Lecture #15
10/17/2017
PHYS 598 AQG

Continuation of

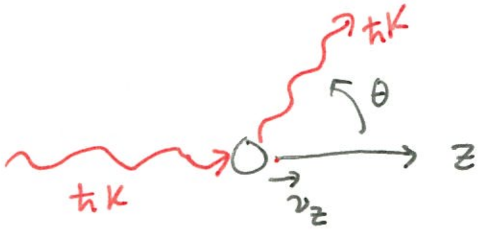
Laser-cooling

+

Discussion on

Magneto-optical trapping

Let's look in detail @ absorption / re-emission



to conserve momentum, need

$$v_z \to v_z'$$

w/

$$m v_z + \hbar K = m v_z' + \hbar K \cos\theta$$

in 3D, over Ω

$$\Delta v_z = \frac{\hbar K}{m}\left(1 - \cos\theta\right)$$

averaging over θ, direction of scattered photon

$$\left\langle \Delta v_z^2 \right\rangle_\Omega = \left(\frac{\hbar K}{m}\right)^2 \int \frac{(1-\cos\theta)^2}{4\pi}\, d\Omega = \frac{1}{2}\left(\frac{\hbar K}{m}\right)^2 \int_{-1}^{1} (1-x)^2\, dx$$

w/ $x = \cos\theta$

$$= \frac{1}{2}\left(\frac{\hbar K}{m}\right)^2 \times \frac{4}{3} = \frac{4}{3}\frac{E_{rec}}{m}$$

w/ $E_{rec} = \dfrac{p_{rec}^2}{2m} = \dfrac{\hbar^2 K^2}{2m}$ the recoil energy

for 2 beams (1D optical molasses)

$$\frac{1}{2} m \frac{d}{dt}\langle v_z^2 \rangle_\Omega = \frac{1}{2} m \left(\frac{4}{3}\frac{E_{rec}}{m}\right)\left(2 R_{scatt}\right) = \frac{4}{3} E_{rec} R_{scatt}$$

for 6 beams (3D optical molasses)

$$\frac{1}{2} m \frac{d}{dt}\langle v_z^2 \rangle_\Omega = 4 E_{rec} R_{scatt}$$

Combining w/ "cooling"

$$\frac{1}{2} m \frac{d}{dt}\langle \bar{v}_z^2 \rangle_\Omega = 4 E_{rec} R - \beta \bar{v}_z^2$$

get $\frac{d}{dt}\langle \bar{v}_z^2 \rangle = \frac{8 E_{rec}}{m} R_{scatt} - \frac{2\beta}{m} \bar{v}_z^2$

$\frac{d}{dt}\langle \bar{v}_z^2 \rangle = 0$ in equilibrium

giving $\underbrace{\bar{v}_z^2 = \frac{4 E_{rec} R}{\beta}}$

ignoring saturation effects
(valid if $1+\left(\frac{2\delta}{\Gamma}\right)^2 \gg s$)

$$R \simeq \frac{\Gamma}{2} s \frac{1}{1+\left(\frac{2\delta}{\Gamma}\right)^2} \qquad \beta \simeq \frac{8 \hbar k^2 \delta s}{\Gamma\left[1+\left(\frac{2\delta}{\Gamma}\right)^2\right]^2}$$

such that

$$\bar{v}_z^2 \simeq \frac{\hbar \Gamma^2}{8 m \delta}\left[1+\left(\frac{2\delta}{\Gamma}\right)^2\right]$$

equipartition

$$\frac{1}{2} m \bar{v}_z^2 = \frac{1}{2} k_B T$$

$$T = \frac{m \bar{v}_z^2}{k_B}$$

$$T = \frac{\hbar\Gamma}{4 k_B} \frac{1+\left(\frac{2\delta}{\Gamma}\right)^2}{2(\delta/\Gamma)} = \frac{\hbar\Gamma}{4 k_B} \underbrace{\left(\frac{\Gamma}{2\delta} + \frac{2\delta}{\Gamma}\right)}_{\substack{\text{min @} \\ 2\delta = \Gamma}} \Longrightarrow \boxed{T_{Dopp} = \frac{\hbar\Gamma}{2 k_B}}$$

for alkalis, $T_{Dopp} \sim$ few $\times 100\ \mu K$ "Doppler limit"

$T_{Dopp}^{Na} \simeq 235\ \mu K$ w/ $\frac{\Gamma}{2\pi} \simeq 9.8$ MHz

At this "Doppler limit," typical velocity is <u>much</u> less than at room temp.

In fact, for $k_B T_{Dopp} = \frac{\hbar\Gamma}{2}$

$$v_{Dopp} \approx \sqrt{\frac{\hbar\Gamma}{m}} = \left(v_{rec}\, v_{cap}\right)^{1/2} \lesssim 1 \ \text{m/s}$$

where

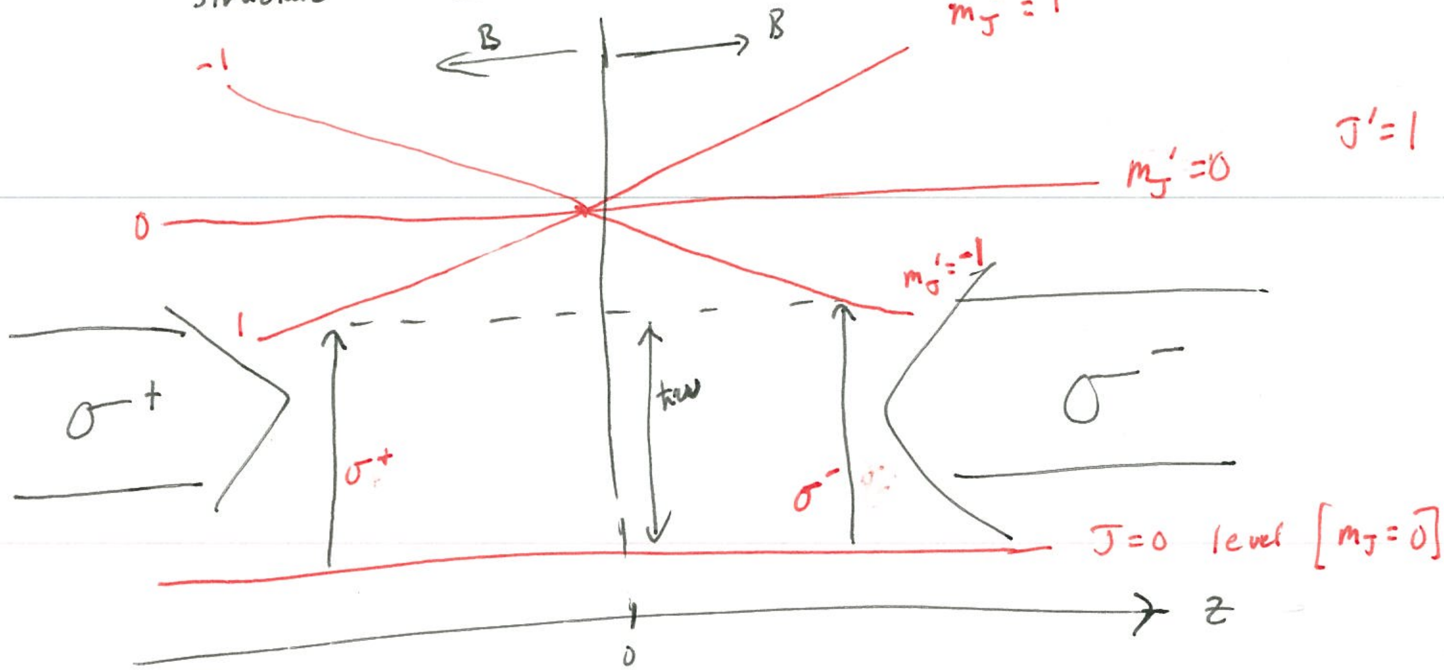
$$v_{rec} = \frac{\hbar k}{m}$$

and $$v_{cap} \approx \frac{\Gamma}{k}$$

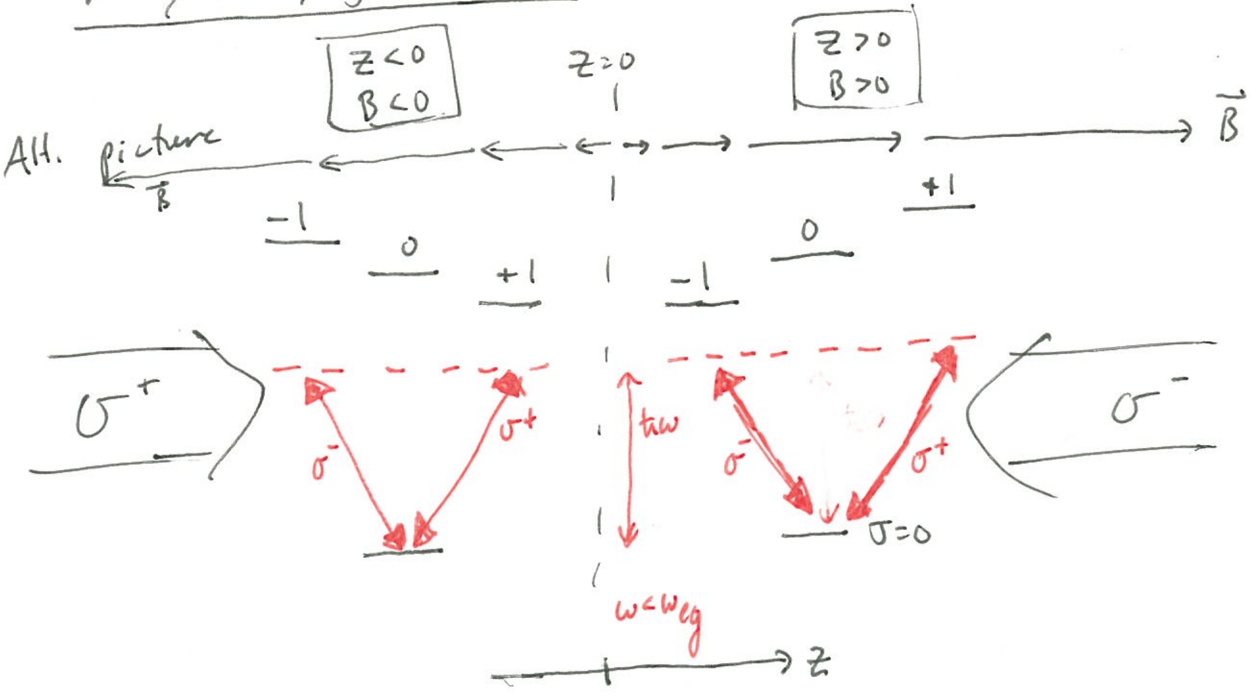
Simplified model for 1D MOT [magneto-optical trap]

simplified · 3D → 1D

real structure → $|g\rangle$ $J=0$ → $|e\rangle$ $J=1$



$\omega < \omega_{eg}$, atoms preferentially scatter from the σ^+ beam for $z<0$ and from the σ^- beam for $z>0$. Gives spatially-varying force (restoring force).



$$\vec{F}_{TOT} = \vec{F}_+ + \vec{F}_-$$

$$\vec{F}_\pm = \pm \frac{\hbar K \Gamma}{2} \frac{s}{1+s+\left(\frac{2\delta_\pm}{\Gamma}\right)^2} \hat{z}$$

$$\delta_\pm = \underbrace{\delta}_{\omega-\omega_{eg}} \mp Kv = \left[\omega - \left(\omega_{eg} \pm \frac{\Delta\mu B}{\hbar}\right) \mp Kv\right] = \underline{\delta \mp Kv \mp \xi z}$$

→ now w/ spatially-varying Zeeman shift

$$\Delta\mu = (g_{F'} m_{F'} - g_F m_F)\mu_B$$

$$B(z) \approx \left(\frac{dB}{dz}\right) z$$ in linear region near center of coils.

$$\xi = \frac{\Delta\mu}{\hbar}\left(\frac{dB}{dz}\right)$$

under similar approximations as for our molasses result ($Kv \ll \Gamma$ and $\xi z \ll \Gamma$)

This can be rewritten (ignoring higher order terms) as

$$\vec{F}_{MOT} \simeq \left(\frac{\partial F}{\partial \delta}\left[-Kv - \xi z\right] - \frac{\partial F}{\partial \delta}\left[Kv + \xi z\right]\right)\hat{z} = -2\frac{\partial F}{\partial \delta}\left[Kv + \xi z\right]\hat{z}$$

w/ $F = \frac{\hbar K \Gamma}{2} \frac{s}{1+s+(\ldots}$

$$\vec{F}_{MOT} \simeq \left(-\beta v - \frac{\xi \beta}{K} z\right)\hat{z}$$

$$\beta = -\frac{8\hbar K^2 s \delta}{\Gamma\left(1+s+\left(\frac{2\delta}{\Gamma}\right)^2\right)^2}$$

effective spring constant $\kappa_z = \frac{\beta \xi}{K} = -\frac{8\Delta\mu\left(\frac{\partial B}{\partial z}\right) K s \delta}{\Gamma\left(1+s+\left(\frac{2\delta}{\Gamma}\right)^2\right)^2}$

typical MOT figures

oscillation frequency $\Rightarrow$ $\omega_{MOT} = \sqrt{\frac{K}{m}} \sim$ few kHz

for $\frac{dB}{dz} \sim 10 \, \frac{G}{cm}$

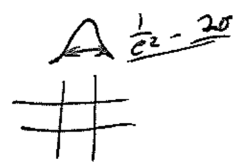
damping rate $\Longrightarrow$ $\frac{\beta}{m} \simeq$ few $\times 100$ kHz   overdamped (typically)

What sets v_c?

In molasses, atoms were only approx. in resonance up to the natural linewidth, so naturally this allowed efficient cooling up to $|\omega_D| = |Kv| \sim \Gamma$, or $|v_c| = \frac{\Gamma}{K}$.

In a MOT, the picture is more complicated, but we can make an analogy to the Zeeman slower

[atoms enter w/ v_{in}, and $\frac{\Delta\mu B_{in}}{\hbar} = Kv_{in}$, i.e. Zeeman shift compensates Doppler shift] for v_{in}, assuming $F_{max} = m\left(\frac{\hbar K\Gamma}{2m}\right) = m a_m$ can stop atoms in $L = v_{in}^2 / a_{max}$.



In the MOT, with beam radius of σ,

stop atoms w/ $v_{in}^2 < a_{max}\sigma$

$$v_{in} < \sqrt{\frac{\hbar K\Gamma}{2m}\sigma}$$

for ^{87}Rb

$$\left[\begin{array}{l} \frac{\Gamma}{2\pi} \sim 6 \text{ MHz} \\ m = 87 \text{ amu} \\ K = 2\pi / 780 \text{ nm} \end{array}\right.$$

and $\sigma = 1$ cm.

cooling + trapping for $v_{in} < v_c = 83$ m/s

Note: To complete the analogy, need $|B_0| = \frac{\hbar K v_c}{\Delta\mu}$ at $r = \sigma$