Week 3: Reading Assignment, Homework Assignment

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All lecture notes, homework, demos, references, *etc*. are available on the P598AEM website. Please spend some time checking these out!

Course Organization:

A. Lectures: Tuesday & Thursday, 12:30-1:50 pm, in 136 Loomis.
B. Weekly Reading and Homework Assignments: HW due following Thursday, in class.
C. Take-Home Midterm Exam: Oct. 10th, due Oct. 17th (in lieu of P598AEM HW 7).
D. Take-Home Final Exam: Dec. 10th, due Dec. 17th.

Reading Assignment For Week 3: Please read/work through P598AEM Lect. Notes 6.
Homework Assignment For Week 3: See/do HW # 3 problems on following pages.

Physics 598AEM Week 3 Homework Assignment

Use ***matrix*** ***calc***. info in P598AEM Lect. Notes 6 to help you do the following HW problems.

1*a*.) Calculate the ***variance*** $\sigma_{A_{FB}}^2 \equiv \underline{\hat{V}}_{A_{FB}} \equiv \underline{\hat{D}}_{A_{FB}/x} \underline{\hat{V}}_x \underline{\hat{D}}_{A_{FB}/x}^T$ {*n.b.* a ***scalar*** quantity – 1×1 matrix} and hence the uncertainty $\sigma_{A_{FB}}$ on the forward-backward asymmetry A_{FB} associated with (an arbitrary) angular distribution of particles (*e.g.* μ^+ in $Z^0 \to \mu^+\mu^-$ production/decay): $A_{FB} \equiv (F-B)/(F+B) \equiv D/N$ where *F* = # of events with μ^+ in the forward hemisphere, *B* = # of events with μ^+ in the backward hemisphere, *N* = *F*+*B* = total # of $Z^0 \to \mu^+\mu^-$ events and *D* = *F* – *B*. Assume that *F* and *B* are ***independent*** random variables ($\Rightarrow$ *n.b.* *N* is ***not*** fixed/constant here!), and that the individual ***variances*** $\sigma_F^2 = F$ and $\sigma_B^2 = B$.

b.) Show that $\sigma_{A_{FB}} = \frac{1}{\sqrt{N}}\sqrt{1-A_{FB}^2}$. Hint: use the relations $\sigma_F^2 = F$, $\sigma_B^2 = B$, $N = F + B$ and $A_{FB} \equiv (F-B)/(F+B)$ in your expression for $\sigma_{A_{FB}}^2$. Then make a plot of $\sqrt{N}\cdot\sigma_{A_{FB}}$ *vs.* A_{FB} where $-1 \le A_{FB} \le +1$. Briefly discuss what's going on in this plot.

c.) Since *N* = *F* + *B* and *D* = *F* – *B*, thus: $g(N,D)\,dNdD = f(F,B)\,dFdB$ $\Rightarrow g(N,D) = f(F,B)/\left|J(N,D/F,B)\right|$. Obtain the elements of the ***covariance*** matrix

$$\underline{\hat{V}}(N,D) = \underline{\hat{D}}_{N,D/F,B}\,\underline{\hat{V}}(F,B)\,\underline{\hat{D}}_{N,D/F,B}^T = \begin{pmatrix} \sigma_N^2 & \text{cov}(N,D) \\ \text{cov}(N,D) & \sigma_D^2 \end{pmatrix} = \begin{pmatrix} \sigma_N^2 & \sigma_N\sigma_D\rho(N,D) \\ \sigma_N\sigma_D\rho(N,D) & \sigma_D^2 \end{pmatrix}$$

d.) Show that the ***covariance*** matrix $\hat{V}(N,D)$ can be written as: $\hat{V}(N,D) = N\begin{pmatrix} 1 & A_{FB} \\ A_{FB} & 1 \end{pmatrix}$.

Briefly discuss the nature of correlations and/or anti-correlations between $N = F + B$ and $D = F - B$ when $A_{FB} \approx -1.0, -0.5, 0.0, +0.5, +1.0$.

2.) Obtain the ***covariance*** matrix $\hat{V}(r,\theta)$ associated with the change of independent random variables (x, y) to their polar coordinate variables (r,θ) via $r^2 = x^2 + y^2$ and $\tan\theta = y/x$. Assume that the ***variance*** matrix $\hat{V}(x, y)$ is given, *i.e.* σ_x^2 and σ_y^2 are known/specified. Draw a 2-D picture of an ellipse centered at $(+x_0, +y_0)$. Briefly discuss the conditions for which the covariance between r and θ will be positive, zero, or negative, *i.e*. discuss the behavior of the correlation coefficient $\rho(r,\theta)$ for the three cases $\sigma_x < \sigma_y$, $\sigma_x = \sigma_y$ and $\sigma_x > \sigma_y$. For the special case of $\rho(r,\theta) = 0$, briefly discuss how σ_r and σ_θ are affected by increasing r from its $\hat{r}$ value, $\hat{r} \equiv \sqrt{x_0^2 + y_0^2}$.