Journal Talk: Experimental Realization of any Discrete Unitary Operator

Team 10: Suhas Sheikh, Melika Shekari, Irina Shenogina, Kaushik Singirikonda, Haoran Sun

M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, Phys. Rev. Lett. 73, 58 (1994)

Introduction

- Any discrete unitary matrix operator can be constructed in lab
  - Sequence of 2x2 transformation matrices of U(2)
- U(2) transformation
  - Well known experimental representation
  - Beam splitters
    - input/output states related by U(2) matrix
  - Interferometer
    - input/output states related by SU(2) matrix
    - Comprised of beam splitters and phase shifters

- 1. B. Yurke, S. L. McCall, and J. R. Klauder, Phys. Rev. A33, 4033 (1986)
- 2. A. Zeilinger, Am. J. Phys. 49, 822 (1981)

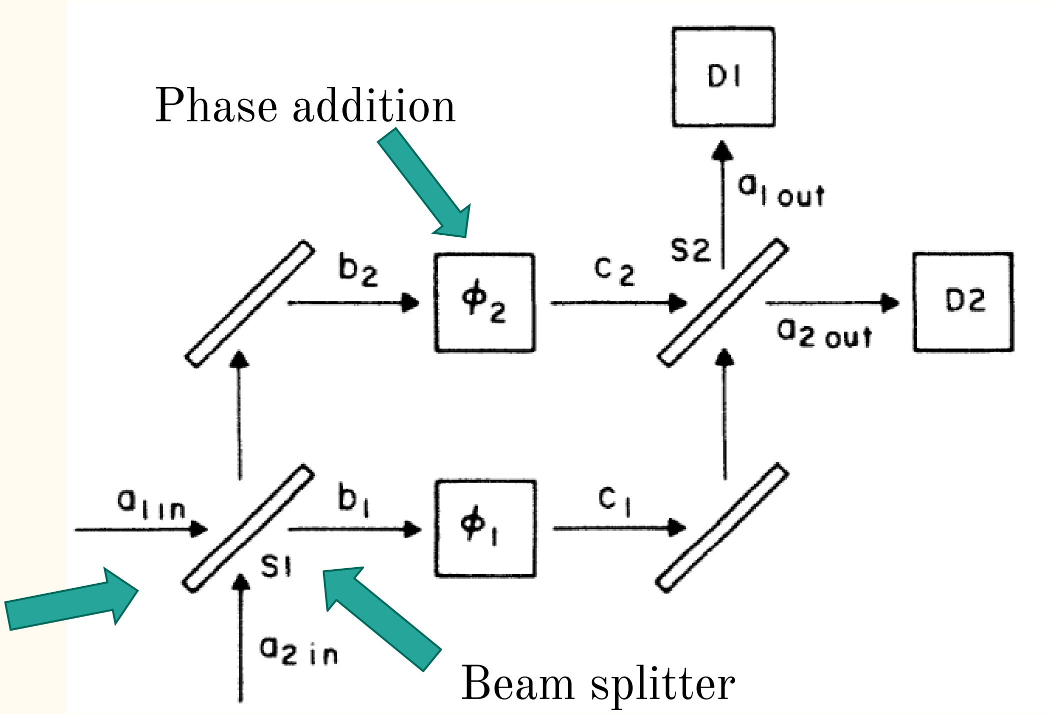
2x2 Unitary Group in Experiment

- Beam splitter
  - Transformation matrix depends on the properties
    - Transmittance $(\sqrt{T} = \cos\omega)$
    - Reflectance $(\sqrt{R} = \sin\omega)$
    - Phase shifter ϕ

$$\begin{pmatrix} k_1' \\ k_2' \end{pmatrix} = \begin{pmatrix} e^{\imath\phi}\sin\omega & e^{\imath\phi}\cos\omega \\ \cos\omega & -\sin\omega \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

Implementing a 2x2 unitary

Pretty straightforward!!



Input state $|\Psi_{in}>$

B. Yurke, S. L. McCall, and J. R. Klauder, Phys. Rev. A33, 4033 (1986)

Unitary Matrices: A primer

1. An invertible square matrix U with complex entries, with

$$U^* U = UU^* = UU^{-1} = I,$$

1. In a physics context (QM): $U^\dagger U = UU^\dagger = I.$

Few properties:

I. Norm/Inner-product preserving: $\langle U\mathbf{x}, U\mathbf{y}\rangle = \langle \mathbf{x}, \mathbf{y}\rangle$

II. Diagonalizable: $U = VDV^*$

III. Unit determinant: $|\det(U)| = 1$ → Eigenvalues lie on the unit circle in the complex plane

IV. Nice parametrization: $U = e^{iH}$,

$$U = \begin{bmatrix} a & b \\ -e^{i\varphi} b^* & e^{i\varphi} a^* \end{bmatrix}, \qquad |a|^2 + |b|^2 = 1$$

Unitary Matrices and Rotations

$$U = e^{i\varphi/2}\begin{bmatrix} e^{i\alpha}\cos\theta & e^{i\beta}\sin\theta \\ -e^{-i\beta}\sin\theta & e^{-i\alpha}\cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\alpha = \psi + \delta \qquad \beta = \psi - \delta$$

$$U = e^{i\varphi/2}\begin{bmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} e^{i\delta} & 0 \\ 0 & e^{-i\delta} \end{bmatrix}$$

U(N) → Set of (NxN) unitary matrices with matrix multiplication as the group operation

Can elements in U(N) be realized experimentally, for a general N?

Why is unitarity important?

1. Time evolution is unitary in quantum mechanics!

$$U(t) = e^{-i\hat{H}t/\hbar}$$

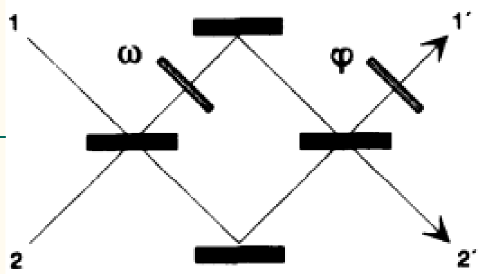
1. Unitarity is rooted in both the Heisenberg and Schrödinger picture descriptions of QM

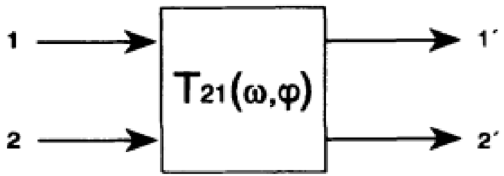
$$A_{\mathrm{H}}(t) = e^{iH_{\mathrm{S}}\ t/\hbar} A_{\mathrm{S}} e^{-iH_{\mathrm{S}}\ t/\hbar} \qquad |\psi_{\mathrm{S}}(t)\rangle = e^{-iH_{\mathrm{S}}\ t/\hbar} |\psi_{\mathrm{S}}(0)\rangle$$

Complete credit to Ibrahim for the video edit (Team 10 approves)!

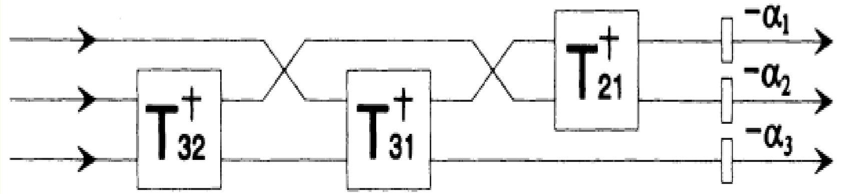
Theory implementation and procedure

$T_{Nq}(\omega_{Nq}, \phi_{Nq})$ for $q = N-1, \ldots, 1$





For a 3x3 unitary, the schematic looks like:



Rotation employed as matrix multiplication:

$$R(N) = T_{N,N-1} \cdots T_{N,1}.$$

dim reduction →

$$U(N) \cdot T_{N,N-1} \cdot T_{N,N-2} \cdots T_{N,1} = \begin{pmatrix} \boxed{U(N-1)} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

A bit of gory detail (an example)

$$U(N) = \begin{pmatrix} \langle 1| \\ \langle 2| \\ \vdots \\ \langle N| \end{pmatrix}$$

Implemented as →

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}^T \cdot R(N)^{-1} = \begin{pmatrix} +e^{-i\phi_1} \cos\omega_1 \\ -e^{-i\phi_2} \cos\omega_2 \sin\omega_1 \\ +e^{-i\phi_3} \cos\omega_3 \sin\omega_2 \sin\omega_1 \\ \vdots \\ \mp e^{-i\phi_{N-1}} \cos\omega_{N-1} \ldots \sin\omega_1 \\ \pm \sin\omega_{N-1} \sin\omega_{N-2} \ldots \sin\omega_1 \end{pmatrix}^T .$$

Advantages of the setup

Using optical systems to realize matrices physically: you get to invert freely!

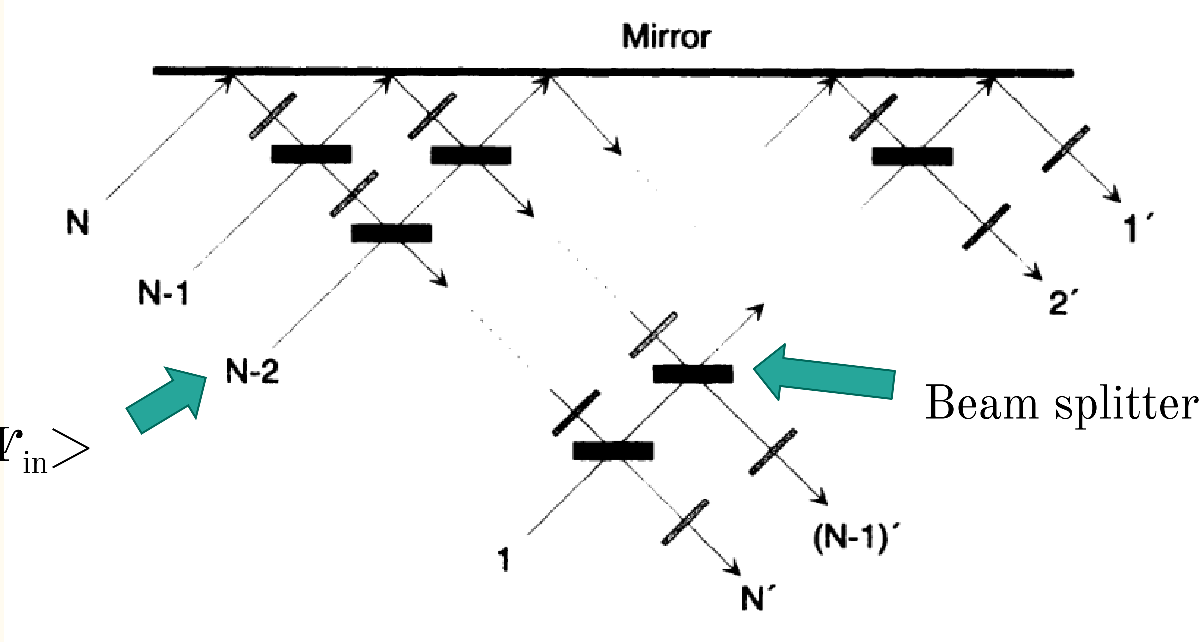
$$U(N) \cdot T_{N,N-1} \cdot T_{N,N-2} \cdots T_{2,1} \cdot D = I(N)$$

↓

$$U(N) = (T_{N,N-1} \cdot T_{N,N-2} \cdots T_{2,1} \cdot D)^{-1}$$

Lots of flexibility in choosing no. of layers of beam splitters and phase shifters

Experimental Implementation (N dimensions)



Beam splitter

Input state $|\Psi_{in}>$

Order of beam splitters does matter!!!

Implementing a 4x4 Unitary

$(\sigma_y \otimes \sigma_x)$

$$H = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}.$$

	ω	ϕ	Note
T_{43}	exchange beams 3 and 4		
T_{42}	$\pi/4$	$\pi/2$	1:1 beam splitter
T_{31}	$\pi/4$	$\pi/2$	1:1 beam splitter
Phases	$\alpha_1 = \alpha_2 = 0$		$\alpha_3 = \alpha_4 = \pi$

Applications

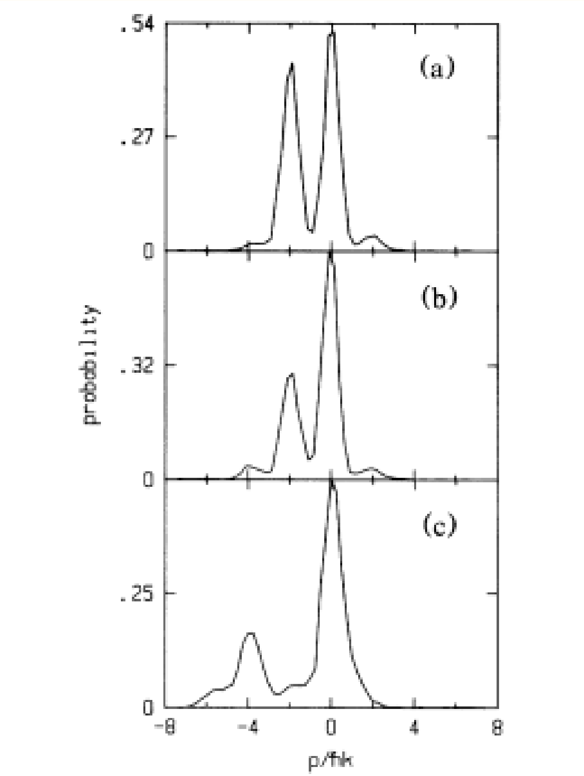
→ building beam splitter analogs of Kochen-Specker type experiments

→ studying Einstein-Podolsky-Rosen correlations in higher-dimensional Hilbert spaces experiments

→ in atom lithography, realizing experimentally any discrete unitary operator acting on an atomic beam.

→ equivalent beam splitter can arrange a laser field

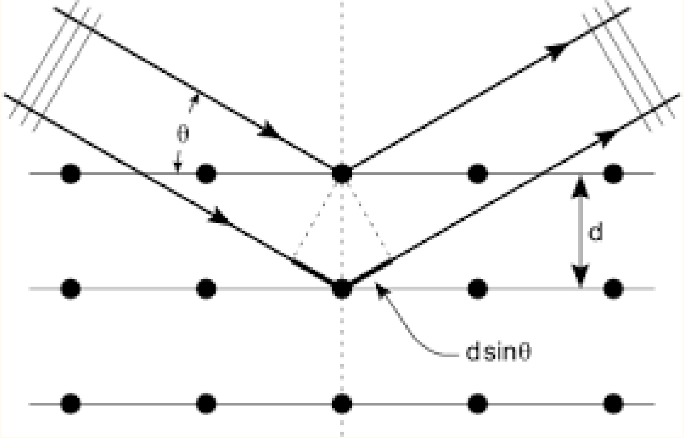
Laser field, first observation of Bragg scattering of sodium atoms from a standing light wave



(a) First order Bragg scattering (P=6 mW)

(b) First-order Bragg scattering (P=10 mW)

(c) Second-order Bragg scattering

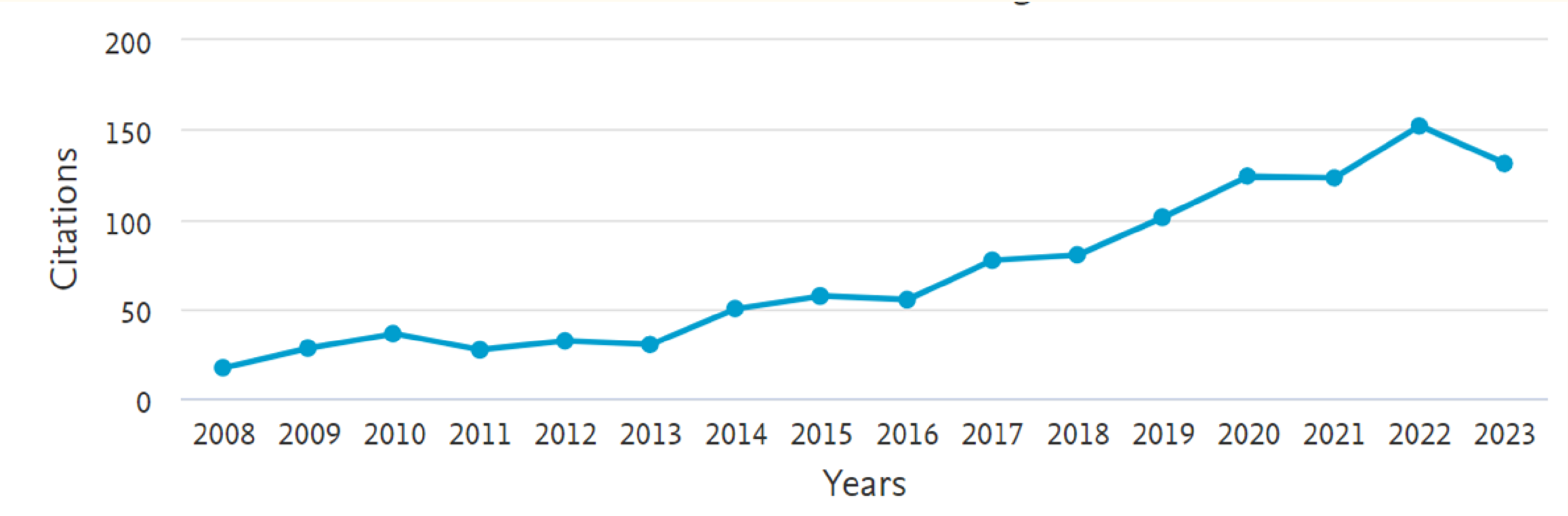


Bragg scattering

P. J. Martin, B. G. Oldaker, A. H. Miklich, and D. E. Pritchard, Phys. Rev. Lett. 60, 515 (1988); J. J. Mc- Clelland, R. E. Scholten, E. C. Palm, and R. J. Celotta, Science 262, 877 (1993).

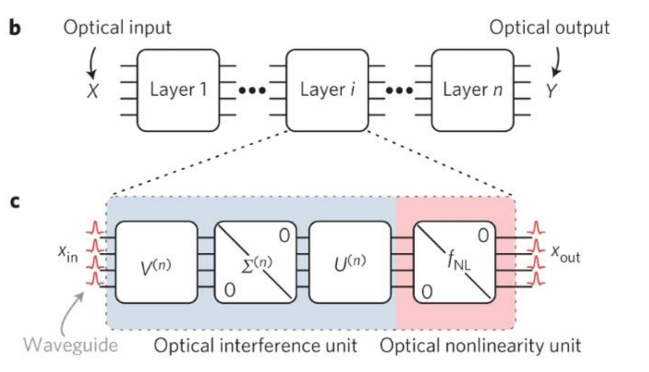
Citation Review & Future Applications

- **1415** total citations on Scopus; **1120** from 2008
- Growing number of citations - Wide Range of Applications in Computing



Citation Review & Future Applications

- New Optical Neural Networks(ONN)
- Optical Interference Unit(OIU)
  - Compose layers of ONN
  - Singular Value Decomposition
  - Purely Optic ANN & Consume no Power(Theoretically)



Article | Published: 12 June 2017

Deep learning with coherent nanophotonic circuits

Yichen Shen, Nicholas C. Harris, Scott Skirlo, Mihika Prabhu, Tom Baehr-Jones, Michael Hochberg, Xin Sun, Shijie Zhao, Hugo Larochelle, Dirk Englund & Marin Soljačić

Nature Photonics **11**, 441–446 (2017) | Cite this article

Citation Review & Future Applications

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RESEARCH ARTICLE

Hartree-Fock on a superconducting qubit quantum computer

GOOGLE AI QUANTUM AND COLLABORATORS, FRANK ARUTE, KUNAL ARYA, RYAN BABBUSH, DAVE BACON, JOSEPH C. BARDIN, RAMI BARENDS, SERGIO BOIXO, MICHAEL BROUGHTON, [...], AND ADAM ZALCMAN +72 authors Authors Info & Affiliations

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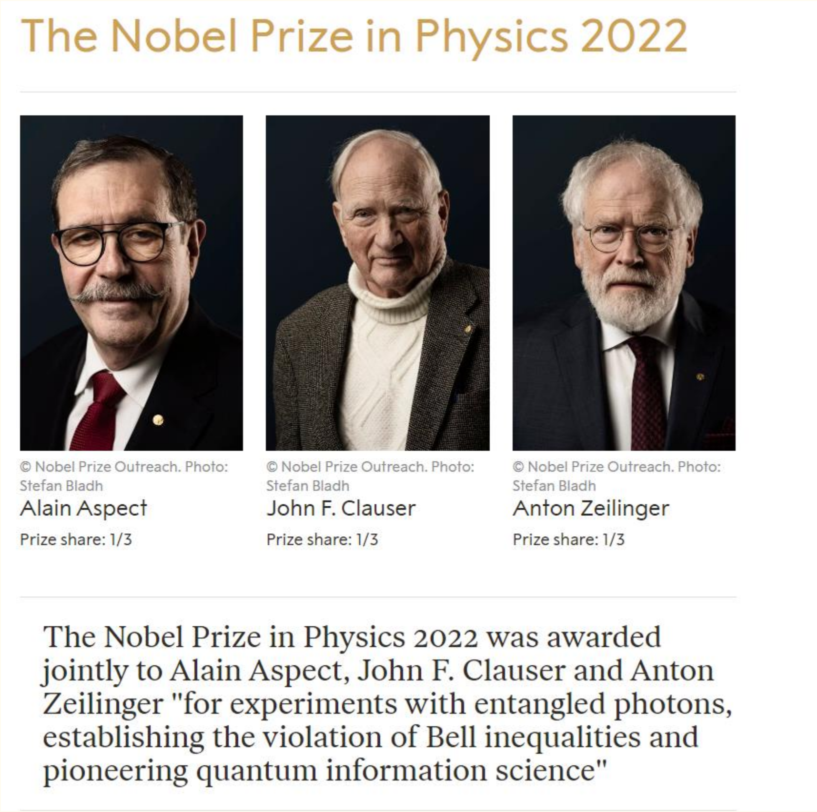
Citation Review & Future Applications

- Quantum Information Science is thriving!
- More possible future applications



Our Opinions On the Paper

1. Well-explained theory in terms of how the optical setup is put together and measurements are carried out;
2. Few other examples would have felt more complete;
3. Meaning of some notations could be explained more in depth;
4. The results are well-supported and encouraged future studies.

We approve!

Thank you!

References

Arute, Frank, et al. “Hartree-Fock on a Superconducting Qubit Quantum Computer.” *Science*, vol. 369, no. 6507, Aug. 2020, pp. 1084–89, https://doi.org/10.1126/science.abb9811.

Martin, Peter, et al. “Bragg Scattering of Atoms from a Standing Light Wave.” *Physical Review Letters*, vol. 60, no. 6, Feb. 1988, pp. 515–18, https://doi.org/10.1103/physrevlett.60.515.

Reck, Michael, et al. “Experimental Realization of Any Discrete Unitary Operator.” *Physical Review Letters*, vol. 73, no. 1, July 1994, pp. 58–61, https://doi.org/10.1103/physrevlett.73.58.

Shen, Yichen, et al. “Deep Learning with Coherent Nanophotonic Circuits.” *Nature Photonics*, vol. 11, no. 7, June 2017, pp. 441–46, https://doi.org/10.1038/nphoton.2017.93.