AC Stark effect / optical dipole traps

Let's recall our treatment of a coupled two-level system, describing a quantum atom interacting with a classical radiation field $\hat{E}$.

We showed that this system could be described by the effective Hamiltonian

$$\tilde{H} = \hbar \left( -\frac{\delta}{2}, \begin{array}{cc} \alpha e^{i\varphi} & -i \varphi \\ -i \varphi & \frac{\delta}{2} \end{array} \right)$$

This system obeyed dynamical evolution according to $i\hbar \frac{\partial}{\partial t} \tilde{\Psi} = \tilde{H} \tilde{\Psi}$,

i.e.

$$\begin{align*}
\dot{\tilde{c}}_g &= \hbar \Delta \tilde{c}_g - i \frac{\varphi}{2} \tilde{c}_e \\
\dot{\tilde{c}}_e &= \hbar \Delta \tilde{c}_e + i \frac{\varphi}{2} \tilde{c}_g
\end{align*}$$

and

$$\tilde{\Psi} = \tilde{c}_g \tilde{1} + \tilde{c}_e \tilde{2}$$

The "rotating frame" transformation

$$\tilde{c}_g = \tilde{c}_g e^{-i \frac{\varphi}{2}}$$

We can describe the system in terms of "dressed" or "field-dressed" eigenstates $\ket{1}$ and $\ket{2}$,

W/ energies

$$\begin{align*}
E_1 &= -\hbar \sqrt{\Delta^2 + (\frac{\varphi}{2})^2} = -\hbar \widetilde{\alpha} \\
E_2 &= \hbar \sqrt{\Delta^2 + (\frac{\varphi}{2})^2} = +\hbar \widetilde{\alpha}
\end{align*}$$

and

$$\begin{align*}
\ket{1} &= -\sin \frac{\varphi}{2} \ket{1} + \cos \frac{\varphi}{2} \ket{2} \\
\ket{2} &= \cos \frac{\varphi}{2} \ket{1} + \sin \frac{\varphi}{2} \ket{2}
\end{align*}$$

$$\tan (\theta) = -\frac{2\varphi}{\Delta}$$
If we start in $|g\rangle$, i.e. $\tilde{c}_g(t=0) = c_g(t=0) = 0$, then the probability to be found in the excited state $|e\rangle$ at some time $t$ follows the Rabi formula:

$$P_e(t) = \frac{\Omega}{\Delta} \sin^2 \left( \frac{\Delta}{2} t \right) \quad \text{w/} \quad \Delta = \sqrt{\Delta^2 + \left(\frac{\Omega}{2}\right)^2}$$

In the large detuning limit $(|\Delta| >> \Omega)$, we find that the actual excitation of the excited state is not significant, i.e. $P_e^{\text{max}} = \frac{\Omega^2}{\Delta^2 + \left(\frac{\Omega}{2}\right)^2} \approx \frac{4\Omega^2}{8^2} \ll 1 \quad \left[ \text{time-averaged } P_e \text{ is } \frac{2\Omega^2}{8^2} \right]$.

While the excited-state fraction of the "dressed" eigenstate may be ignorable, there is still an energy shift of state $|e\rangle$ due to the radiation:

This is the "Light Shift" or "Dipole Potential" shown for $\delta < 0$. 

\[ E \]

- $\hbar \omega / 2$

\[ c \]

- $\hbar \omega / 2$

\[ + \hbar \omega \]

\[ \omega \]
For large negative detunings, $\delta < 0$, and $|\delta| > |\Omega|$, the state $|11\rangle$ is mostly $|g\rangle$, while for large positive detunings it is mostly $|e\rangle$.

Alternatively

$|12\rangle = |e\rangle$ for $\delta < 0$
$|11\rangle = |g\rangle$ for $\delta > 0$

For $\delta < 0$, $|\delta| > |\Omega|$, the ground state energy is effectively shifted by

$$\Delta E_1 = E_1(\Omega) - E_1(\Omega = 0) = -\hbar \Delta E_{\text{shift}} = -\hbar \left[ \sqrt{\Omega^2 + (\frac{2\delta}{\hbar})^2} - |\delta| \right]$$

$$\Delta E_1 = -\frac{\hbar |\delta|}{2} \left[ \sqrt{1 + \left(\frac{2\Omega}{\hbar}\right)^2} - 1 \right] \approx -\frac{\hbar |\delta|}{2} \left[ 1 + \frac{1}{2} \left(\frac{2\Omega}{\hbar}\right)^2 \right]$$

for $|\delta|/\hbar < 1$

$$\Delta E_1 = -\hbar \frac{\Omega^2}{18|\delta|} \text{ or } \frac{\hbar |\delta|}{\delta}$$
Representation of dressed-state's light shift

(Real-detuned)
\( \delta < 0 \)

\[ |e\rangle \quad \downarrow \quad |g\rangle \]

\[ \hbar \omega \frac{\gamma}{\delta} \]

(Blue-detuned)
\( \delta > 0 \)

\[ |e\rangle \quad \downarrow \quad |g\rangle \]

\[ \hbar \omega \frac{\gamma}{\delta} \]

So, light shift is \( \Delta E_i = U = \frac{\hbar \omega^2}{\delta} \), or using

\[ \frac{\Delta^2}{\omega^2} = \frac{\mu^2 |E_0|^2}{\hbar^2} \]

w/ \( \mu = -e \langle \hat{r} \hat{r} \cdot \hat{e}_1 | \rangle \)

\[ |E_0|^2 = \frac{2I}{\varepsilon_0 c} \]

\[ \gamma = \frac{\mu^2 \omega^3}{3\pi \varepsilon_0 \hbar c^3} \]

\[ U = \frac{6\pi c^2}{\omega^3} \left( \frac{\gamma}{\delta} \right) I \]

Using
\[ I_{\text{sat}} = \frac{\hbar c \pi}{3 \lambda^3} \Gamma \]

and \( \omega \lambda = 2\pi c \)

Note: off from most conventions by a factor of \( \frac{1}{4} \), due to use of \( \omega \) instead of \( \omega^2 \), giving \( \frac{\omega^2}{\delta} \) instead of \( \frac{\omega^2}{4\delta} \).
Great, we get a light shift (which can be used to make) optical potentials.

Do we have to worry about scattering?

That is, \( \langle P_e \rangle \neq 0 \), and our excited state decays.

We can estimate the effective scattering rate as

\[
\Gamma_{sc} \approx \langle P_e \rangle \Gamma = \frac{1}{2} \left| \frac{\omega}{\kappa} \right|^2 \Gamma = \frac{1}{2} \left( \frac{4 \omega^2}{\kappa^2} \right) \Gamma = \left( \frac{\Gamma}{2} \right) \left( \frac{\Gamma}{8} \right) \frac{1}{I_{sat}}
\]

So, comparing the conservative effect

\[
U = \left( \frac{\Gamma}{2} \right) \left( \frac{\Gamma}{8} \right) \frac{1}{I_{sat}}
\]

d to the non-conservative part

\[
\Gamma_{satt} = \left( \frac{\Gamma}{2} \right) \left( \frac{\Gamma}{8} \right) \frac{1}{I_{sat}} = U \left( \frac{\Gamma}{8} \right)
\]

we see that going to large detunings can mitigate the influence of spontaneous decay.

→ You need more intensity to get the same \( U \) if \( \left( \frac{\Gamma}{8} \right) \) is smaller, but \( \Gamma_{satt} \) will be smaller [for the same \( U \)], and \( \text{heating due to scattering} \) will be less important. \( \text{["heating rule"} \Rightarrow H \approx 2 \text{Free } \Gamma_{satt} \text{ emission} \)
Optical dipole traps

So far we've shown that light/radiation can lead to an energy shift of the ground state, where

\[ U \propto \frac{I}{8} \text{ and } I \text{ is the intensity of our field.} \]

If our light is not a plane-wave, then \( I = I(\mathbf{r}) \) will have some spatial variation, and so will the ground state energy \( U(\mathbf{r}) \).

Thus, we can make some energy landscape for our atoms. In the simplest case, we can think about using this to trap our atoms \( \rightarrow \) conservatively, with minimal heating for large \( \frac{1}{\Delta I} \) \( \text{for } \Delta I < 0 \).

\begin{align*}
\text{For Gaussian beam } (H_{\infty}) \\
I(\mathbf{r}) &= \frac{2P}{\pi W(\mathbf{z})^2} e^{-\frac{2r^2}{W(\mathbf{z})^2}}
\end{align*}

\( P = \text{total power} \)
\( r = \sqrt{x^2 + y^2} \; \text{radial coord.} \)
\( W(\mathbf{z}) = \text{Waist } \rightarrow W(\mathbf{z}) = W_0 \sqrt{1 + \left(\frac{z - z_0}{2r}\right)^2} \)

\( W_0 = \text{Waist at focal position, } z_0 \)
\( z_R = \pi W_0^2 / \lambda \text{ the Rayleigh range} \)

**Simplest case**

- Single, focused Gaussian beam

\[ 1e^x \]
\[ 1e^y \]
\[ z \]
\[ 8<0 \]
\[ \text{atoms trapped near the focus} \]
This gives us a "dipole trap" or "far-off-resonance trap" (FORT) 
in which to confine atoms:

\[ U(r) = \frac{\hbar^2}{2m} \frac{2P}{I_{\text{sat}}} \frac{2}{W(2)} e^{-\frac{2(x^2+y^2)}{W^2}} \]

Near the center, where low-energy particles will spend 
most of their time, this can look roughly harmonic.

\[ e^{-\frac{2p^2}{W^2}} \approx 1 - \frac{2p^2}{W^2} \quad \text{for} \quad p \ll W \quad \text{w/} \quad W = W(z) \]

\[ \frac{1}{W^2} = \frac{1}{W_0^2} \frac{1}{1 + \left(\frac{Z}{Z_R}\right)^2} \approx \frac{1}{W_0^2} \left[ 1 - \left(\frac{Z}{Z_R}\right)^2 \right] \quad \text{for} \quad Z \ll Z_R = \frac{\pi W_0^2}{\lambda} \]

So \[ e^{-\frac{2p^2}{W^2}} \approx 1 - \frac{2p^2}{W^2} \approx 1 - 2p^2 \frac{1 - \left(\frac{Z}{Z_R}\right)^2}{W_0^2} \approx 1 - \frac{2p^2}{W_0^2} \quad \text{if} \quad Z \ll Z_R \]

\[ \Rightarrow \quad \frac{e^{-\frac{2p^2}{W^2}}}{W^2} \approx \frac{1}{W_0^2} \left[ 1 - \left(\frac{Z}{Z_R}\right)^2 \right] \left[ 1 - \frac{2p^2}{W_0^2} \right] \approx \left[ 1 - \frac{2p^2}{W_0^2} - \left(\frac{Z}{Z_R}\right)^2 \frac{1}{W_0^2} \right] \approx \left[ 1 - \frac{2p^2}{W_0^2} - \frac{\lambda^2 Z^2}{\pi^2 W_0^4} \right] \frac{1}{W_0^2} \]
This gives us a potential

$$U = -U_0 \left[ 1 - \frac{2p^2}{W_0^2} - \frac{\lambda^2 z^2}{\pi W_0^4} \right]$$

where the peak depth is

$$U_0 = \frac{2p}{\pi W_0^2} \left( \frac{h'}{2} \right) \frac{1}{181 \text{ I sat}}$$

to get harmonic frequencies, set

$$\frac{1}{2} m w^2 p^2 = \frac{2U_0 p^2}{W_0^2} \Rightarrow \omega_p = \sqrt{\frac{4U_0}{m W_0^2}}$$

$$\frac{1}{2} m w^2 z^2 = \frac{U_0 z^2}{Z_{Re}^2} = \frac{U_0 \lambda^2 z^2}{\pi W_0^4} \Rightarrow \omega_z = \sqrt{\frac{2U_0}{m z^2}} = \sqrt{\frac{2\lambda^2 U_0}{\pi W_0^2 W_K^2}}$$

can make a stiffer body (higher \( \omega \))

by increasing \( U_0 \) (power) or decreasing \( W_0 \) (size, mass).

Also, typically \( W_0 \gg \lambda \) \cite{Rayleigh criterion limits it to \( W_0 \sim \lambda \)}.

so typically \( \omega_z = \omega_p \sqrt{\frac{\lambda}{2\pi W_0}} \ll \omega_p \) \cite{less "stiff" along z}. 