Lecture 2: Fine structure, Lamb shift

Readings: Foot 1.4, 2.3

Last Time: eigenstates and energies for hydrogen-like atoms

lots of energy degeneracy, only *n*-dependence

$$\psi_{nlm} = R_{nl}(r) Y_{lm}(\theta, \varphi)$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}$$

$$E_n = -\frac{Z^2}{n^2} hcR_\infty (\mu / m) = -\frac{1}{2}\mu c^2 \left(\frac{Z\alpha}{n}\right)^2$$

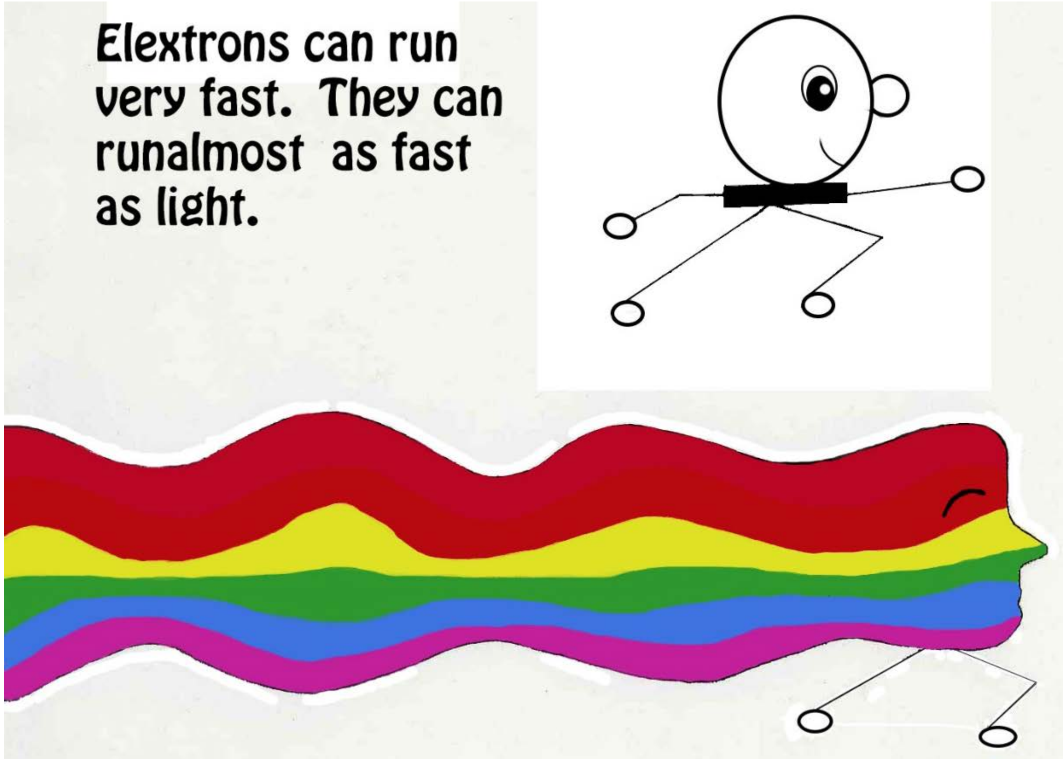
Today: corrections to these energies due to stuff we ignored

[e.g. relativity, electron spin, QED, etc.]

new terms will be come in at higher powers of α

Relativistic effects: (1) KE correction

electrons *can* run very fast, and so we need to consider relativistic corrections to their kinetic energy ($p^2/2m$)



Relativistic effects: (1) KE correction

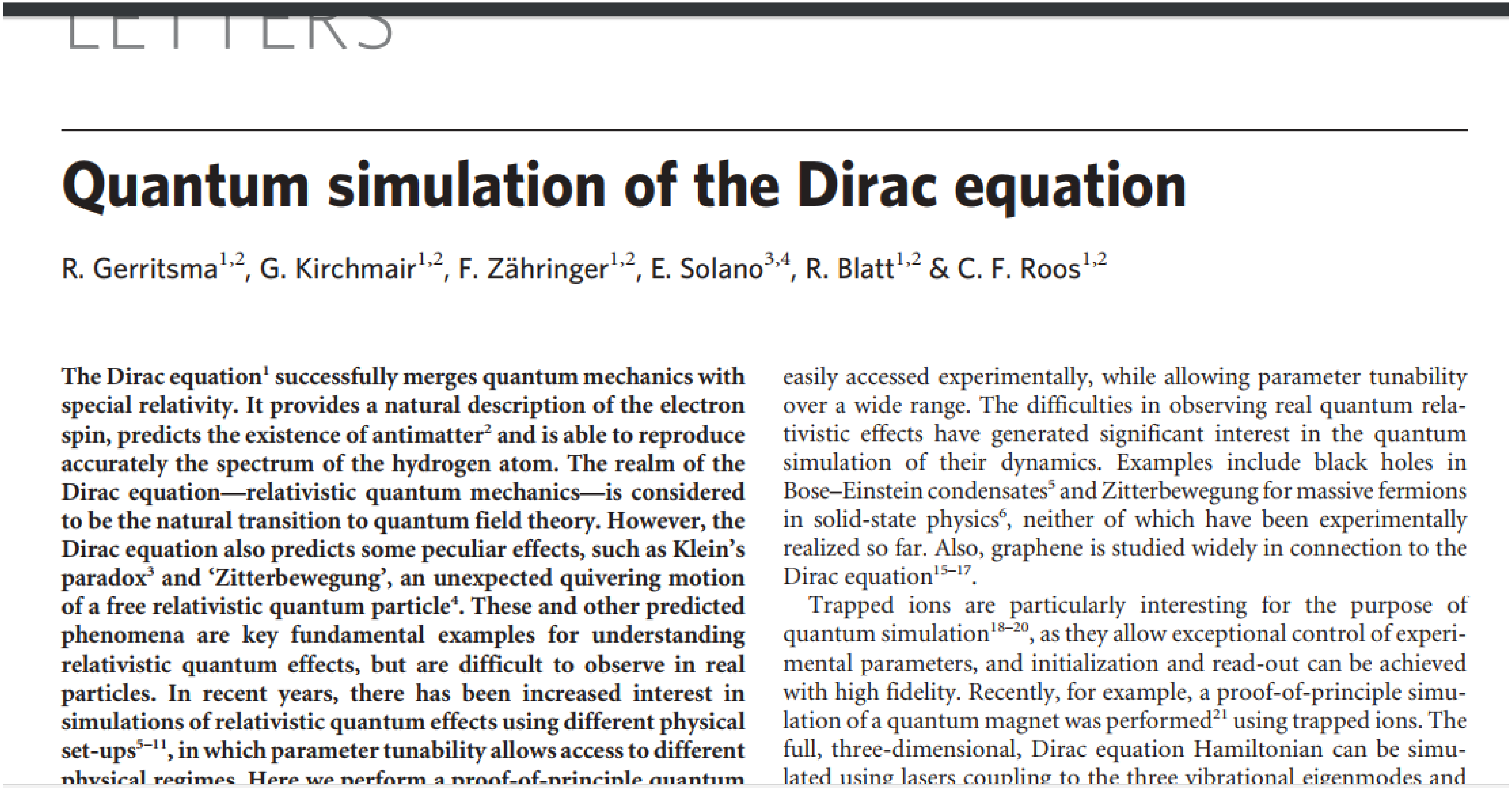
$$KE = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2} - mc^2 = \ldots$$

Relativistic effects: (2) Darwin term

The electron actually wiggles very fast, on a short length scale (Zitterbewegung)

Effectively feels a “time-averaged” or “wiggle-averaged” potential, modifying the energy when the electron is near the nucleus

Relativistic effects: (2) Darwin term



LETTERS

Quantum simulation of the Dirac equation

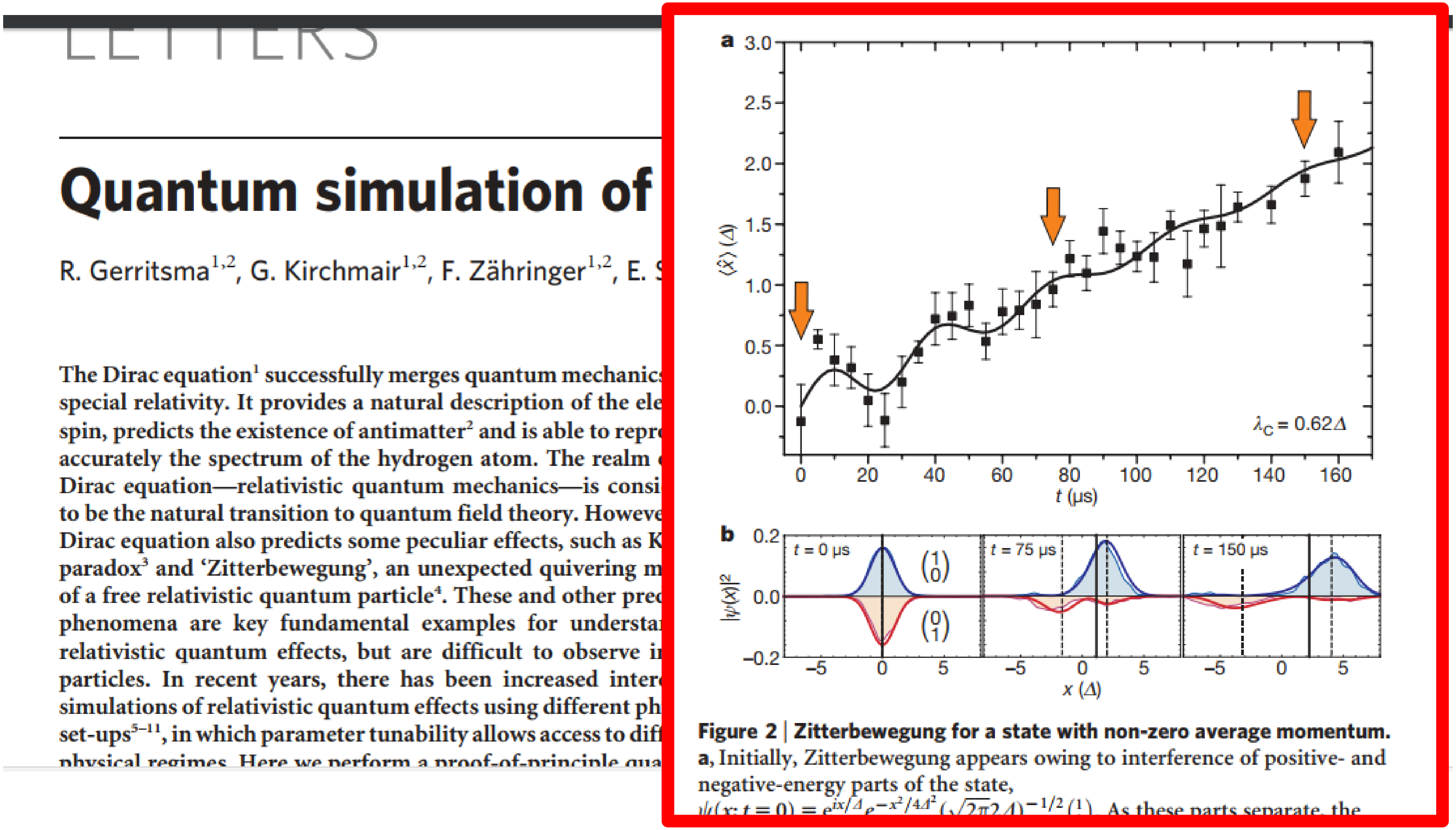
R. Gerritsma[1,2], G. Kirchmair[1,2], F. Zähringer[1,2], E. Solano[3,4], R. Blatt[1,2] & C. F. Roos[1,2]

The Dirac equation[1] successfully merges quantum mechanics with special relativity. It provides a natural description of the electron spin, predicts the existence of antimatter[2] and is able to reproduce accurately the spectrum of the hydrogen atom. The realm of the Dirac equation—relativistic quantum mechanics—is considered to be the natural transition to quantum field theory. However, the Dirac equation also predicts some peculiar effects, such as Klein's paradox[3] and 'Zitterbewegung', an unexpected quivering motion of a free relativistic quantum particle[4]. These and other predicted phenomena are key fundamental examples for understanding relativistic quantum effects, but are difficult to observe in real particles. In recent years, there has been increased interest in simulations of relativistic quantum effects using different physical set-ups[5–11], in which parameter tunability allows access to different physical regimes. Here we perform a proof-of-principle quantum

easily accessed experimentally, while allowing parameter tunability over a wide range. The difficulties in observing real quantum relativistic effects have generated significant interest in the quantum simulation of their dynamics. Examples include black holes in Bose–Einstein condensates[5] and Zitterbewegung for massive fermions in solid-state physics[6], neither of which have been experimentally realized so far. Also, graphene is studied widely in connection to the Dirac equation[15–17].

Trapped ions are particularly interesting for the purpose of quantum simulation[18–20], as they allow exceptional control of experimental parameters, and initialization and read-out can be achieved with high fidelity. Recently, for example, a proof-of-principle simulation of a quantum magnet was performed[21] using trapped ions. The full, three-dimensional, Dirac equation Hamiltonian can be simulated using lasers coupling to the three vibrational eigenmodes and

Relativistic effects: (2) Darwin term



LETTERS

Quantum simulation of

R. Gerritsma[1,2], G. Kirchmair[1,2], F. Zähringer[1,2], E. S

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Figure 2 | Zitterbewegung for a state with non-zero average momentum. **a,** Initially, Zitterbewegung appears owing to interference of positive- and negative-energy parts of the state, $\psi(x;t=0)=e^{ix/\Delta}e^{-x^2/4\Delta^2}(\sqrt{2\pi}2\Delta)^{-1/2}$ As these parts separate, the

Relativistic effects: (2) Darwin term

New Journal of Physics
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Direct observation of zitterbewegung in a Bose–Einstein condensate

L J LeBlanc[1,3,7], **M C Beeler**[1,4], **K Jiménez-García**[1,2,5], **A R Perry**[1], **S Sugawa**[1], **R A Williams**[1,6] **and I B Spielman**[1,7]
[1] Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, MD 20899, USA
[2] Departamento de Física, Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional, México, DF 07360, Mexico
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New Journal of Physics **15** (2013) 073011 (11pp)
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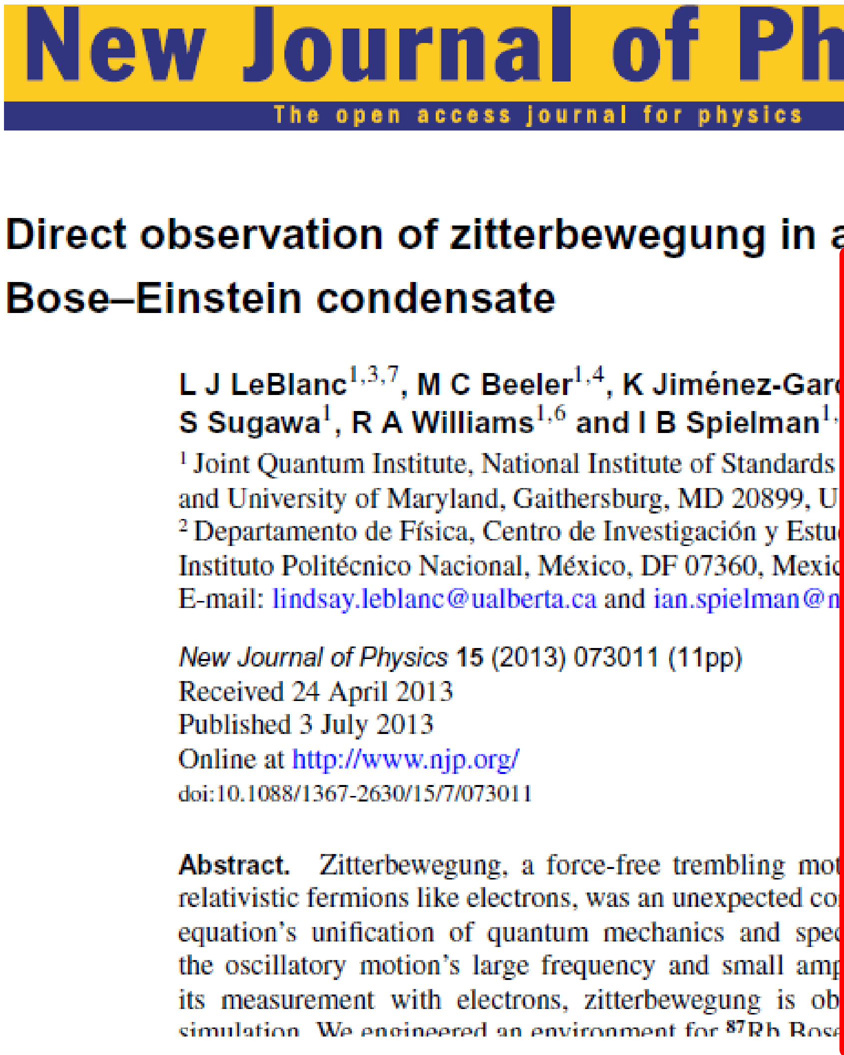
Abstract. Zitterbewegung, a force-free trembling motion first predicted for relativistic fermions like electrons, was an unexpected consequence of the Dirac equation's unification of quantum mechanics and special relativity. Though the oscillatory motion's large frequency and small amplitude have precluded its measurement with electrons, zitterbewegung is observable via quantum simulation. We engineered an environment for ^{87}Rb Bose–Einstein condensates

Relativistic effects: (2) Darwin term



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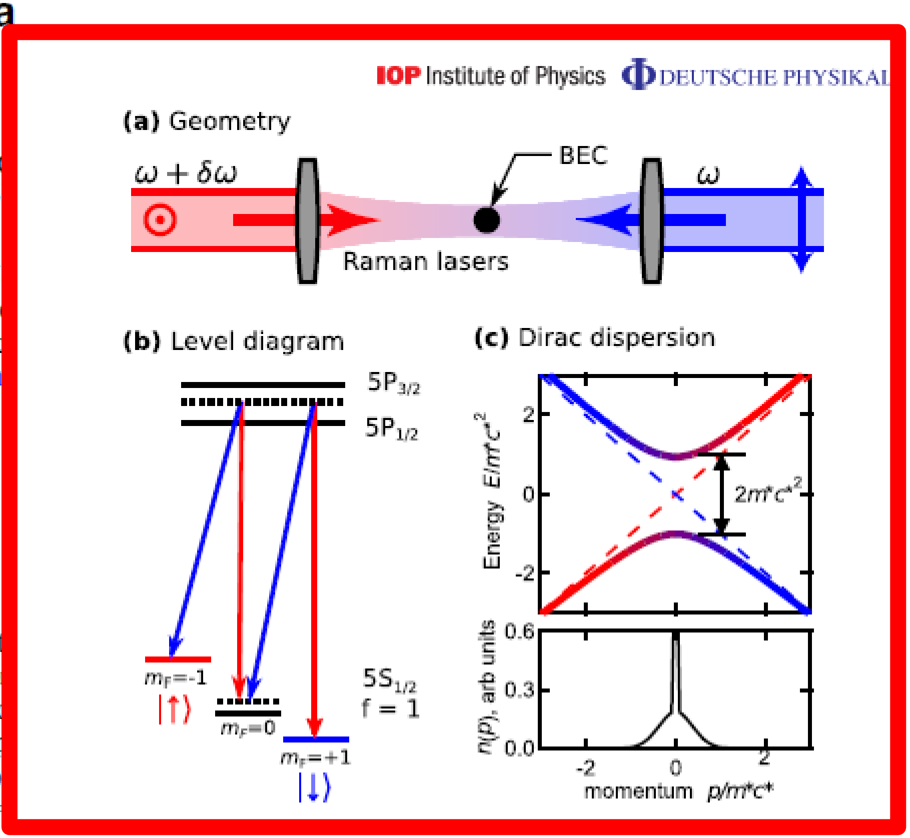
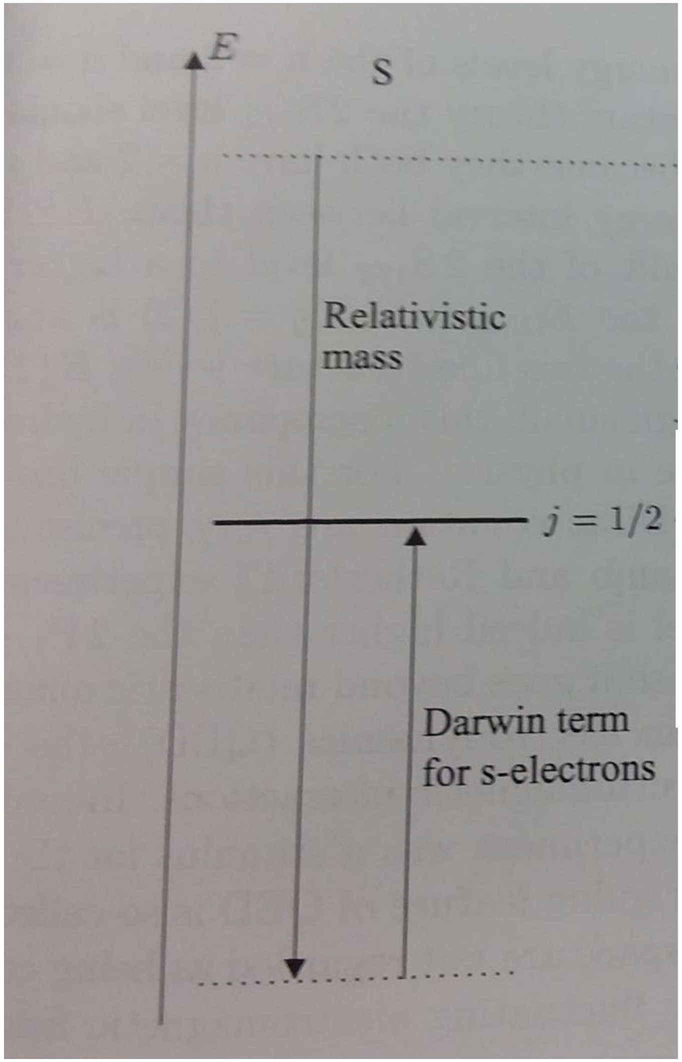


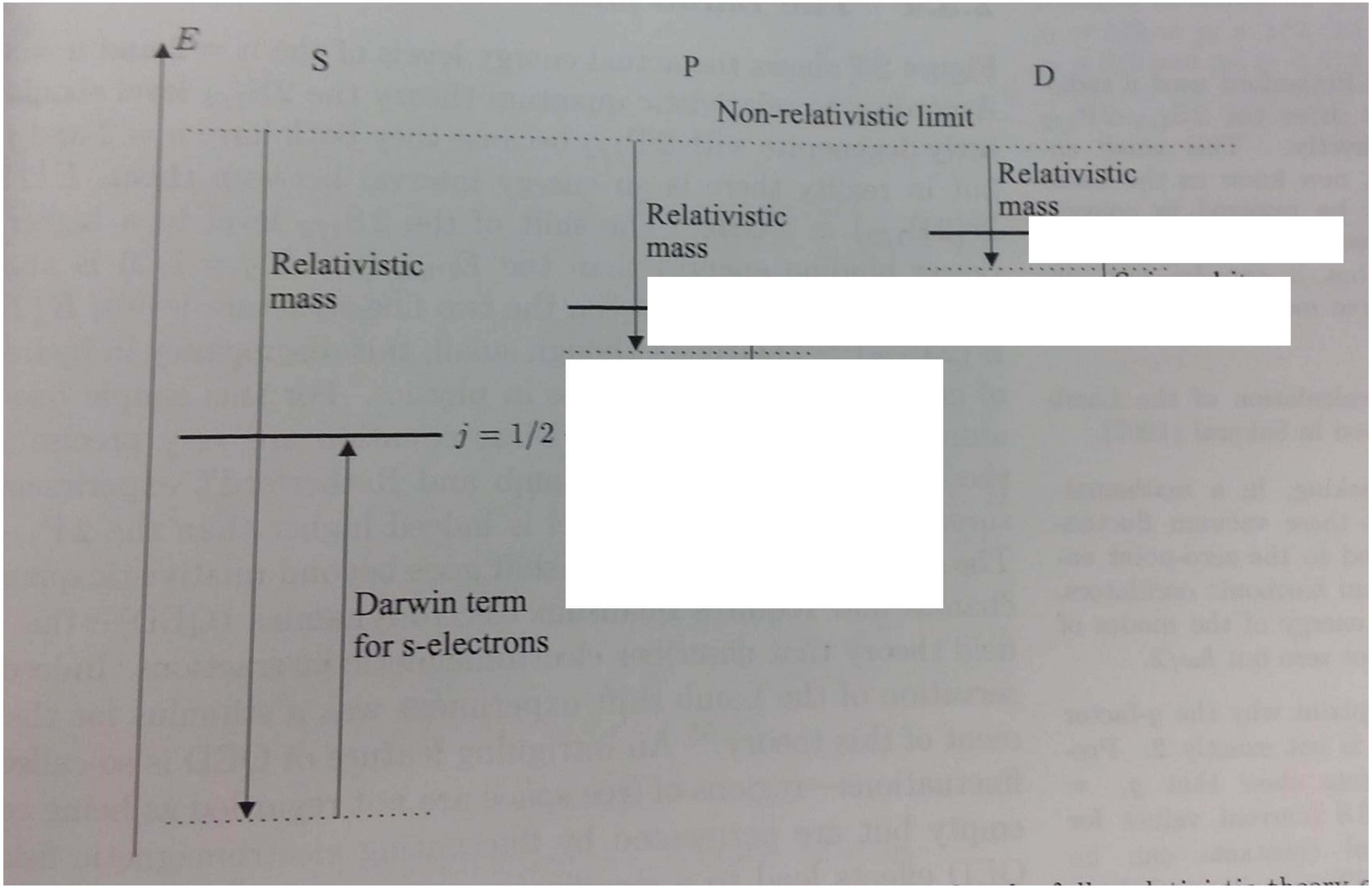
Relativistic effects: (2) Darwin term

$$H_D = \frac{\pi}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)\xi_{wiggle}^2\,\delta(\vec{r})$$

Relativistic effects: (1&2)



Relativistic effects: (1&2)

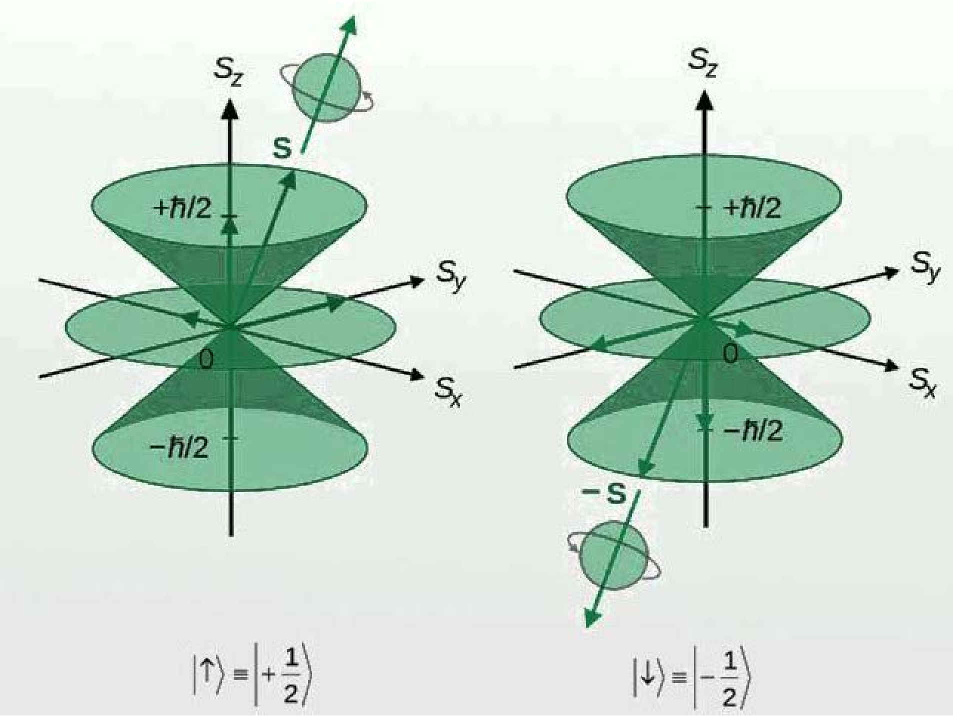


Relativistic effects: (3) Spin-Orbit

$$\vec{B} = -\frac{\vec{v}}{c^2} \times \vec{E}$$

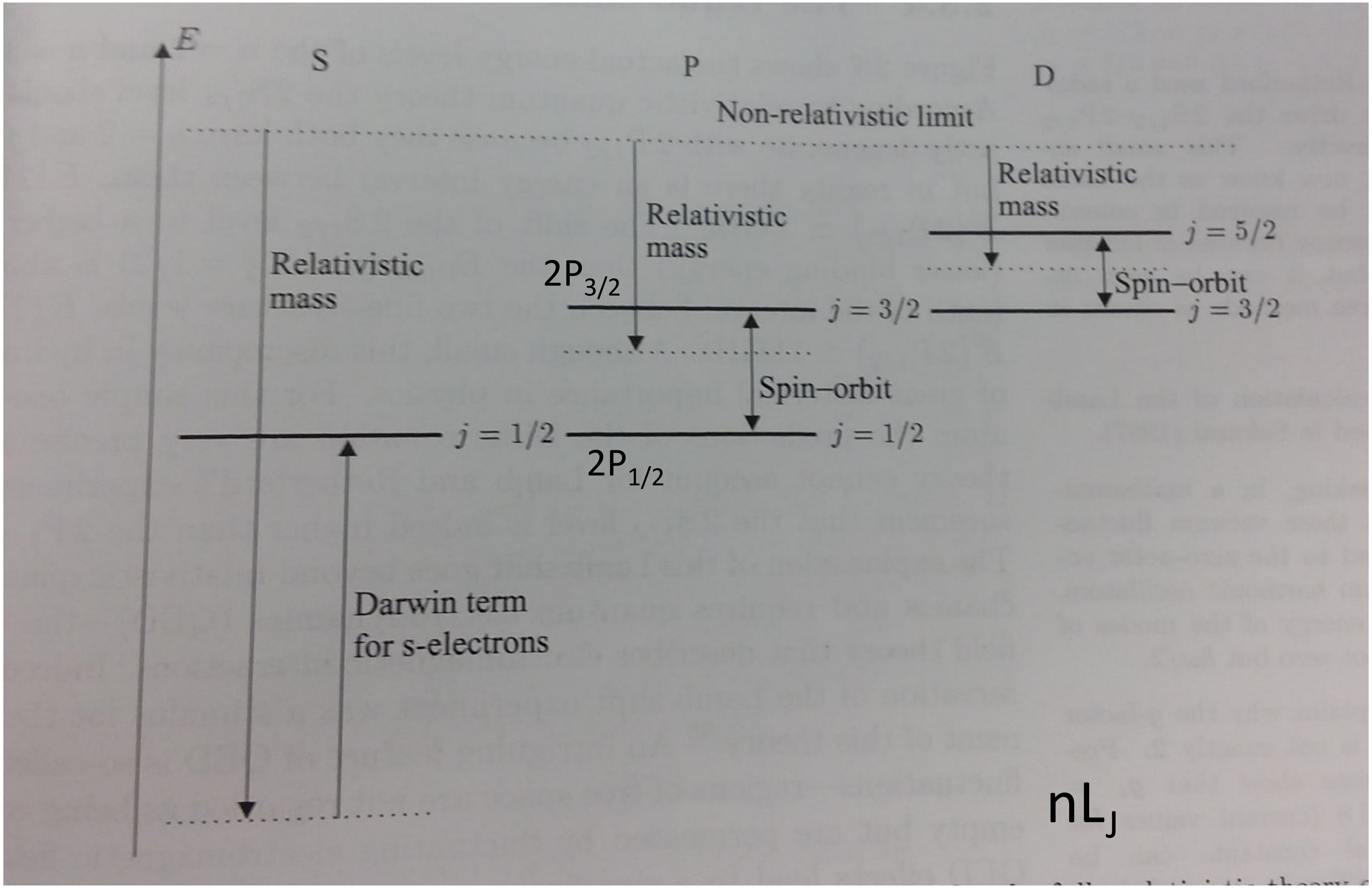
$$\vec{\mu} = -g_s \mu_B \vec{s}$$

$$\Delta E = -\vec{\mu} \cdot \vec{B}$$



wikipedia

Fine structure

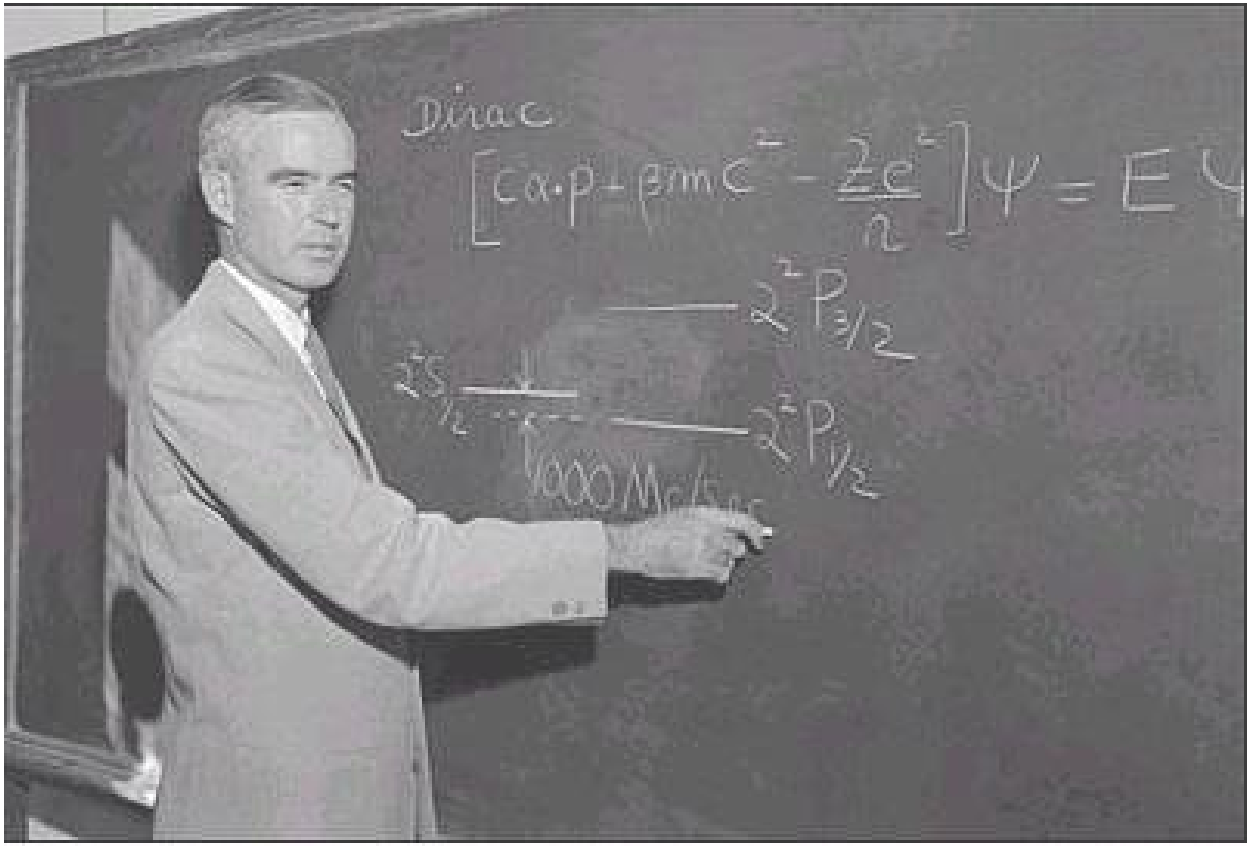


nL_J

Lamb shift – interactions with the vacuum

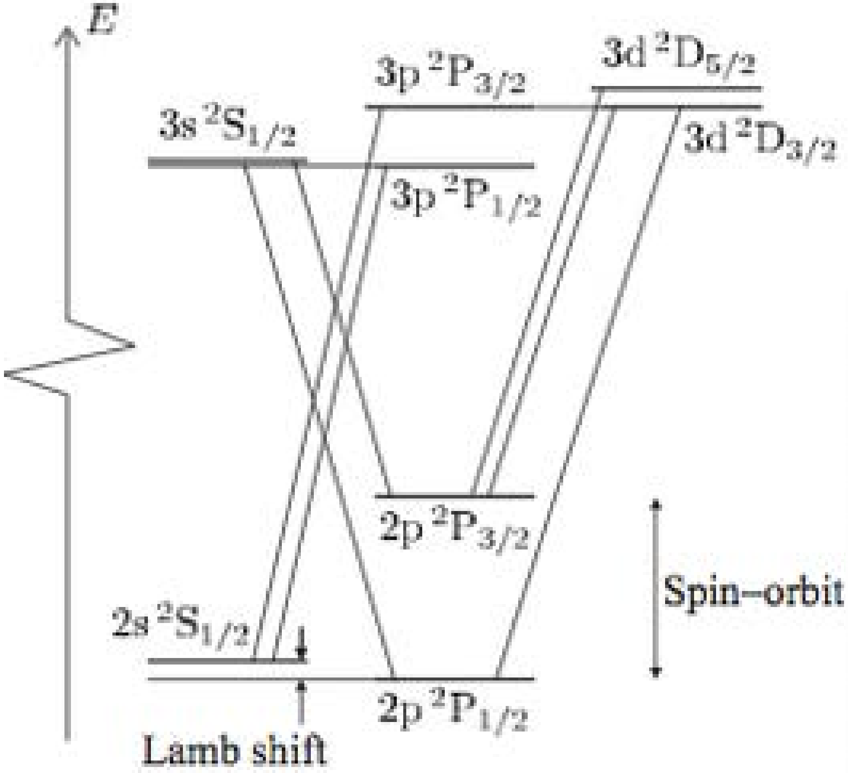
Observed energy shift between states with same *J* value

Also, no explanation for spontaneous emission



Willis E. Lamb Jr (1913 - 2008)

Lamb shift – interactions with the vacuum



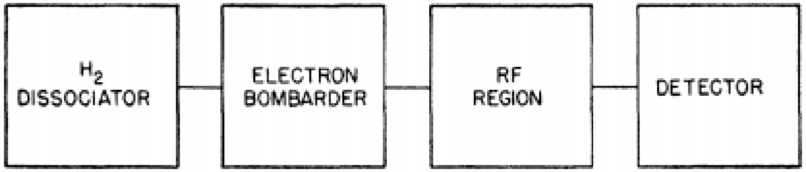
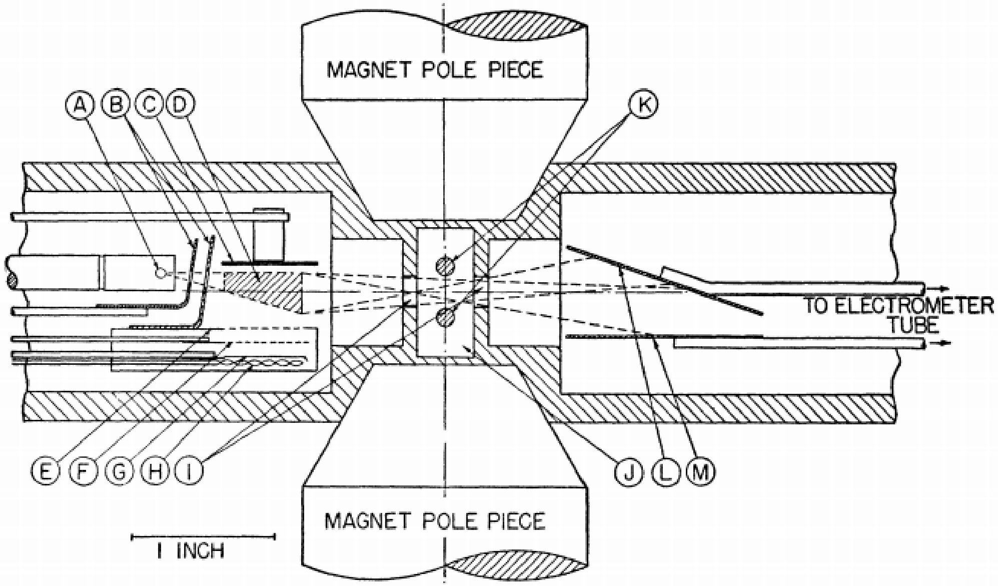
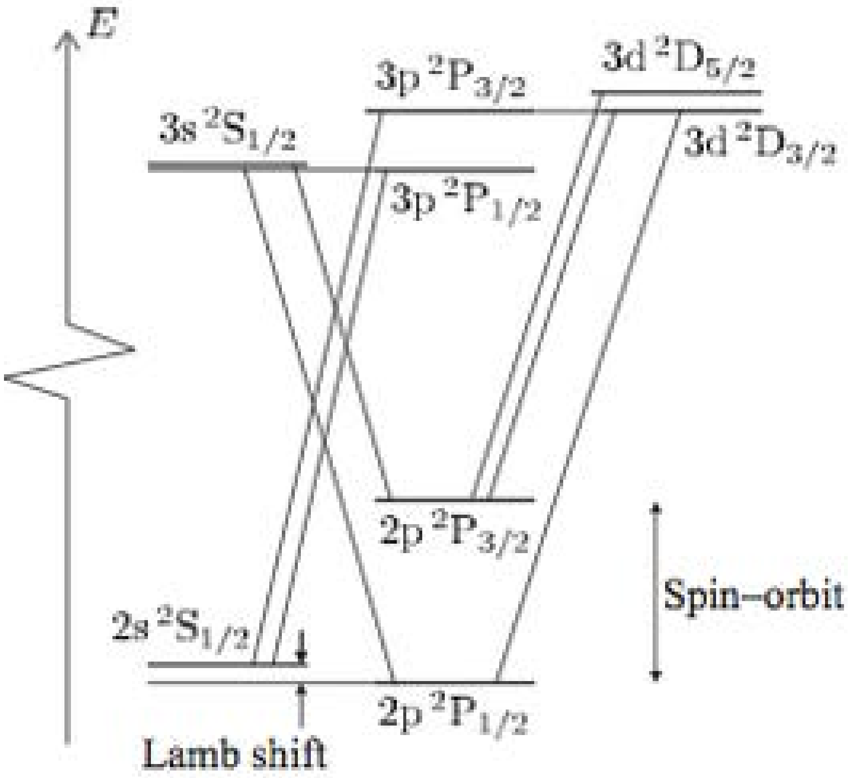


Fig. 4. Modified schematic block diagram of apparatus.



Lamb shift – interactions with the vacuum



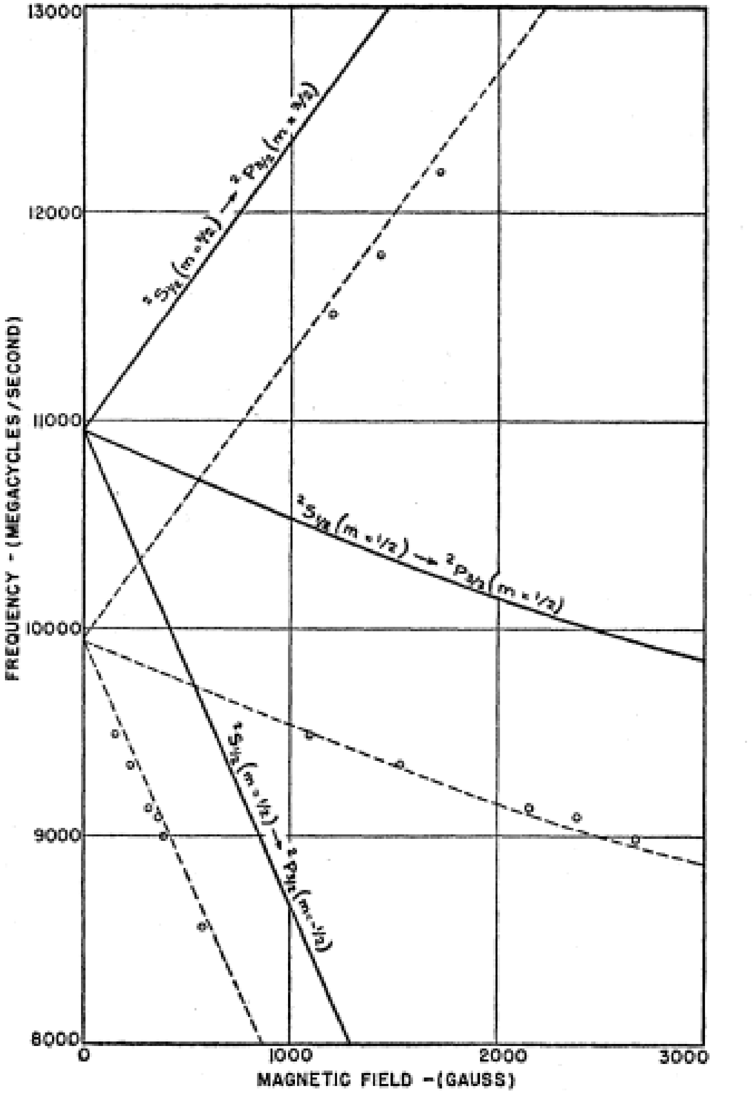
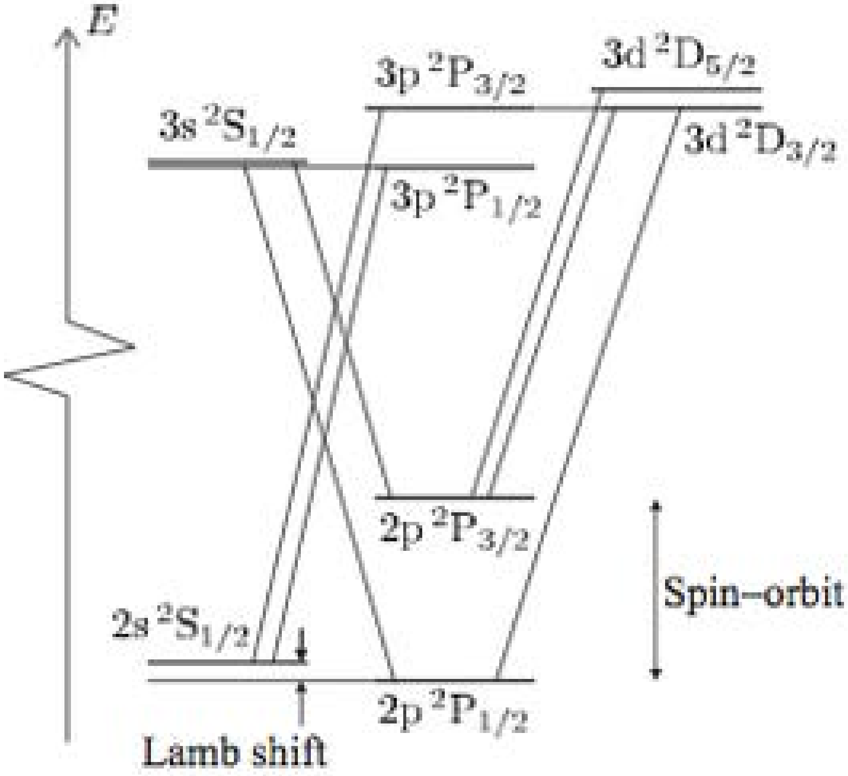


FIG. 2. Experimental values for resonance magnetic

Lamb shift – interactions with the vacuum



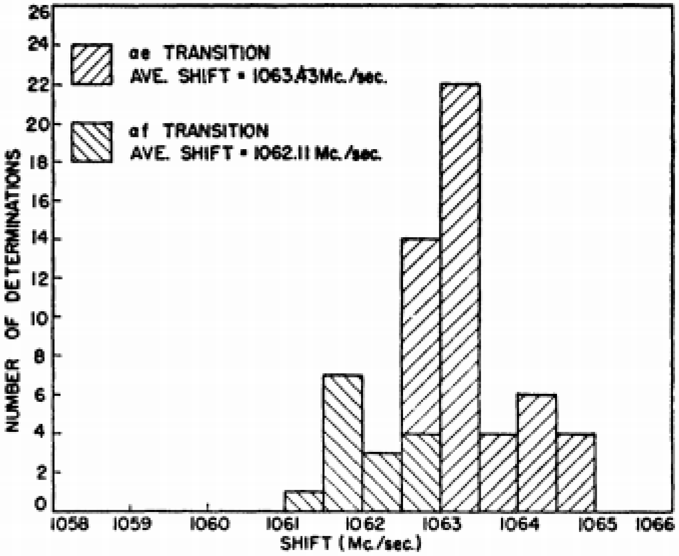
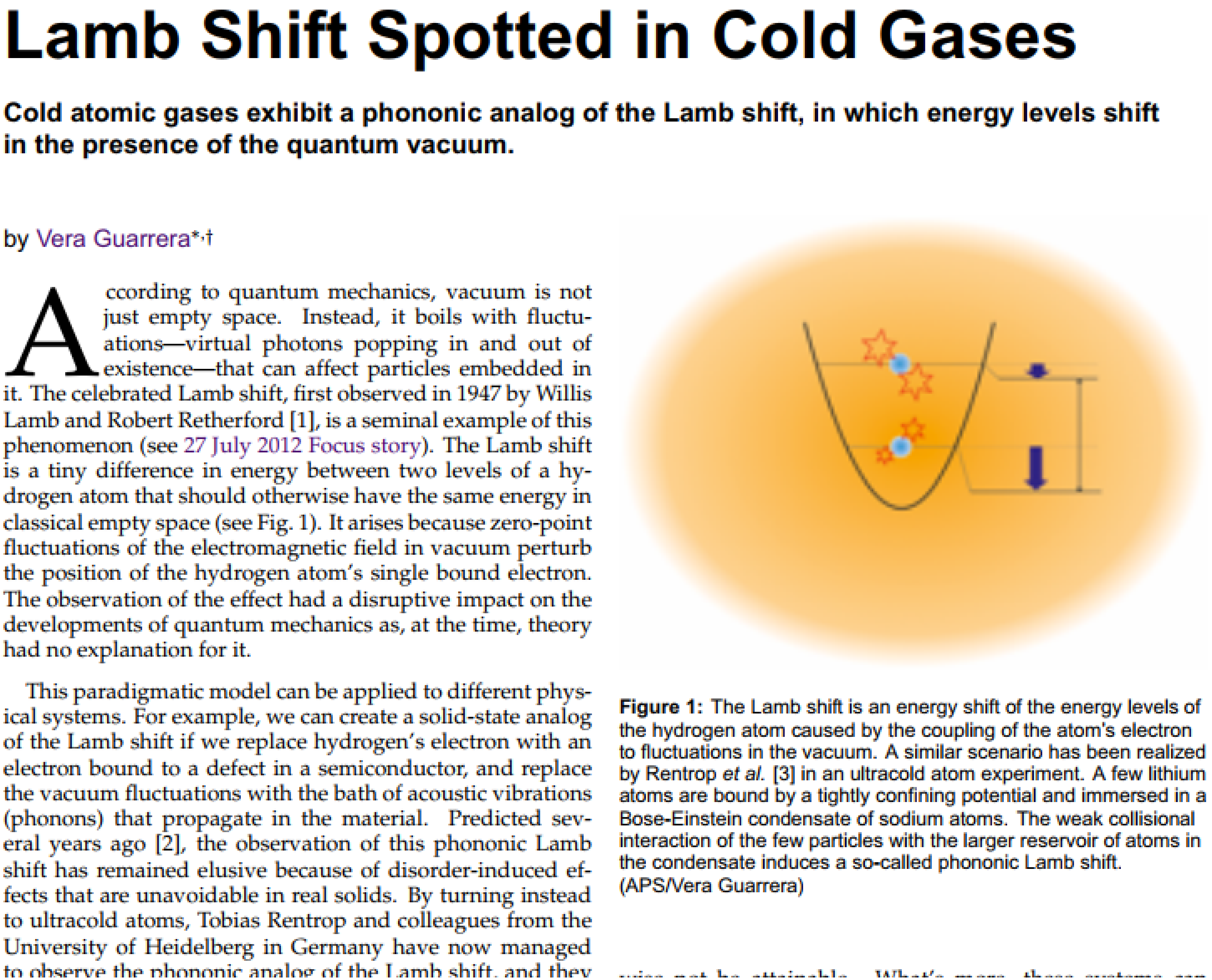


FIG. 41. Histogram showing distribution of results obtained from 61 determinations of $2^2S_{\frac{1}{2}}$ level shift. The difference between results obtained from αe and αf would presumably be removed if the corrections for the errors listed in Sec. 45 were applied. The finally determined level shift may be expected to lie in the range 1062±5 Mc/sec.

value from Udem paper 1045.0079(72) MHz

Lamb shift – coupling to virtual phonons



Lamb Shift Spotted in Cold Gases

Cold atomic gases exhibit a phononic analog of the Lamb shift, in which energy levels shift in the presence of the quantum vacuum.

by Vera Guarrera*,†

According to quantum mechanics, vacuum is not just empty space. Instead, it boils with fluctuations—virtual photons popping in and out of existence—that can affect particles embedded in it. The celebrated Lamb shift, first observed in 1947 by Willis Lamb and Robert Retherford [1], is a seminal example of this phenomenon (see 27 July 2012 Focus story). The Lamb shift is a tiny difference in energy between two levels of a hydrogen atom that should otherwise have the same energy in classical empty space (see Fig. 1). It arises because zero-point fluctuations of the electromagnetic field in vacuum perturb the position of the hydrogen atom's single bound electron. The observation of the effect had a disruptive impact on the developments of quantum mechanics as, at the time, theory had no explanation for it.

This paradigmatic model can be applied to different physical systems. For example, we can create a solid-state analog of the Lamb shift if we replace hydrogen's electron with an electron bound to a defect in a semiconductor, and replace the vacuum fluctuations with the bath of acoustic vibrations (phonons) that propagate in the material. Predicted several years ago [2], the observation of this phononic Lamb shift has remained elusive because of disorder-induced effects that are unavoidable in real solids. By turning instead to ultracold atoms, Tobias Rentrop and colleagues from the University of Heidelberg in Germany have now managed to observe the phononic analog of the Lamb shift, and they

Figure 1: The Lamb shift is an energy shift of the energy levels of the hydrogen atom caused by the coupling of the atom's electron to fluctuations in the vacuum. A similar scenario has been realized by Rentrop *et al.* [3] in an ultracold atom experiment. A few lithium atoms are bound by a tightly confining potential and immersed in a Bose-Einstein condensate of sodium atoms. The weak collisional interaction of the few particles with the larger reservoir of atoms in the condensate induces a so-called phononic Lamb shift. (APS/Vera Guarrera)

wise not be attainable. What's more, these systems can

Precision measurements in AMO physics

www.sciencemag.org/news/2010/09/superaccurate-clocks-confirm-your-hair-aging-faster-your-toenails

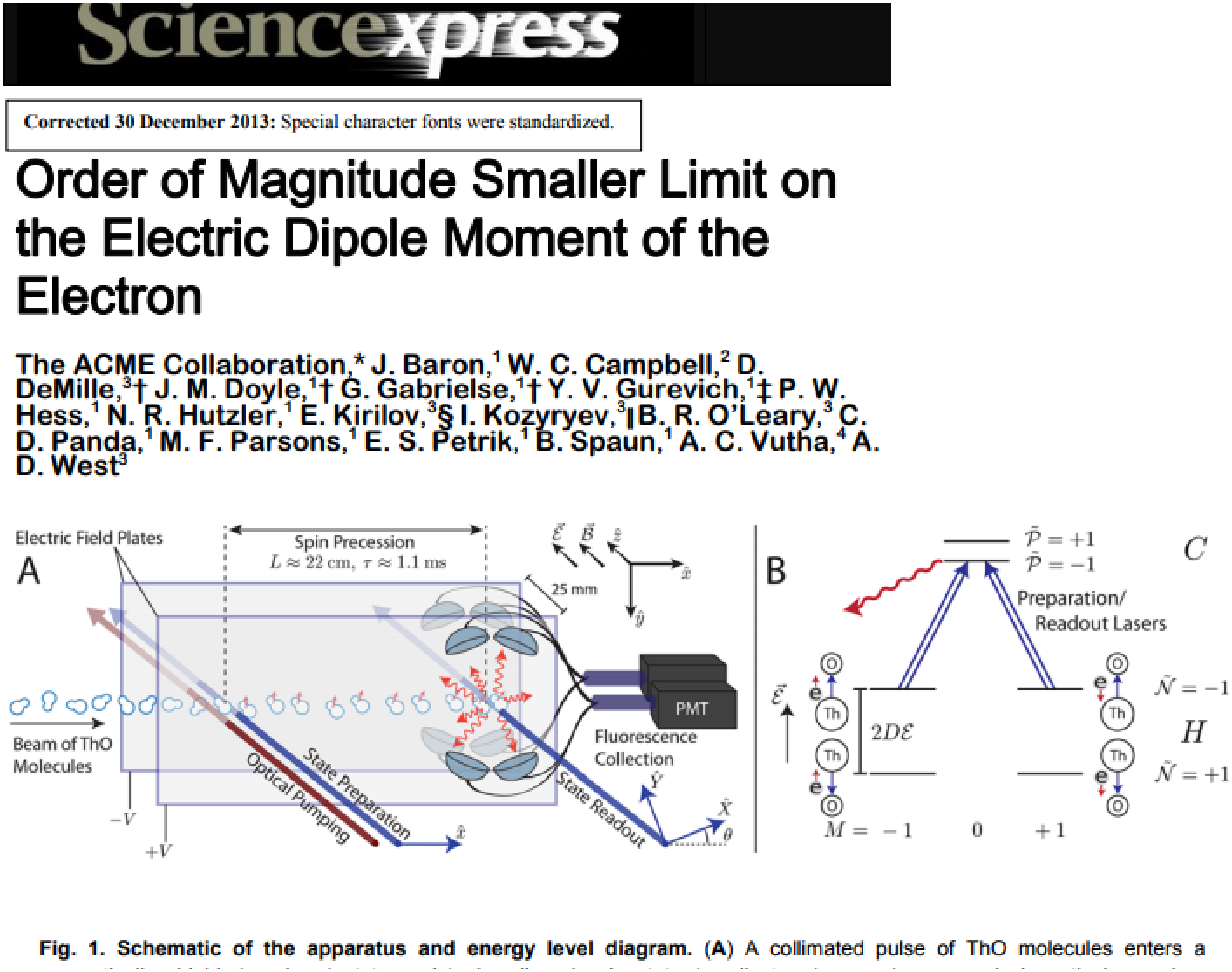
Out of sync. Because of the effects of gravity, a clock high on a wall should run ever so slightly faster than a watch just below it.

Loel Barr/NIST

Superaccurate Clocks Confirm Your Hair Is Aging Faster Than Your Toenails

By Adrian Cho | Sep. 23, 2010 , 5:13 PM

Precision measurements in AMO physics



Sciencexpress

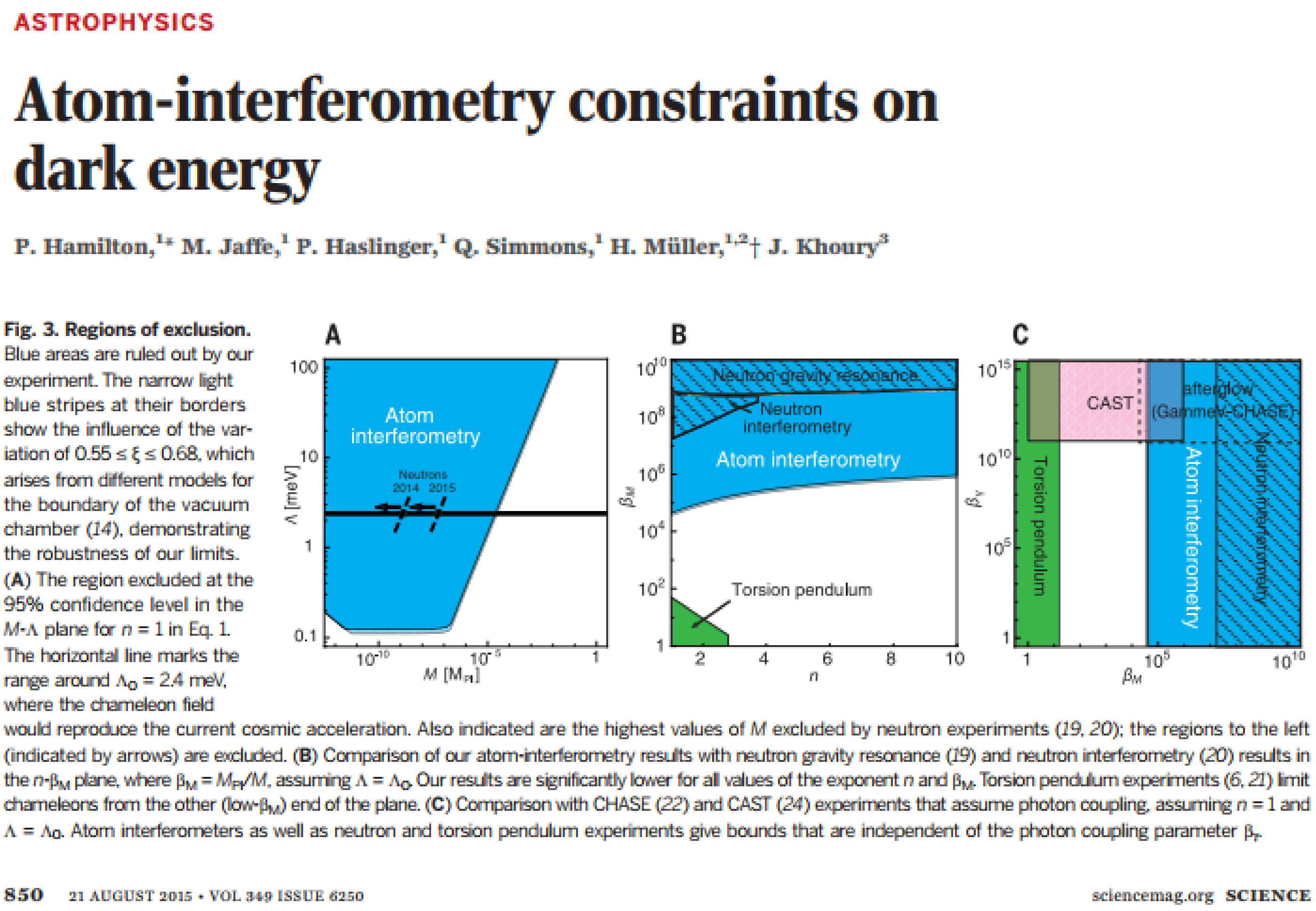
Corrected 30 December 2013: Special character fonts were standardized.

Order of Magnitude Smaller Limit on the Electric Dipole Moment of the Electron

The ACME Collaboration,* J. Baron,[1] W. C. Campbell,[2] D. DeMille,[3]† J. M. Doyle,[1]† G. Gabrielse,[1]† Y. V. Gurevich,[1]‡ P. W. Hess,[1] N. R. Hutzler,[1] E. Kirilov,[3]§ I. Kozyryev,[3]‖ B. R. O'Leary,[3] C. D. Panda,[1] M. F. Parsons,[1] E. S. Petrik,[1] B. Spaun,[1] A. C. Vutha,[4] A. D. West[3]

Fig. 1. Schematic of the apparatus and energy level diagram. (A) A collimated pulse of ThO molecules enters a

Precision measurements in AMO physics



ASTROPHYSICS

Atom-interferometry constraints on dark energy

P. Hamilton,[1*] M. Jaffe,[1] P. Haslinger,[1] Q. Simmons,[1] H. Müller,[1,2]† J. Khoury[3]

Fig. 3. Regions of exclusion. Blue areas are ruled out by our experiment. The narrow light blue stripes at their borders show the influence of the variation of $0.55 \leq \xi \leq 0.68$, which arises from different models for the boundary of the vacuum chamber (*14*), demonstrating the robustness of our limits. (**A**) The region excluded at the 95% confidence level in the M-Λ plane for $n = 1$ in Eq. 1. The horizontal line marks the range around $\Lambda_O = 2.4$ meV, where the chameleon field would reproduce the current cosmic acceleration. Also indicated are the highest values of M excluded by neutron experiments (*19*, *20*); the regions to the left (indicated by arrows) are excluded. (**B**) Comparison of our atom-interferometry results with neutron gravity resonance (*19*) and neutron interferometry (*20*) results in the n-β_M plane, where $\beta_M = M_{Pl}/M$, assuming $\Lambda = \Lambda_O$. Our results are significantly lower for all values of the exponent n and β_M. Torsion pendulum experiments (*6*, *21*) limit chameleons from the other (low-β_M) end of the plane. (**C**) Comparison with CHASE (*22*) and CAST (*24*) experiments that assume photon coupling, assuming $n = 1$ and $\Lambda = \Lambda_O$. Atom interferometers as well as neutron and torsion pendulum experiments give bounds that are independent of the photon coupling parameter β_γ.

850 21 AUGUST 2015 • VOL 349 ISSUE 6250

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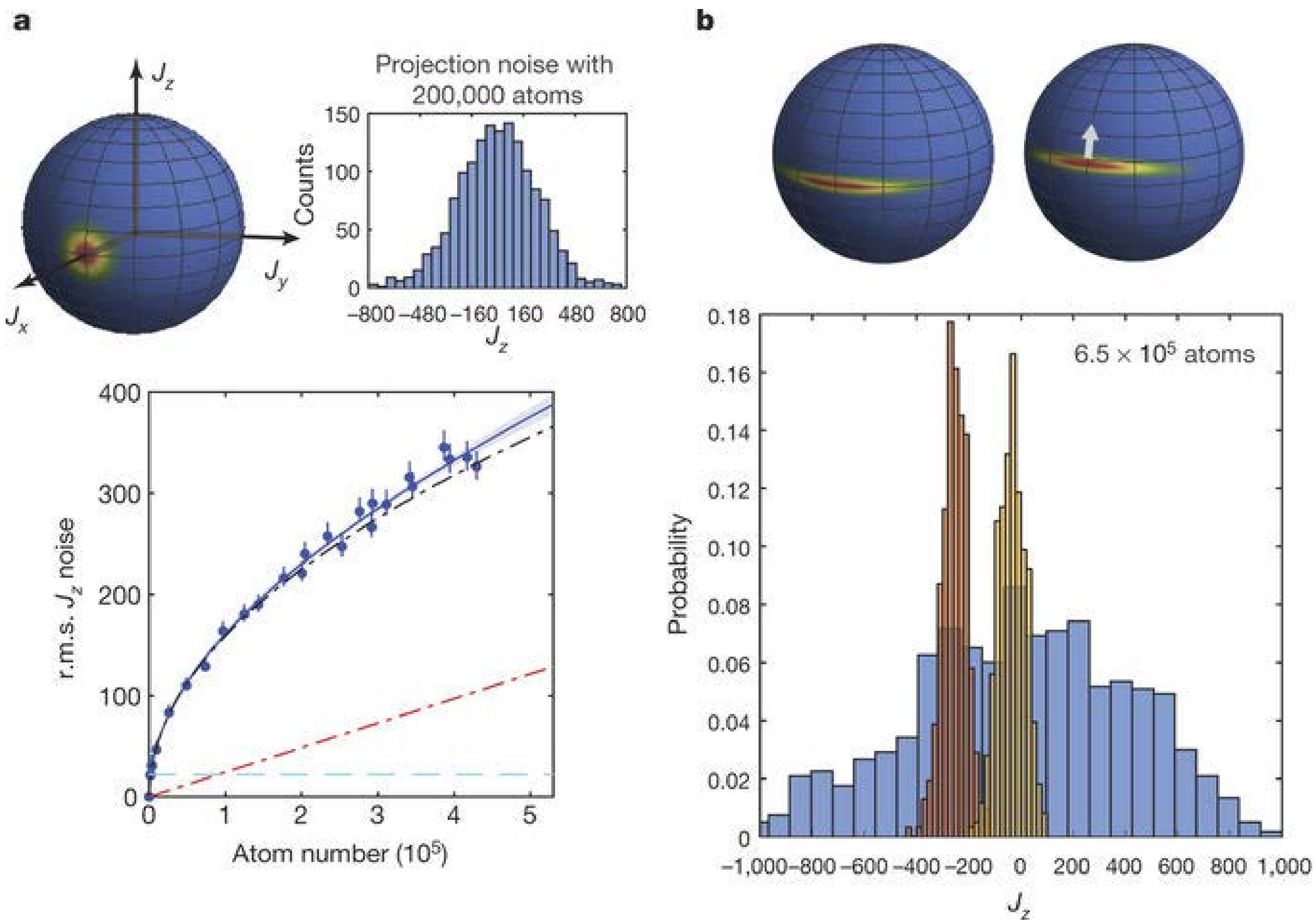
Quantum-limited measurements



Kasevich group, Nature 2016

Looking ahead

Readings: Foot Ch.3

Next week: multi-electron atoms, and then hyperfine structure

Homework due next Friday, office hours on Wed / Thurs