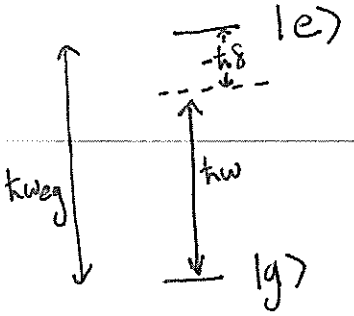
AC Stark effect / optical dipole traps

Lecture #17
10/24/2017
PHYS 598 AQG

Let's recall our treatment of a two level system, describing a quantum atom coupled interacting w/ a classical radiation field @ ang. frequency ω,



We showed that this system could be described by the effective Hamiltonian

$\delta = \omega - \omega_{eg}$

$$\tilde{H} = \hbar \begin{pmatrix} -\delta/2 & \Omega e^{-i\varphi} \\ \Omega e^{i\varphi} & \delta/2 \end{pmatrix}$$

$\boxed{\text{in the following, we'll set } \varphi = 0}$

This system obeyed dynamical evolution according to $i\hbar\dot{\tilde{\Psi}} = \tilde{H}\tilde{\Psi}$,

w/ $\tilde{\Psi} = \tilde{c}_g|\tilde{g}\rangle + \tilde{c}_e|\tilde{e}\rangle$

i.e.
$$\begin{cases} i\hbar\dot{\tilde{c}}_e = \hbar\Omega\tilde{c}_g - \frac{\hbar\delta}{2}\tilde{c}_e \\ i\hbar\dot{\tilde{c}}_g = \hbar\Omega\tilde{c}_e + \frac{\hbar\delta}{2}\tilde{c}_g \end{cases}$$

and ~ "rotating frame" transformation
$$\begin{cases} \tilde{c}_g = c_g e^{-i\delta t/2} \\ \tilde{c}_e = c_e e^{i\delta t/2} \end{cases}$$

We can describe the system in terms of "dressed" or "field-dressed" eigenstates $|1\rangle$ and $|2\rangle$, w/ energies

$$E_1 = -\hbar\sqrt{\Omega^2 + (\delta/2)^2} = -\hbar\tilde{\Omega}$$

$$E_2 = \hbar\sqrt{\Omega^2 + (\delta/2)^2} = +\hbar\tilde{\Omega}$$

and

$$|1\rangle = -\sin\tfrac{\theta}{2}|\tilde{e}\rangle + \cos\tfrac{\theta}{2}|\tilde{g}\rangle$$

$$|2\rangle = \cos\tfrac{\theta}{2}|\tilde{e}\rangle + \sin\tfrac{\theta}{2}|\tilde{g}\rangle$$

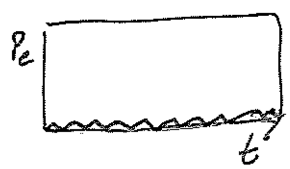
$$\tan(\theta) = -\frac{2\Omega}{\delta}$$

If we start in $|g\rangle$, i.e. w/ $\tilde{c}_g(t=0) = c_g(t=0) = 0$, then the probability to be found in the excited state $|e\rangle$ at some time t follows the Rabi form

$$P_e(t) = \left|\frac{\Omega}{\tilde{\Omega}}\right|^2 \sin^2(\tilde{\Omega} t) \quad \text{w/} \quad \tilde{\Omega} = \sqrt{\Omega^2 + (\delta/2)^2}$$

In the large detuning limit $(|\delta| \gg \Omega)$, we find that the actual excitation of the excited state is not significant, i.e. $P_e^{max} = \frac{\Omega^2}{\Omega^2 + (\delta/2)^2} \approx \frac{4\Omega^2}{\delta^2} \ll 1$

[time-averaged $\bar{P}_e$ is $\frac{2\Omega^2}{\delta^2}$

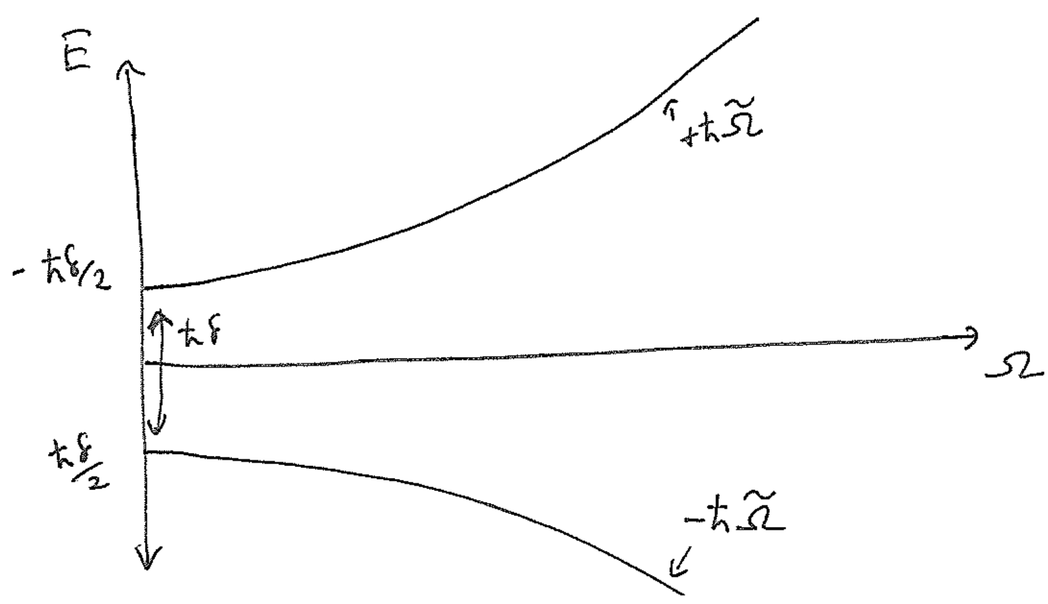


]

While the excited-state fraction of the "dressed" eigenstate may be ignorable, there is still an energy shift of state $|1\rangle$ due to the radiation

→ This is the "Light Shift" or "Dipole Potential"

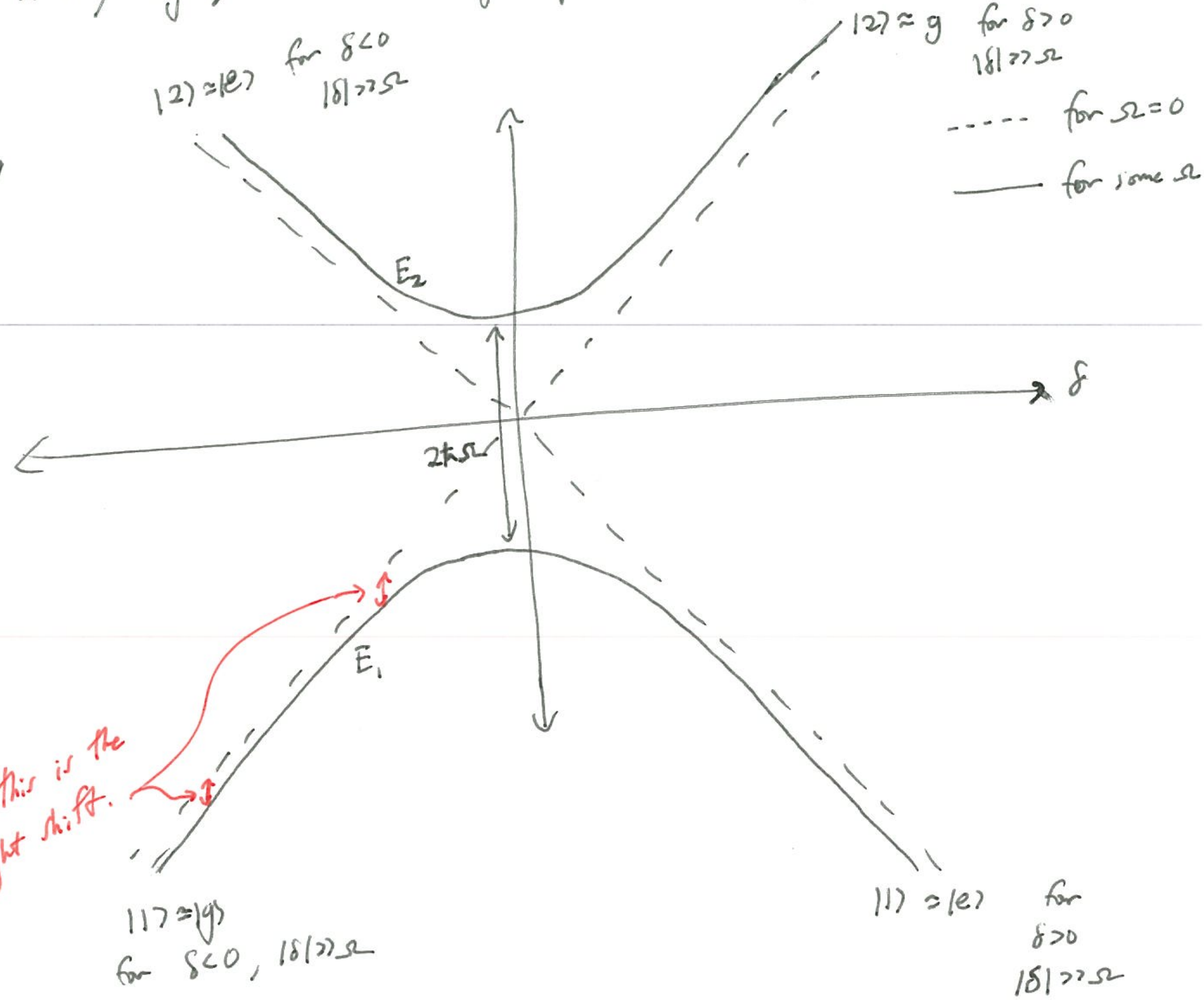
Shown for $\delta < 0$



(red-detuned) $\omega < \omega_{eg}$

For large negative detunings, $\delta < 0$, and $|\delta| \gg \Omega$, the state $|1\rangle$ is mostly $|g\rangle$, while for large positive detunings it is mostly $|e\rangle$.

Alternatively



For $\delta < 0$, $|\delta| \gg \Omega$, the ground state energy is effectively shifted by

$$\Delta E_1 = E_1(\Omega) - E_1(\Omega=0) = -\hbar\tilde{\Omega} - \left(\frac{\hbar\delta}{2}\right) = -\hbar\left[\sqrt{\Omega^2 + \left(\frac{\delta}{2}\right)^2} - \frac{|\delta|}{2}\right]$$

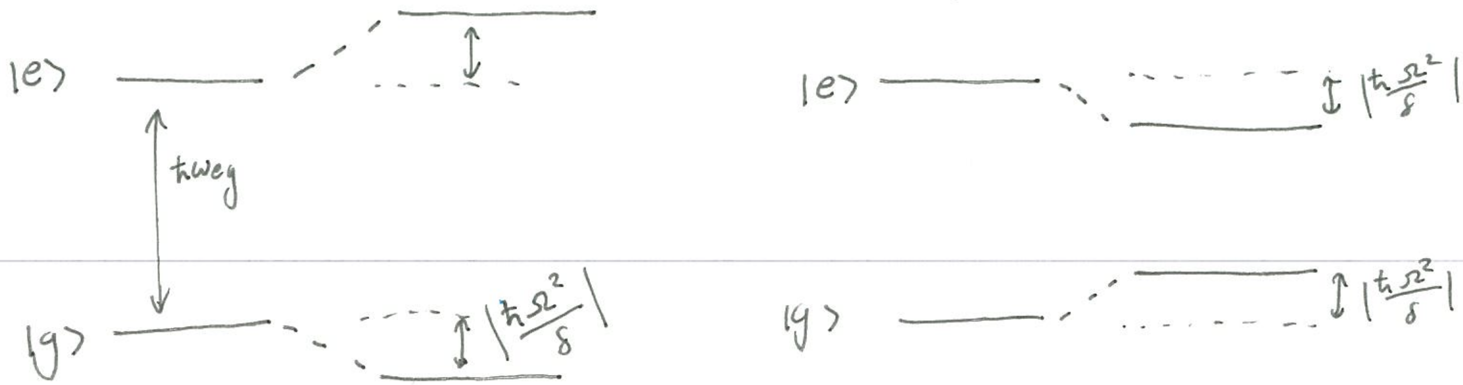
$$\Delta E_1 = -\frac{\hbar|\delta|}{2}\left[\sqrt{1+\left(\frac{2\Omega}{\delta}\right)^2} - 1\right] \approx -\frac{\hbar|\delta|}{2}\left[1 + \frac{1}{2}\left(\frac{2\Omega}{\delta}\right)^2 - 1\right] \quad \text{for } |\Omega/\delta| \ll 1$$

$$\Delta E_1 = -\hbar\frac{\Omega^2}{|\delta|} \quad \text{or} \quad \hbar\frac{\Omega^2}{\delta}$$

Representation of dressed-state's light shift

(red-detuned) $\quad\quad\quad\quad\quad\quad\quad\quad$ (blue-detuned)

$\underbrace{\delta < 0}$ $\quad\quad\quad\quad\quad\quad\quad\quad$ $\underbrace{\delta > 0}$



So, light shift is $\Delta E_1 = U = \frac{\hbar\Omega^2}{\delta}$, or using

$$\Omega^2 = \frac{\mu^2 |E_0|^2}{\hbar^2}$$

w/ $\mu = -e\langle e|\vec{r}\cdot\hat{\varepsilon}|g\rangle$

$$|E_0|^2 = \frac{2I}{\varepsilon_0 c}$$

and

$$\Gamma = \frac{\mu^2\omega^3}{3\pi\varepsilon_0\hbar c^3}$$

$\Rightarrow$ get $U = \frac{6\pi c^2}{\omega^3}\left(\frac{\Gamma}{\delta}\right) I$

using $\boxed{I_{sat} = \frac{hc\pi}{3\lambda^3}\Gamma}$

$$\boxed{U = \left(\frac{\hbar\Gamma}{2}\right)\left(\frac{\Gamma}{\delta}\right)\frac{I}{I_{sat}}}$$

and $\omega\lambda = 2\pi c$

Note: off from most conventions by factor of 4, due to use of Ω instead of $\Omega/2$, giving $\frac{\hbar\Omega^2}{\delta}$ instead of $\frac{\hbar\Omega^2}{4\delta}$.

Great, we get a light shift (which can be used to make optical potentials).

Do we have to worry about scattering?

That is, $\langle P_e \rangle \neq 0$, and our excited state decays.

We can estimate the effective scattering rate as

$$\Gamma_{sc} \approx \langle P_e \rangle \Gamma = \frac{1}{2}\left|\frac{\Omega}{\tilde{\Omega}}\right|^2 \Gamma \approx \frac{1}{2}\left(\frac{4\Omega^2}{\delta^2}\right)\Gamma = \left(\frac{\Gamma}{2}\right)\left(\frac{\Gamma}{\delta}\right)^2 \frac{I}{I_{sat}}$$

So, comparing the conservative effect

$$U = \left(\frac{\hbar\Gamma}{2}\right)\left(\frac{\Gamma}{\delta}\right)\frac{I}{I_{sat}}$$

to the non-conservative part

$$\hbar\Gamma_{scatt} = \left(\frac{\hbar\Gamma}{2}\right)\left(\frac{\Gamma}{\delta}\right)^2 \frac{I}{I_{sat}} = U\left(\frac{\Gamma}{\delta}\right)$$

we see that going to large detunings can mitigate the influence of spontaneous decay.

→ You need more intensity to get the same U if $\left(\frac{\Gamma}{\delta}\right)$ is smaller,

but Γ_{scatt} will be smaller [for the same U], and heating due to scattering will be less important. ["heating rate" ⇒ $H \approx \frac{2}{3} E_{rec}\Gamma_{scatt}$ energy deposition rate due to spont. ~~emission~~ emission

Optical dipole traps

So far we've shown that light / radiation can lead to an energy shift of the ground state, where

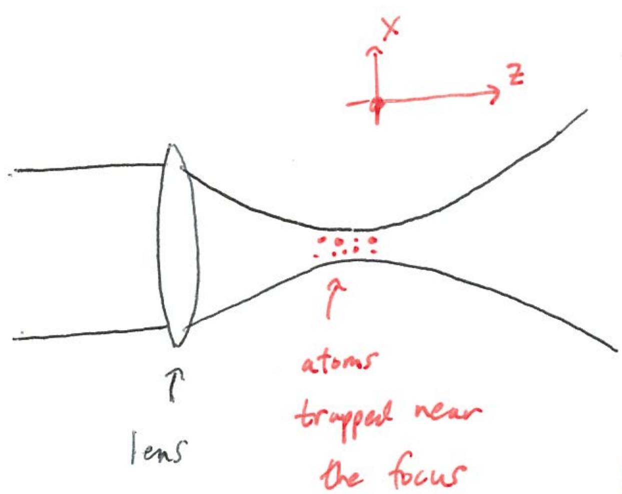
$$\mathcal{U} \propto \frac{I}{\delta} \quad \text{and } I \text{ is the intensity of our field.}$$

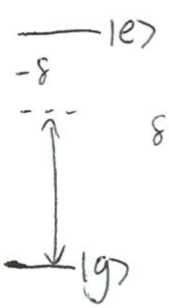
If our light is not a plane-wave, then $I = I(\vec{r})$ will have some spatial variation, and so will the ground state energy shift $\mathcal{U}(\vec{r})$.

Thus, we can make some energy landscape for our atoms, in the simplest case, we can think about using this to trap our atoms → conservatively, with minimal heating for large $|\delta|$. (for $\delta < 0$)

Simplest case

single, focused Gaussian beam





For Gaussian beam (HG_{00})

$$\boxed{I(\vec{r}) = \frac{2P}{\pi W(z)^2} e^{-2\rho^2/W(z)^2}}$$

w/ propagation along z

P = total power

$\rho = \sqrt{x^2+y^2}$ radial coor.

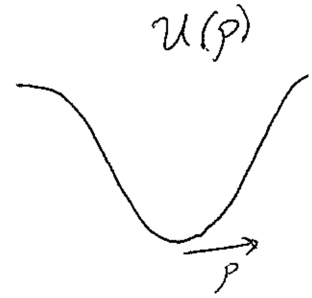
Waist → $W(z) = W_0 \sqrt{1 + \left(\frac{z-z_0}{z_R}\right)^2}$

W_0 = waist at focal position, z_0

$z_R = \pi W_0^2/\lambda$ the Rayleigh range

This gives us a "dipole trap" or "far-off-resonance trap" (FORT)

in which to confine atoms.



$$\mathcal{U}(\vec{r}) = \left(\frac{\hbar\Gamma}{2}\right)\left(\frac{\Gamma}{\delta}\right)\frac{2P}{I_{sat}\,\pi\,W(z)^2}\,e^{-\frac{2(x^2+y^2)}{W^2(z)}}$$

Near the center, where low-energy particles will spend most of their time, this can look roughly harmonic.

$$\left[\, e^{-2\rho^2/W^2} \simeq 1 - \frac{2\rho^2}{W^2} \quad \text{for } \rho \ll W \qquad \text{w/ } W = W(z)\right.$$

$$\left[\, \frac{1}{W^2} = \frac{1}{W_0^2}\,\frac{1}{1+\left(\frac{z}{z_R}\right)^2} \simeq \frac{1}{W_0^2}\left[1-\left(\frac{z}{z_R}\right)^2\right] \quad \text{for } z \ll z_R = \frac{\pi W_0^2}{\lambda}\right.$$

So $$e^{-\frac{2\rho^2}{W^2}} \simeq 1 - \frac{2\rho^2}{W^2} \simeq 1 - 2\rho^2\left[1-\left(\frac{z}{z_R}\right)^2\right]\frac{1}{W_0^2} \simeq \underbrace{1 - \frac{2\rho^2}{W_0^2}}_{\text{if } z \ll z_R}$$

$$\Rightarrow \frac{e^{-2\rho^2/W^2}}{W^2} \simeq \frac{1}{W_0^2}\left[1-\left(\frac{z}{z_R}\right)^2\right]\left[1-\frac{2\rho^2}{W_0^2}\right] \simeq \left[1 - \frac{2\rho^2}{W_0^2} - \left(\frac{z}{z_R}\right)^2\right]\frac{1}{W_0^2}$$

$$\simeq \left[1 - \frac{2\rho^2}{W_0^2} - \frac{\lambda^2 z^2}{\pi^2 W_0^4}\right]\frac{1}{W_0^2}$$

This gives us a potential

$$U = -U_0 \left[1 - \frac{2\rho^2}{w_0^2} - \frac{\lambda^2 z^2}{\pi w_0^4} \right]$$

where the <u>peak</u> depth is

$$U_0 = \frac{2P}{\pi w_0^2} \left(\frac{\hbar \Gamma}{2} \right) \frac{\Gamma}{|\delta|} \frac{1}{I_{sat}}$$

to get harmonic frequencies, set

$$\frac{1}{2} m \omega_\rho^2 \rho^2 = \frac{2 U_0 \rho^2}{w_0^2} \Rightarrow \omega_\rho = \sqrt{\frac{4 U_0}{m w_0^2}}$$

$$\frac{1}{2} m \omega_z^2 z^2 = \frac{U_0 z^2}{z_R^2} = \frac{U_0 \lambda^2 z^2}{\pi w_0^4} \Rightarrow \omega_z = \sqrt{\frac{2 U_0}{m z_R^2}} = \sqrt{\frac{2 \lambda^2 U_0}{\pi w_0^2 w_0^2 m}}$$

Can make a <u>stiffer</u> trap (higher ω)
by increasing U_0 (power) or decreasing w_0 (size, waist).

Also, typically $w_0 \gg \lambda$ [Rayleigh criterion limits it to $w_0 \sim \lambda$],

so typically $\omega_z = \omega_\rho \sqrt{\frac{\lambda}{2\pi w_0}} \ll \omega_\rho$ [less "stiff" along z]