Week 5: Reading Assignment, Homework Assignment

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All lecture notes, homework, demos, references, *etc*. are available on the P598AEM website. Please spend some time checking these out!

Course Organization:

A. Lectures: Tuesday & Thursday, 12:30-1:50 pm, in 136 Loomis.
B. Weekly Reading and Homework Assignments: HW due following Thursday, in class.
C. Take-Home Midterm Exam: Oct. 10^{th}, due Oct. 17^{th} (in lieu of P598AEM HW 7).
D. Take-Home Final Exam: Dec. 10^{th}, due Dec. 17^{th}.

Reading Assignment For Week 5: Please read/work through P598AEM Lect. Notes 9-11.
Homework Assignment For Week 5: See/do HW # 5 problems on following pages.

Physics 598AEM Week 5 Homework Assignment

1.) If $\underline{y} \equiv \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ are a pair of ***non-independent*** random variables, we showed in P598AEM Lect. Notes 9 (*p*. 2-4) that one can ***always*** obtain a corresponding set of ***independent*** random variables $\underline{x} \equiv \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ by making an ***orthogonal transformation***: $\underline{x} = \underline{C}\,\underline{y}$ from the *y*-basis to the *x*-basis, where $\underline{C}$ is an ***orthogonal*** matrix, having the properties that: $\underline{C}\underline{C}^T = \underline{C}\underline{C}^{-1} = \mathbb{1} = \underline{C}^{-1}\underline{C} = \underline{C}^T\underline{C}$ *i.e*. that $\underline{C}^T = \underline{C}^{-1}$, and that: $\det \underline{C} = \pm 1$, $\det \underline{C}^T = \mp 1$ where $\mathbb{1}$ is the unit/identity matrix.
If the (2×2) *y*-basis covariance matrix $\underline{\hat{V}}_{\underline{y}}$ is specified:

$$\underline{\hat{V}}_{\underline{y}} = \begin{pmatrix} \sigma_{y_1}^2 & \mathrm{cov}(y_1,y_2) \\ \mathrm{cov}(y_2,y_1) & \sigma_{y_2}^2 \end{pmatrix} = \begin{pmatrix} \sigma_{y_1}^2 & \sigma_{y_1}\sigma_{y_2}\rho(y_1,y_2) \\ \sigma_{y_2}\sigma_{y_1}\rho(y_2,y_1) & \sigma_{y_2}^2 \end{pmatrix}$$

a.) Show that $\underline{\hat{V}}_{\underline{x}} = \underline{C}\underline{\hat{V}}_{\underline{y}}\underline{C}^T$ is indeed ***diagonal*** for the case of a ***2-D rotation*** as an ***orthogonal transformation***, *i.e*. $\underline{C} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ ***when***: $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{y_1}\sigma_{y_2}\rho(y_1,y_2)}{\sigma_{y_1}^2 - \sigma_{y_2}^2}\right)$.

b.) Explicitly show/verify, for ***this*** $\underline{C}$, that: $\underline{C}\underline{C}^T = \underline{C}\underline{C}^{-1} = \mathbb{1} = \underline{C}^{-1}\underline{C} = \underline{C}^T\underline{C}$ *i.e*. that: $\underline{C}^T = \underline{C}^{-1}$, and that: $\det \underline{C} = \pm 1$, $\det \underline{C}^T = \pm 1$.

c.) Explicitly calculate $\det \underline{\hat{V}}_{\underline{y}}$ and $\det \underline{\hat{V}}_{\underline{x}}$. Are they equal to each other?

2.) <u>For the ***Binomial Distribution***</u>:
a.) Make plots of $P(n;N,p)$ *vs*. *n* (the # of successes in *N* trials), for a ***<u>fixed</u>*** value of the success probability/trial of p = 1/2, for N = 2, 6, 10, 20 and 40 trials.

b.) Make plots of the corresponding ***Cumulative Binomial Distribution*** associated with each in 2*a*.)

3.) <u>For the ***Binomial Distribution***</u>:
a.) Make plots of $P(n;N,p)$ *vs*. *p* for ***<u>fixed</u>*** $n = N/2$, for N = 2, 6, 10, 20 and 40 trials.
b.) For $0 \le p \le 1$ considered as a continuous random variable, then $P(n;N,p)$ can be considered as a P.D.F. for the ***Binomial Distribution***, but while $\int_0^1 P(n;N,p)\,dp \neq 1$, $\sum_{n=0}^{N}\left\{\int_0^1 P(n;N,p)\,dp\right\} = 1$.
To keep it simple, make plots of the corresponding "Cumulative Binomial Distribution" associated with each of 3*a*.) above, *i*.*e*. with ***<u>fixed</u>*** $n = N/2$, for N = 2, 6, 10, 20 and 40 trials. Briefly describe what you infer, physical-meaning-wise from these curves…