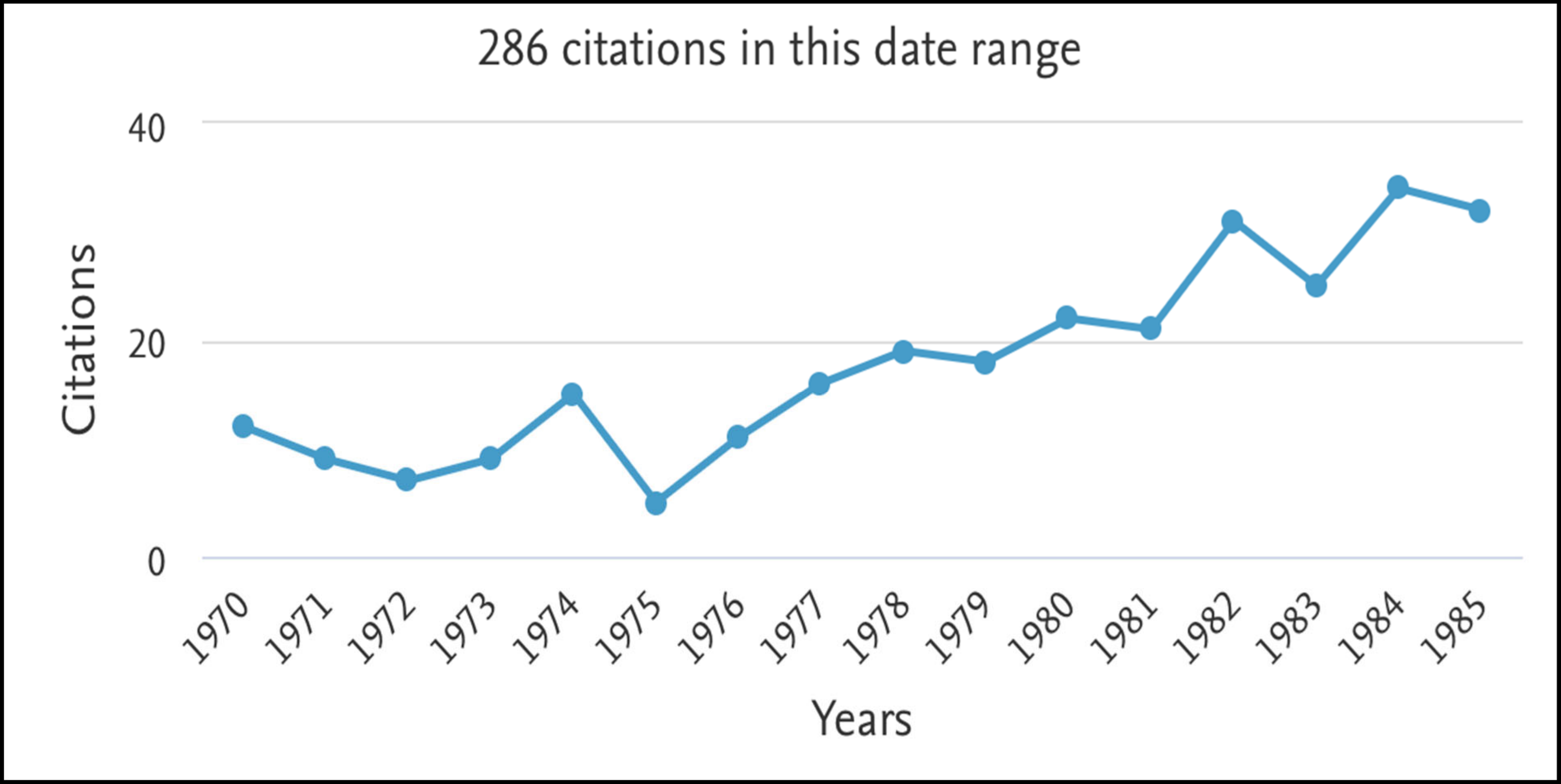
Associated phase shift of e^- wave function: $\Delta S/\hbar = -(e/c\hbar)\oint \bar{A} \cdot dx$

$\oint \bar{A} \cdot dx = \int H \cdot ds = \mathbf{\Phi}$ (total magnetic flux inside the circuit, Stoke's thm)

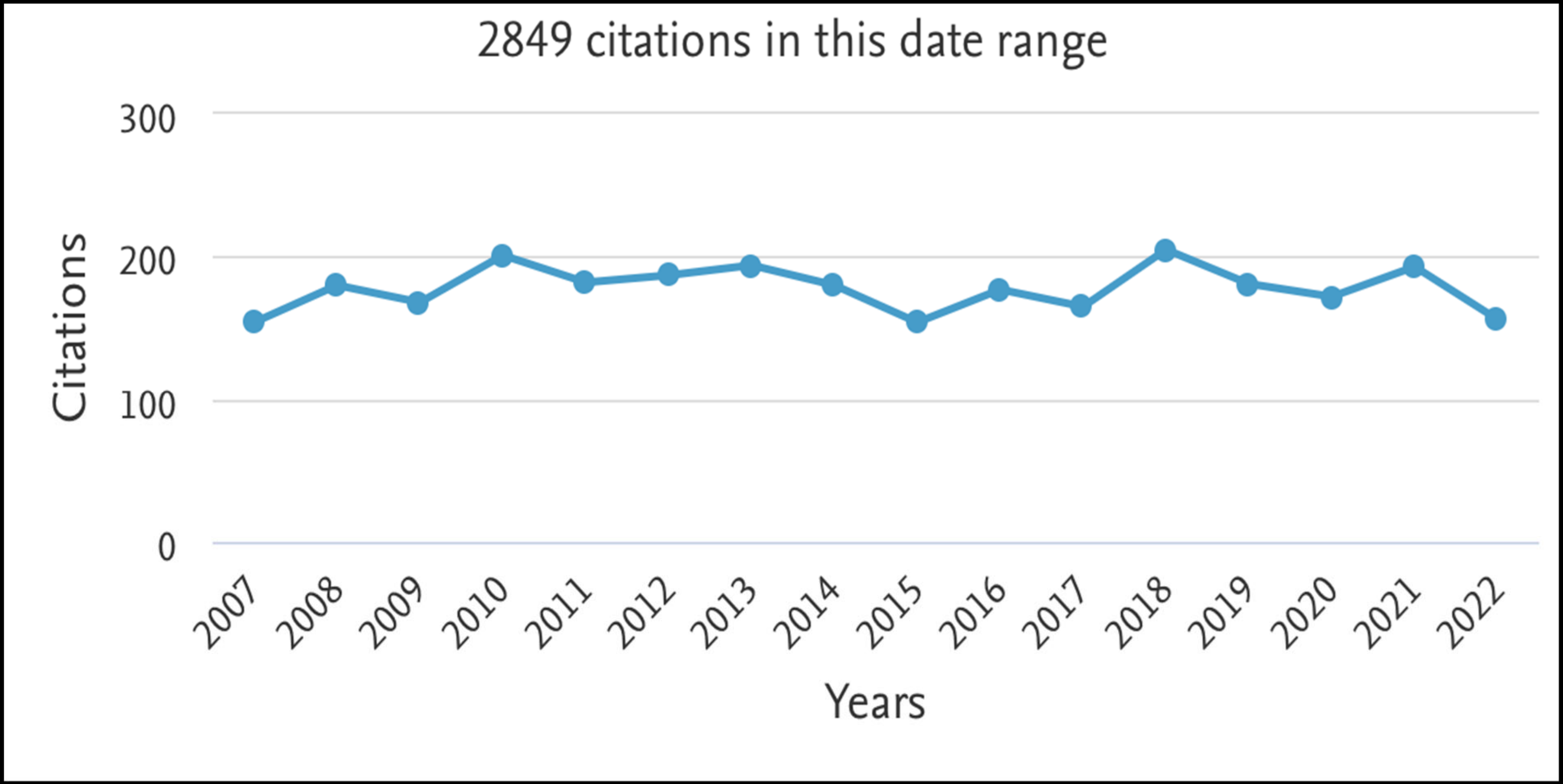
Although there are no magnetic forces acting in the places where the electron beam passes, this effect arises.

Citation - Early Years

- Scopus: 5,041 total citation

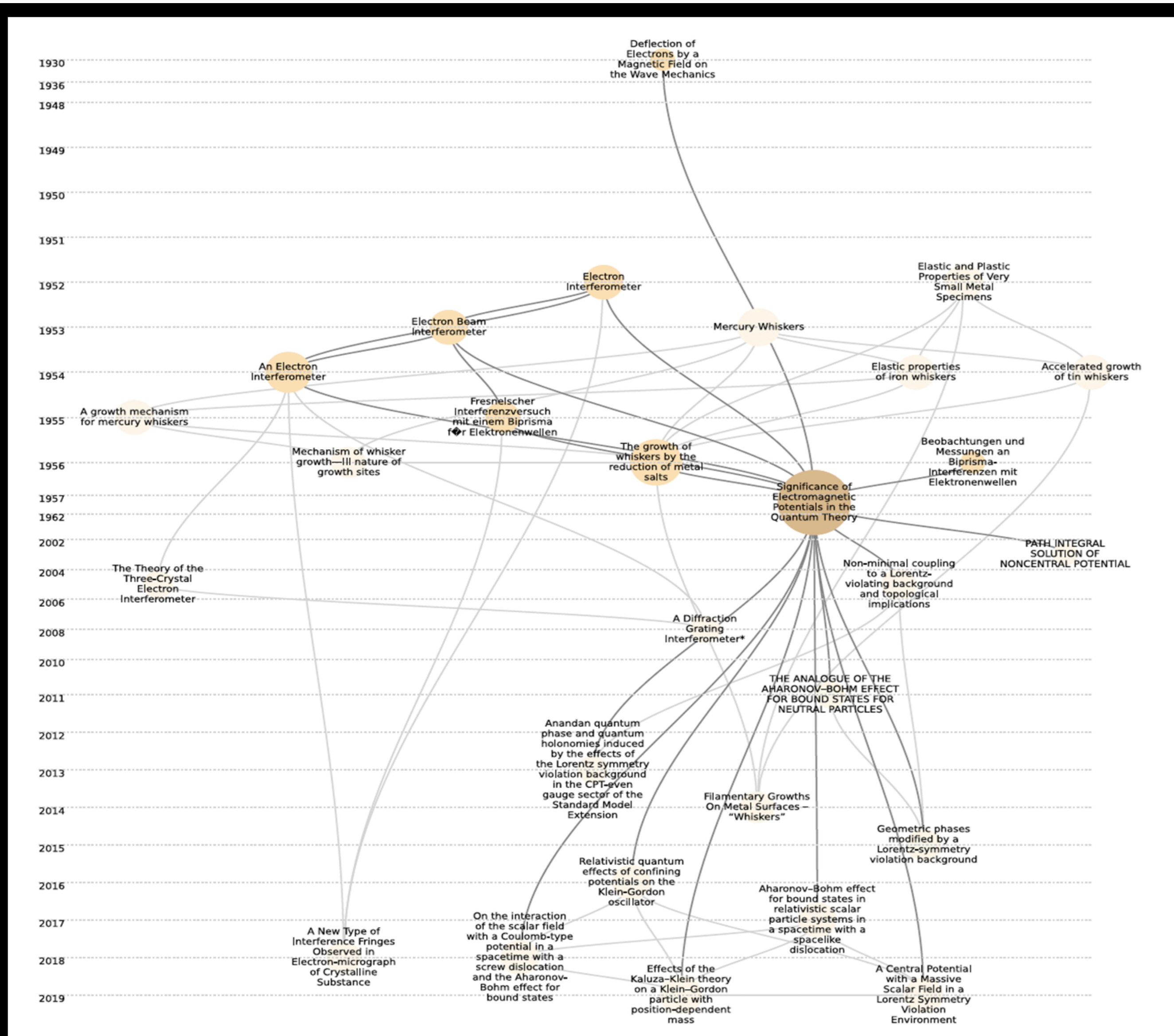


Citation - Recent Years



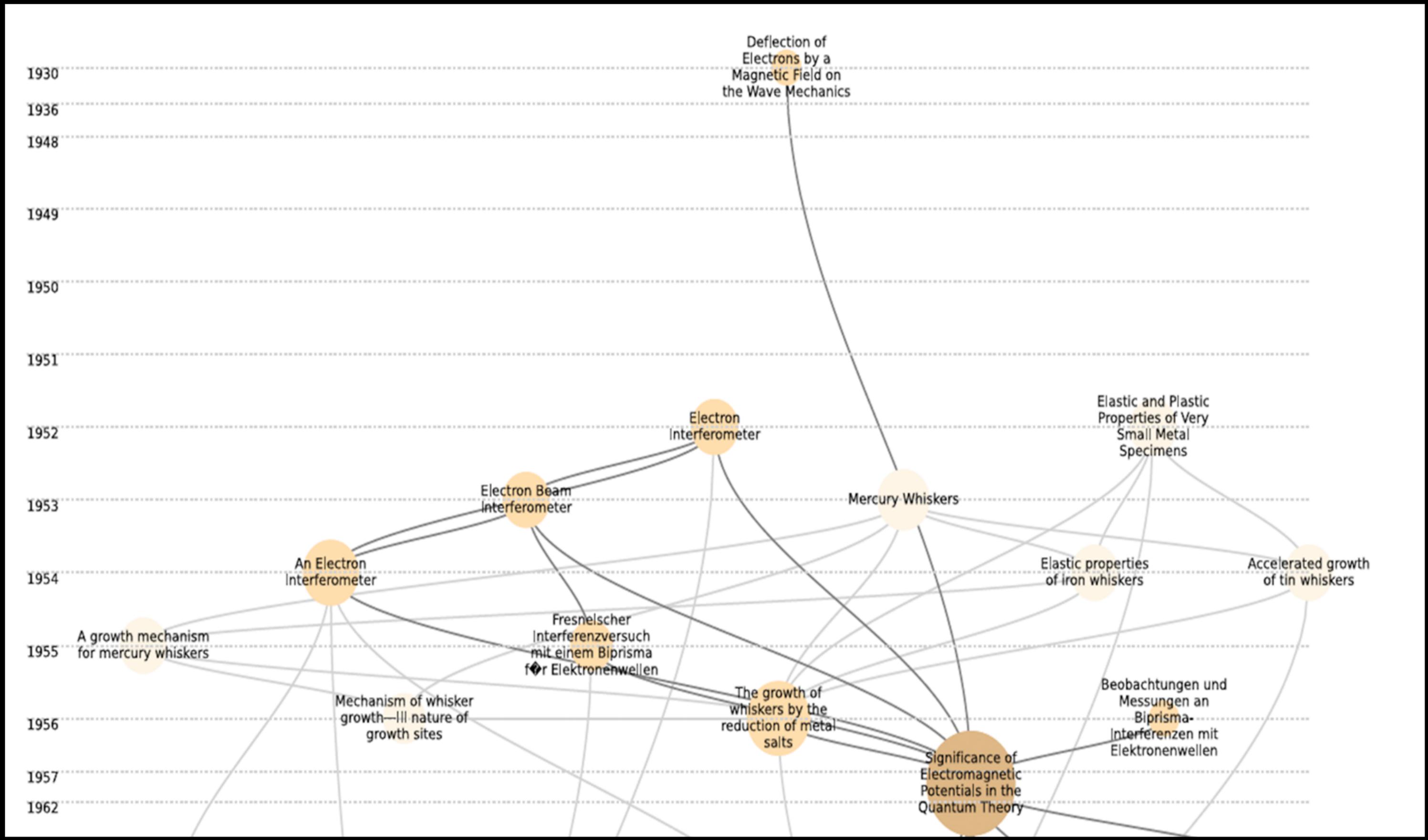
Overall View

- Connection to previous and later papers



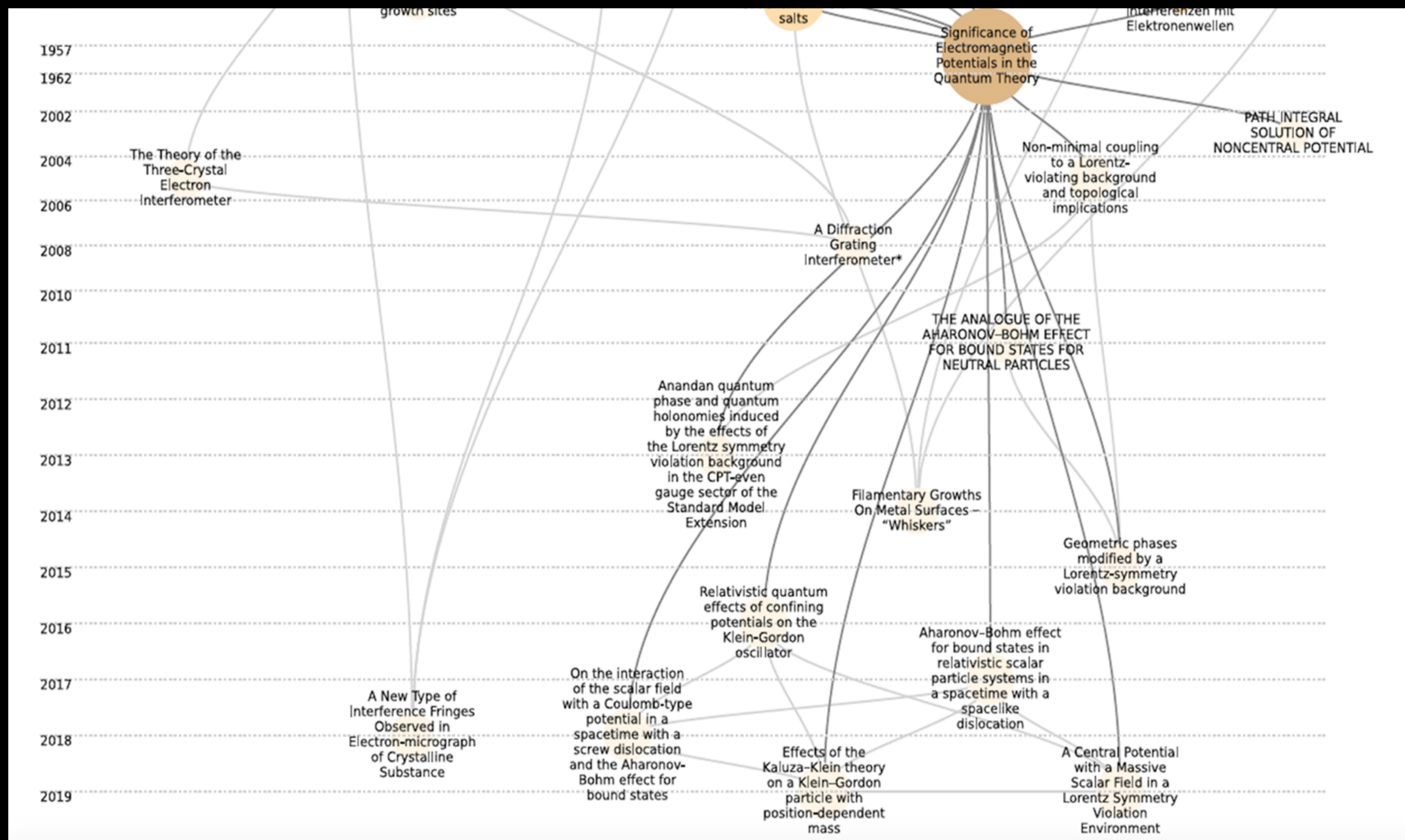
Fields/Experiments Leading to This Paper

- Deflection of electrons by a magnetic field
- Electron beam interferometer
- Metal whisker formation
- Properties of small metal specimens



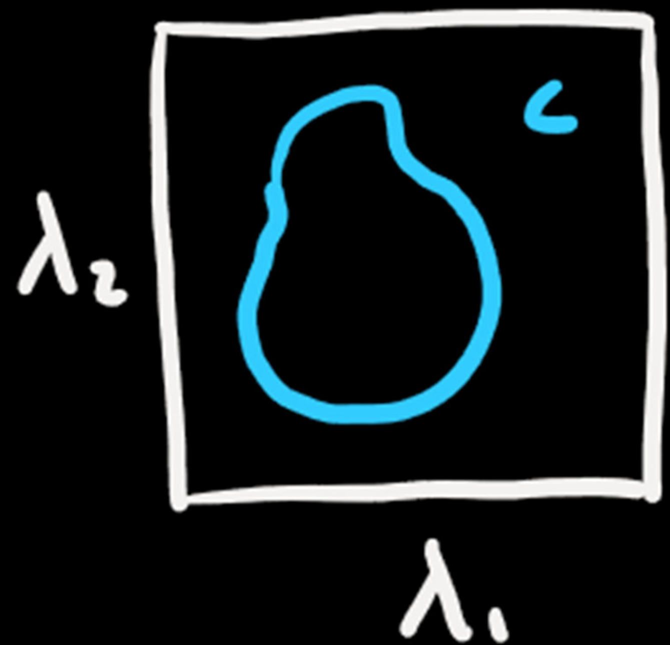
Fields Impacted by This Paper

- Electron microscopy
- Neutral interactions such as photon-photon
- Quantum mechanical effects such as quantum computing
- Relativistic quantum effects
- Gauge fields
- Gravitational AB effect
- Mathematics



Generalization: Geometric phase

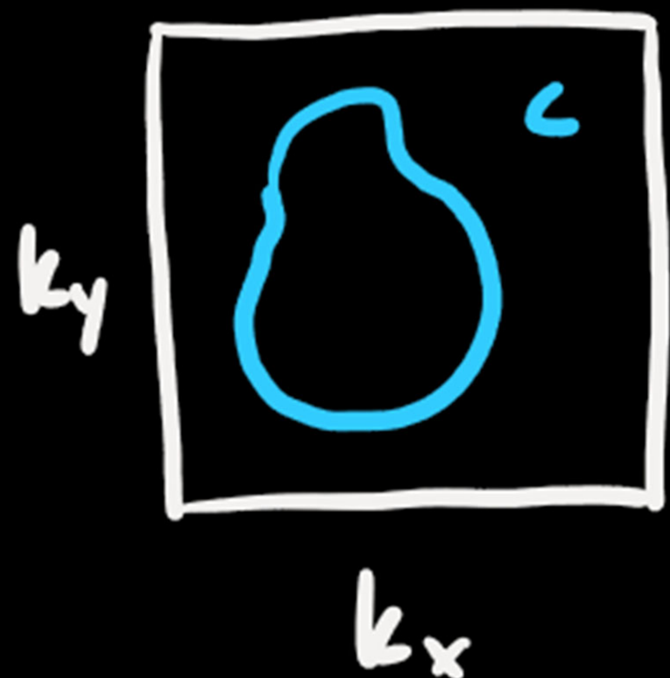
- Consider parameter-dependent Hamiltonian $H(\lambda)\boldsymbol{\psi}(\lambda)=E(\lambda)\boldsymbol{\psi}(\lambda)$
- λ can be momentum, position, etc.
- Closed path C in parameter space
- $\boldsymbol{\psi}(\lambda)\rightarrow\exp(\boldsymbol{\gamma}(C))\boldsymbol{\psi}(\lambda)$,
  $\boldsymbol{\gamma}(C)=i\int\mathcal{A}(\lambda)d\lambda$
- AB $\rightarrow$ $\mathcal{A}(\lambda)$ is $\mathcal{A}(x)$, the vector potential



Born, M., Fock, V. Beweis des Adiabatensatzes. Z. *Physik* 51, 165–180 (1928).

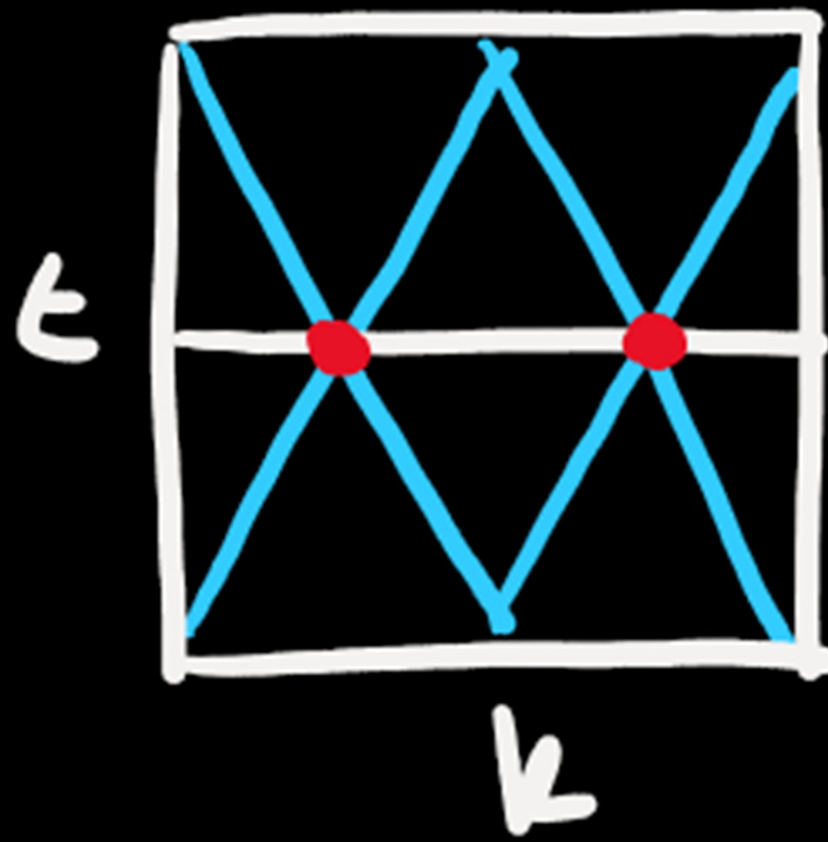
Generalization: Geometric phase

- In crystal, λ=k (p=ħk crystal momentum)
- γ(C): Berry phase
- F=∇×$\mathcal{A}$(k): momentum space magnetic field called Berry curvature



Generalization: Geometric phase

- Allows study of “magnetic monopoles”
- Band crossings are sources of Berry curvature
- These appear in *Weyl semimetals*
- Other systems have a host of interesting features related to their Berry curvature



Critiques

The paper…

- Lacks a more in depth discussion of locality
- Assumes physicality of potentials
- Doesn't consider the topological effects in higher dimensions
- Doesn't include the quantum mechanical potential sources
- Is too math focused

Conclusions of the paper

- AB implies potentials are physical in QM
- Potentials are fundamental, rather than fields
- Potentials lead to phase factors

Broader conclusions

- The more general geometric phase appears in many fields
- Geometric phase can be used to classify properties of many condensed matter systems
- Ideas developed are wide-ranging and powerful beyond their initial scope