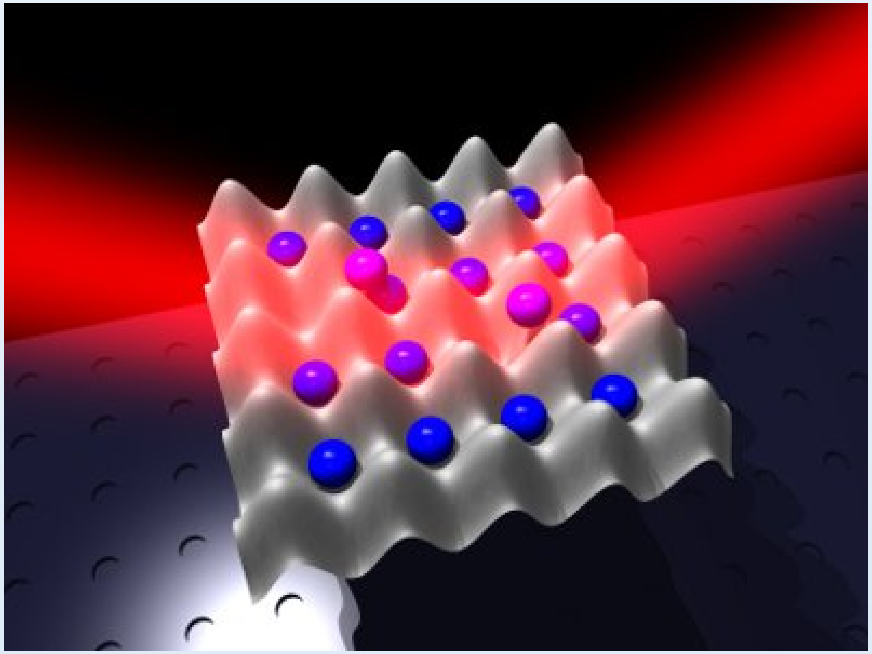
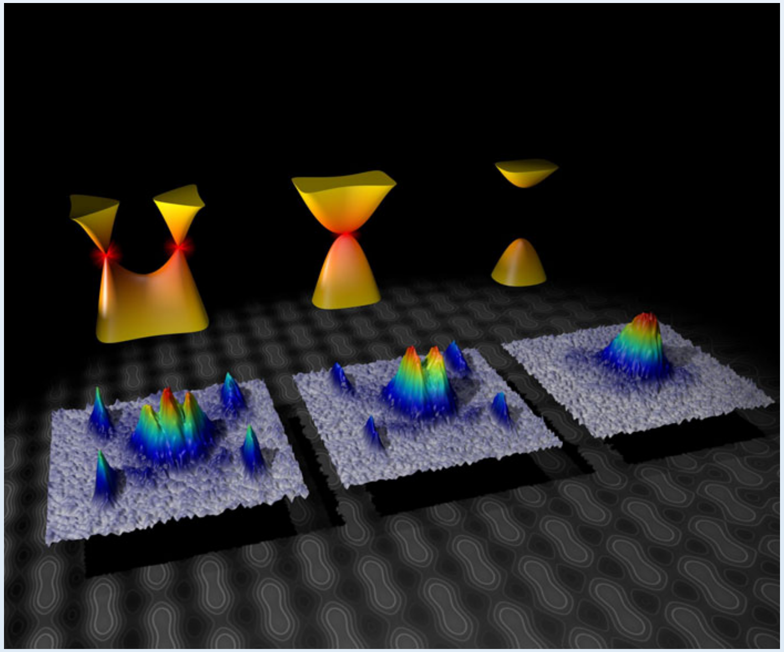
Cold atoms in optical lattices



www.lens.unifi.it



Tarruel, Nature – Esslinger group

Optical lattices – the big picture

We have a textbook model, which is basically exact, describing how a large collection of atoms will behave in free space

$$H = T + V + U$$

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + g_{ij} \sum_{i,j} \delta\left(\vec{r}_i - \vec{r}_j\right)$$

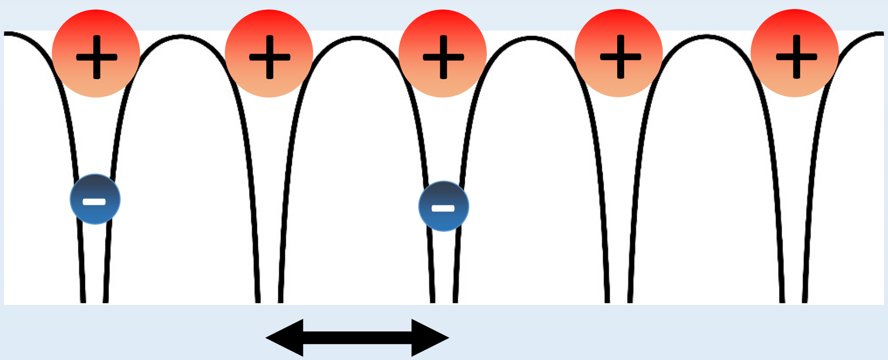
pair-wise contact interactions

We can add a potential term (V), modify the kinetic energy term (T), and even get qualitatively new, effective interaction terms (U) by putting the atoms into **optical lattices**

Optical lattices – the big picture

One obvious analogy: motion of atomic matter waves in laser crystals
~ motion of electron waves in ionic crystal lattices

Electron matter in solid-state crystals



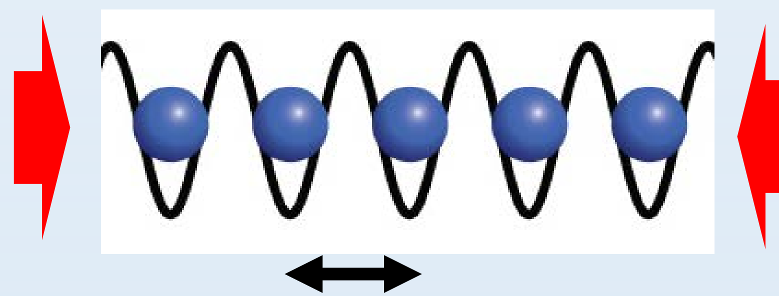
~ 0.2-0.8 nm

Extremely dilute – less dense than H_2O

Ultracold – nK and pK temperatures

No disorder (no phonons or defects)

Atomic matter in light crystals



λ/2 ~ 0.2-0.8 μm

"exact" microscopic model 

 **low energy scales**

$E \propto 1/mL^2$

Optical lattices – the big picture

Not just limited to studying electronic systems – many problems can be mapped from continuum to lattice (example: lattice QCD)

IOP Publishing
Reports on Progress in Physics
Rep. Prog. Phys. 79 (2016) 014401 (30pp)
doi:10.1088/0034-4885/79/1/014401

Report on Progress

Quantum simulations of lattice gauge theories using ultracold atoms in optical lattices

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annalen der physik
Review Article

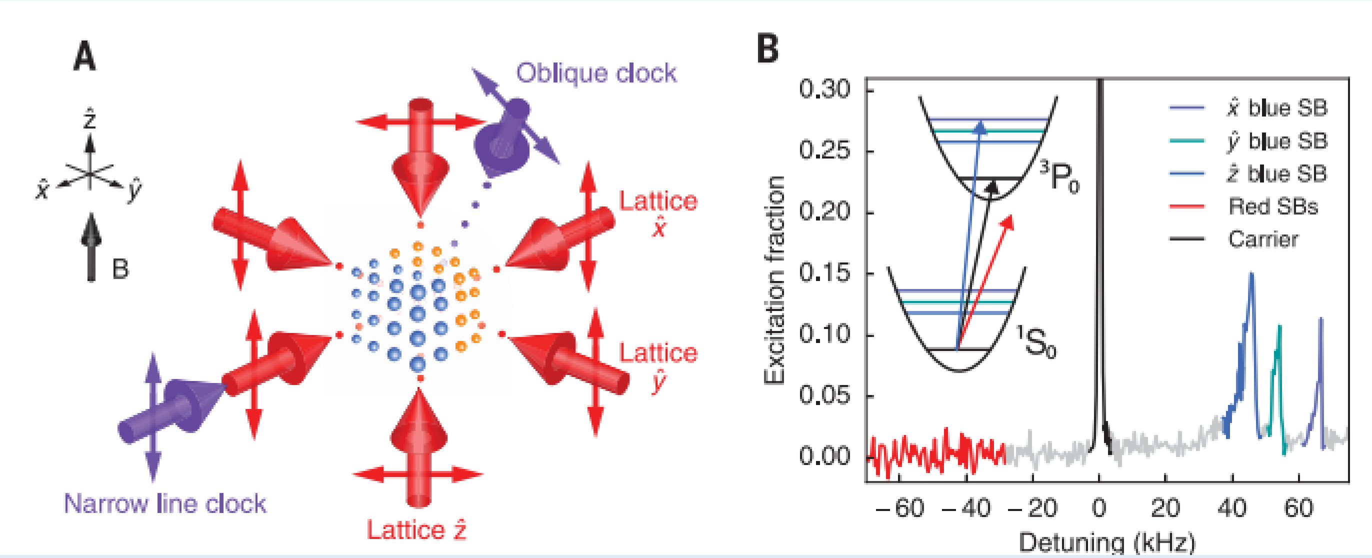
Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories**

Uwe-Jens Wiese[1,2,*]

Received 6 May 2013, revised 11 June 2013, accepted 20 June 2013
Published online 24 July 2013

Optical lattices – the big picture

Aside from quantum simulation / studying many-body physics, also just a great way to keep atoms from moving

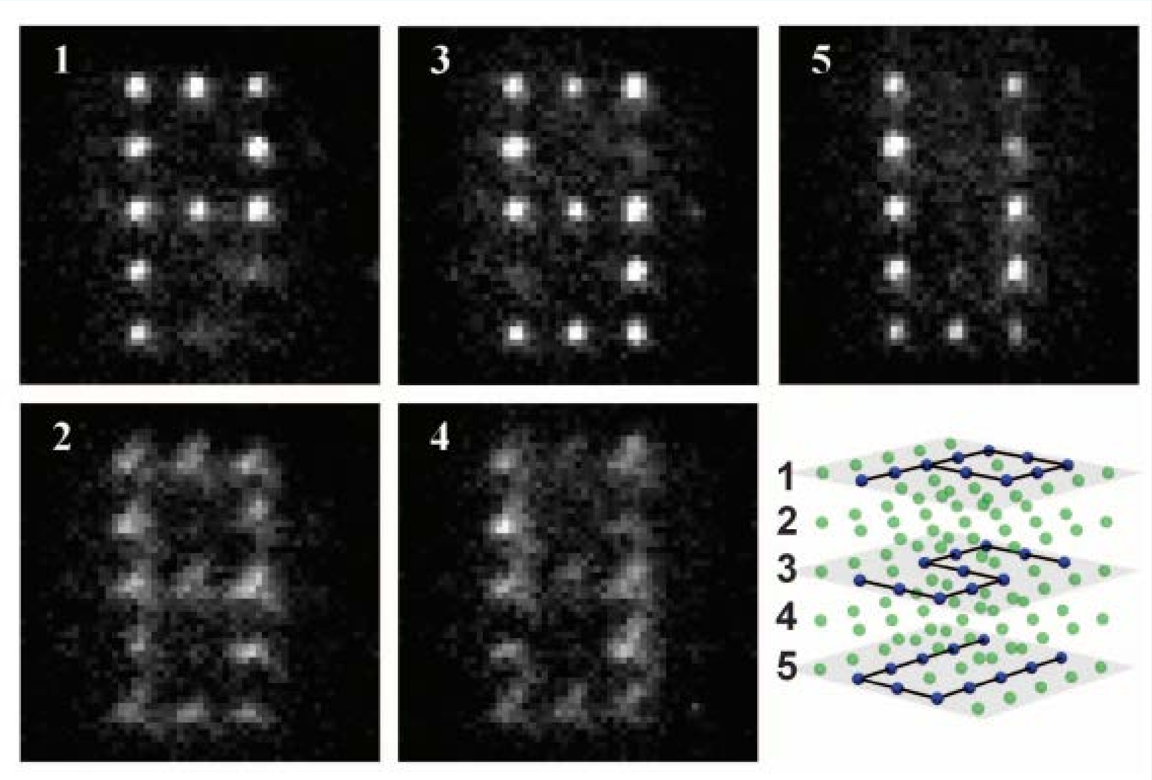


Campbell et al., Science 358, 90–94 (2017)

rid of Doppler shifts
for atomic clocks

Optical lattices – the big picture

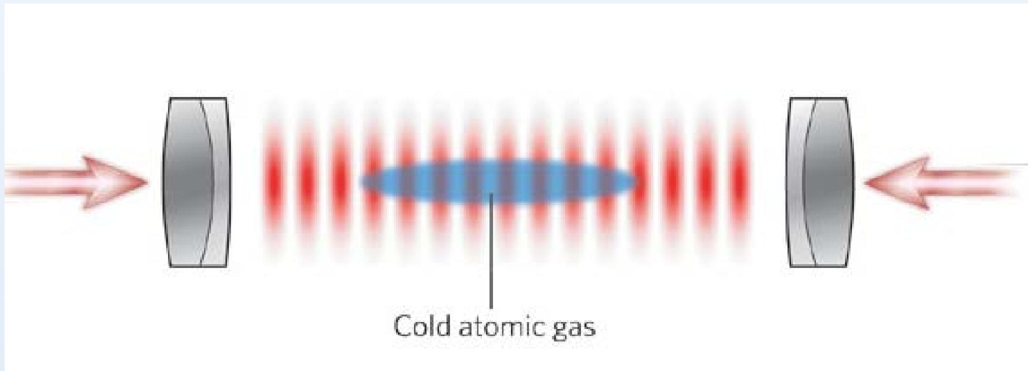
Aside from quantum simulation / studying many-body physics, also just a great way to keep atoms from moving



Wang et al., Science 352, 1562–1565 (2016)

trapping of qubits

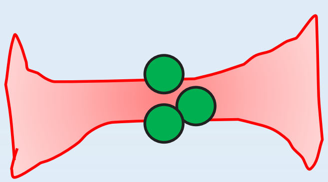
Optical lattices – laser interference

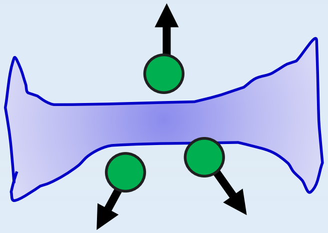


Immanuel Bloch
Nature (2008)

$$V(\vec{r},t) = \alpha(\omega) I(\vec{r},t)$$

$$\alpha(\omega) \propto \frac{1}{\omega - \omega_0}$$

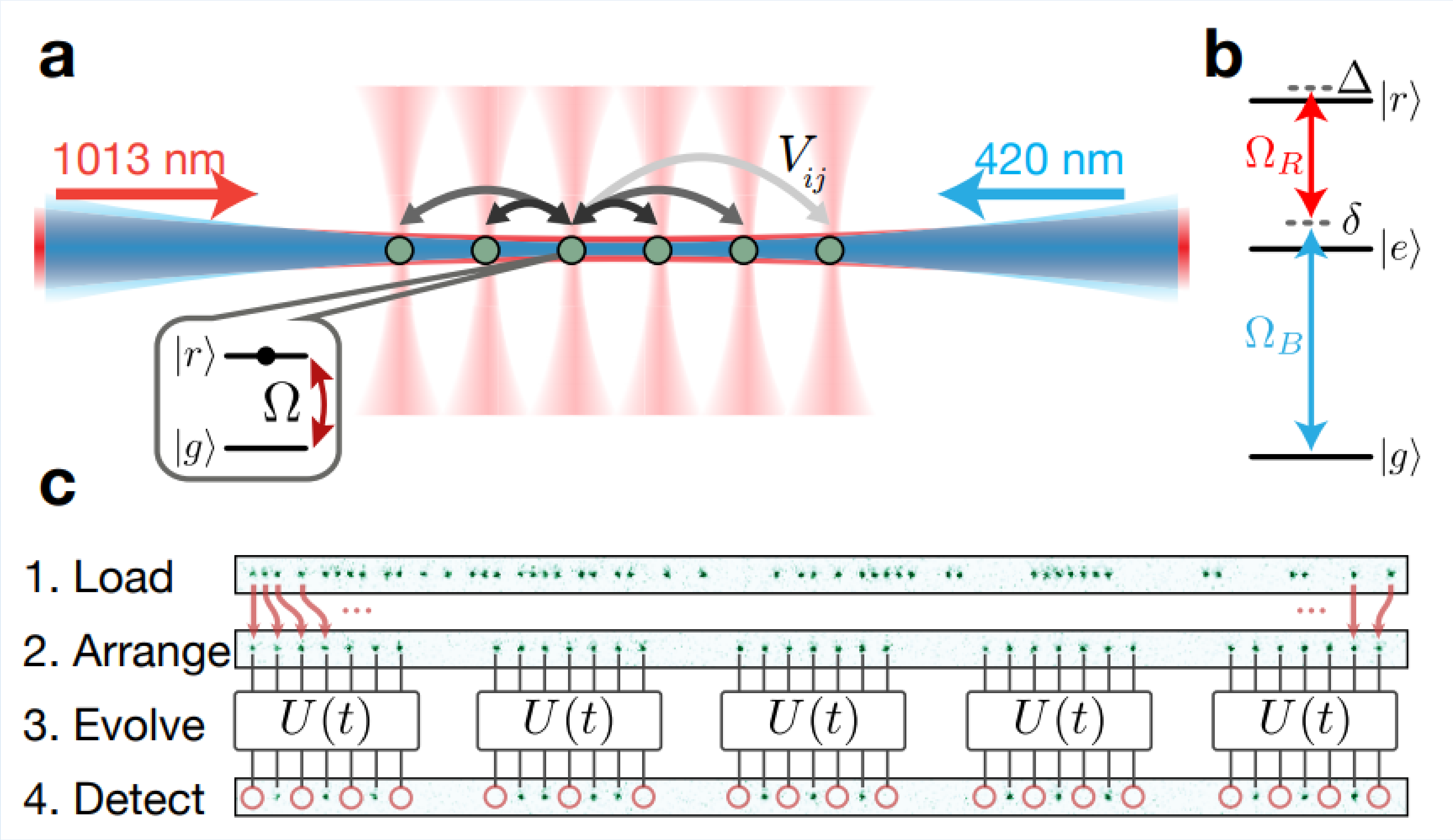




$$I(\vec{r},t)$$

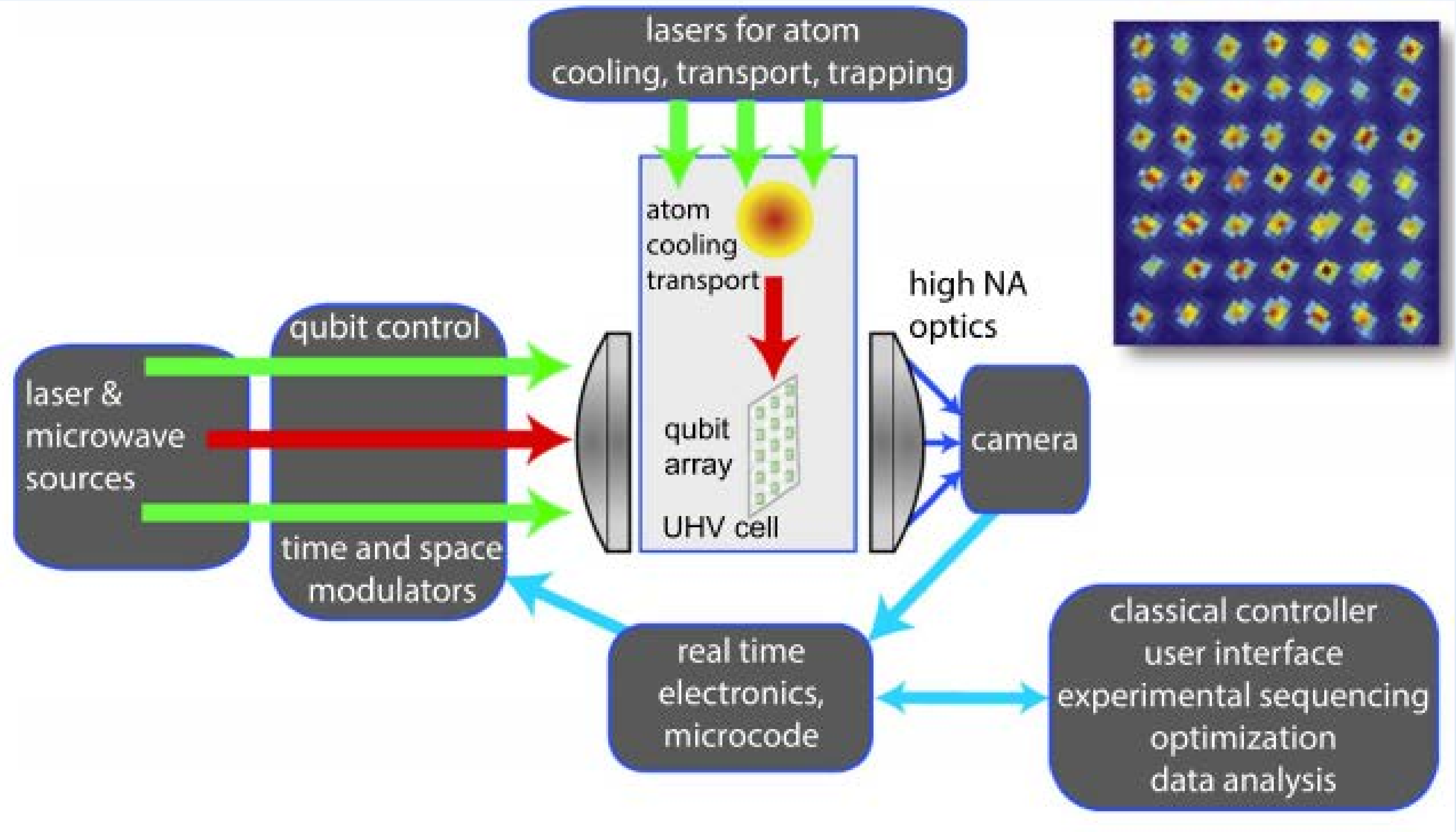
typically formed by laser interference

Optical lattices – laser interference



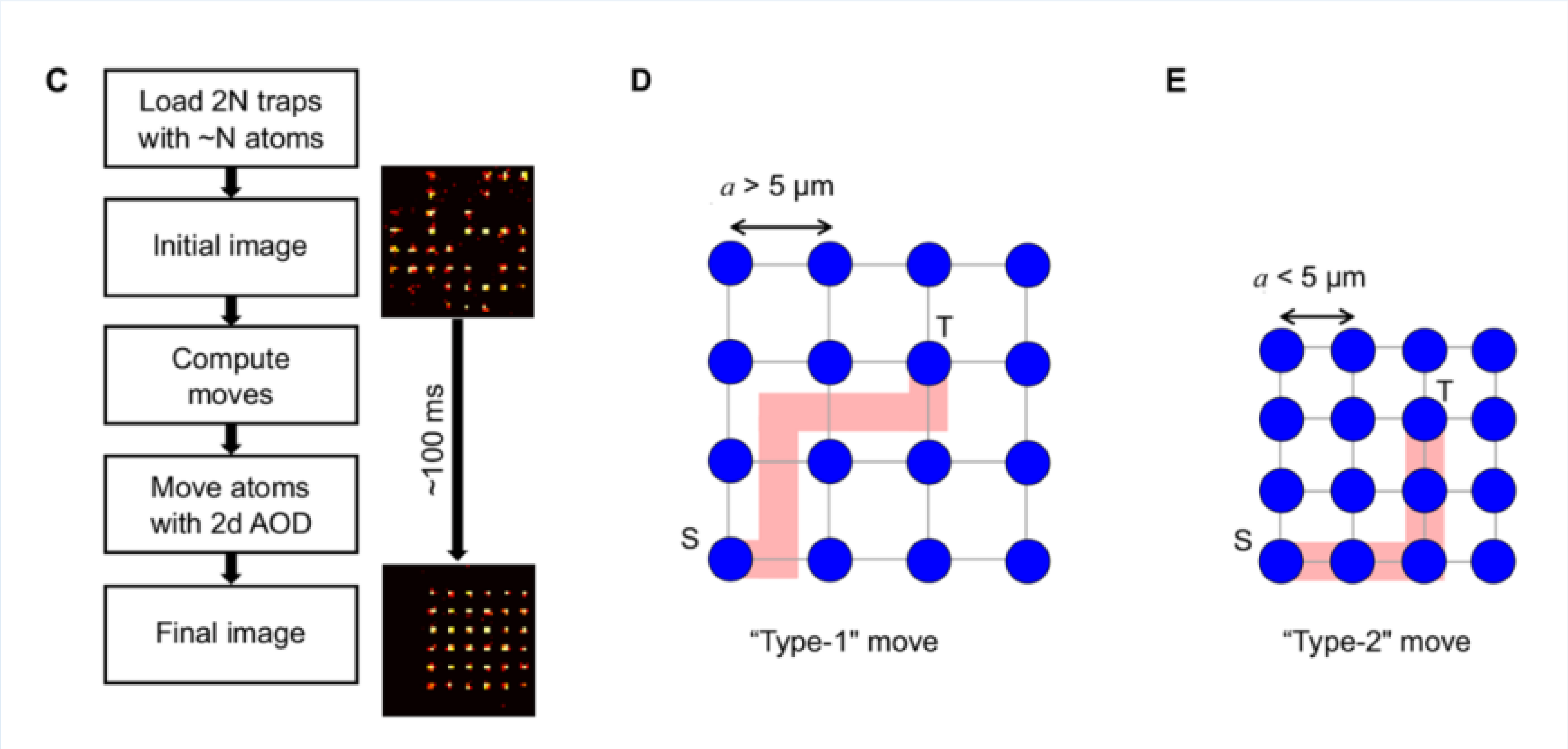
Greiner/Vuletić/Lukin collab, CUA

Optical lattices – laser ~~interference~~



Saffman group, Wisconsin

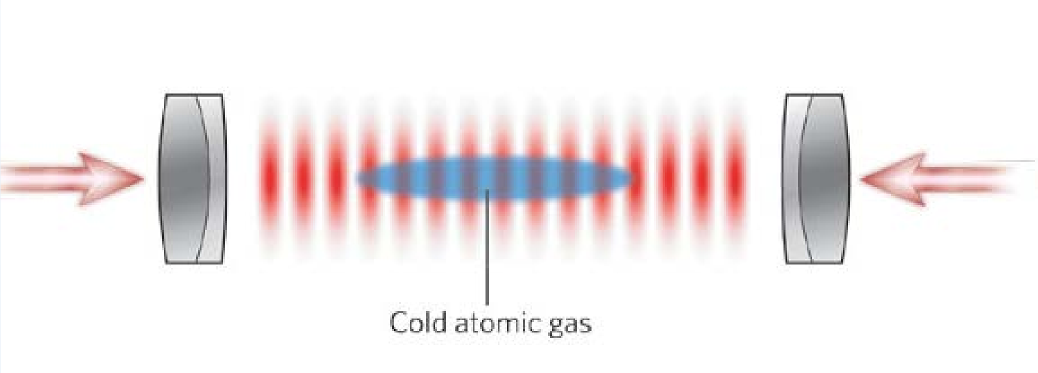
Optical lattices – laser ~~interference~~



Browaeys group, Palaiseau

Note: such potentials have large inter-well spacing, not particularly well-suited to studying coherent tunneling (exceptions: Jochim / Regal groups)

Optical lattices – laser interference



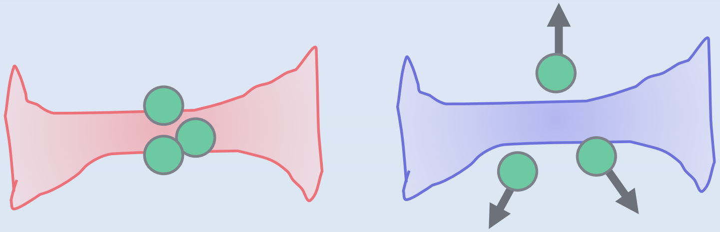
Immanuel Bloch
Nature (2008)

$$V(\vec{r},t) = \alpha(\omega) I(\vec{r},t)$$

$$I(\vec{r},t) = \left|\sum_n E_n(\vec{r},t)\,\hat{\epsilon}_n\right|^2$$

typically formed by laser interference

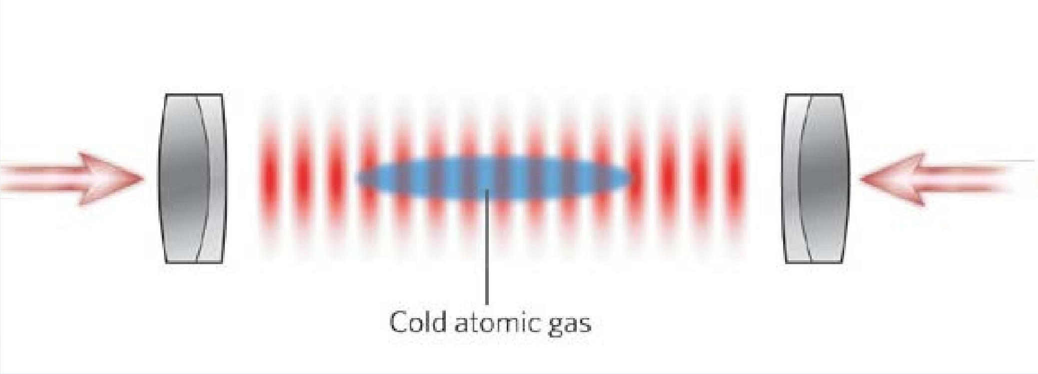
$$\alpha(\omega) \propto \frac{1}{\omega - \omega_0}$$



Lattice pattern depends on:

- frequencies
- polarizations
- directions of propagation
- relative phases
- …

Simplified 1D optical lattice



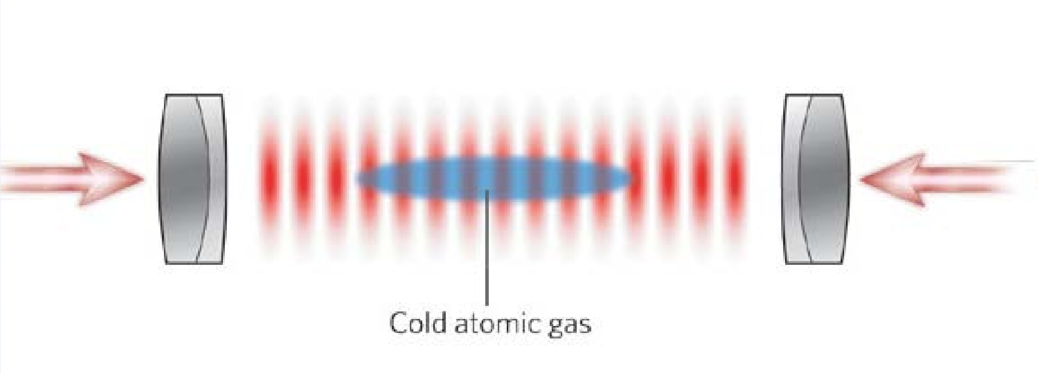
Immanuel Bloch
Nature (2008)

$$V(\vec{r},t) \propto \left| E_1 e^{i(kz-\omega t+\varphi_1)} + E_2 e^{i(-kz-\omega t+\varphi_2)} \right|^2$$

$$V(\vec{r},t) = V_{\text{offset}} + V_0 \cos^2\left(\frac{\pi(z-z_0)}{d}\right)$$

Note: for real laser beams (~Gaussian, not plane-wave), also get confinement (or deconfinement) in radial direction [i.e. V_0 is a slowly-varying function of $\vec{r}$)

Simplified 1D optical lattice



Immanuel Bloch
Nature (2008)

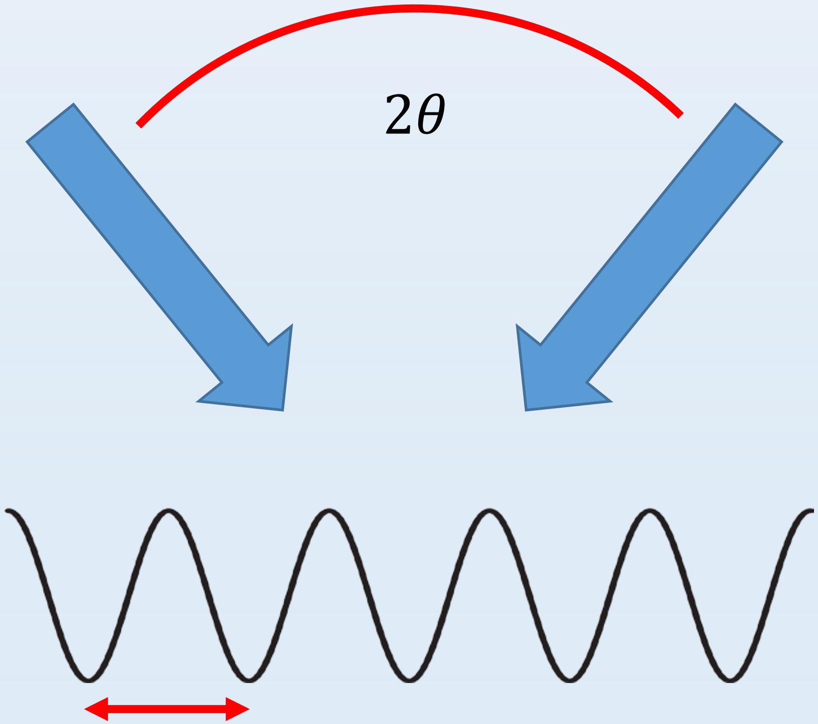
$$V(\vec{r},t) \propto \left|E_1 e^{i(kz-\omega t+\varphi_1)} + E_2 e^{i(-kz-\omega t+\varphi_2)}\right|^2$$

$$V(\vec{r},t) = V_{\text{offset}} + V_0 \cos^2\left(\frac{\pi(z-z_0)}{d}\right) = V'_{\text{offset}} + \frac{V_0}{2}\cos\left(\frac{2\pi z'}{d}\right)$$

Note: for real laser beams (~Gaussian, not plane-wave), also get confinement (or deconfinement) in radial direction [i.e. V_0 is a slowly-varying function of $\vec{r}$)

Relevant energy scale

The recoil energy --- for counter-propagating beams, $E_R = \frac{\hbar^2 k_L^2}{2m}$

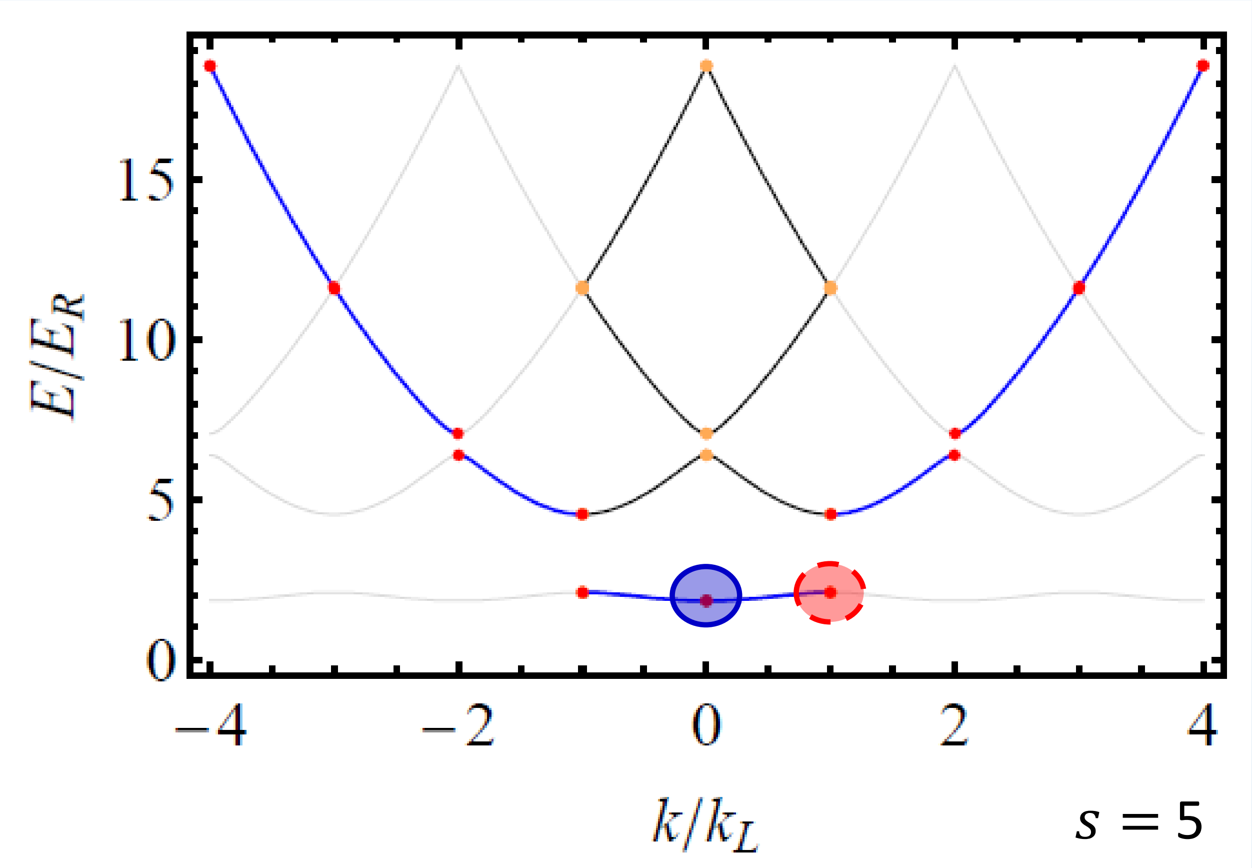


$$E_R = \frac{h^2}{2md^2}$$

$$s = V_0/E_R$$

$$d = \frac{\lambda}{2\sin\theta}$$

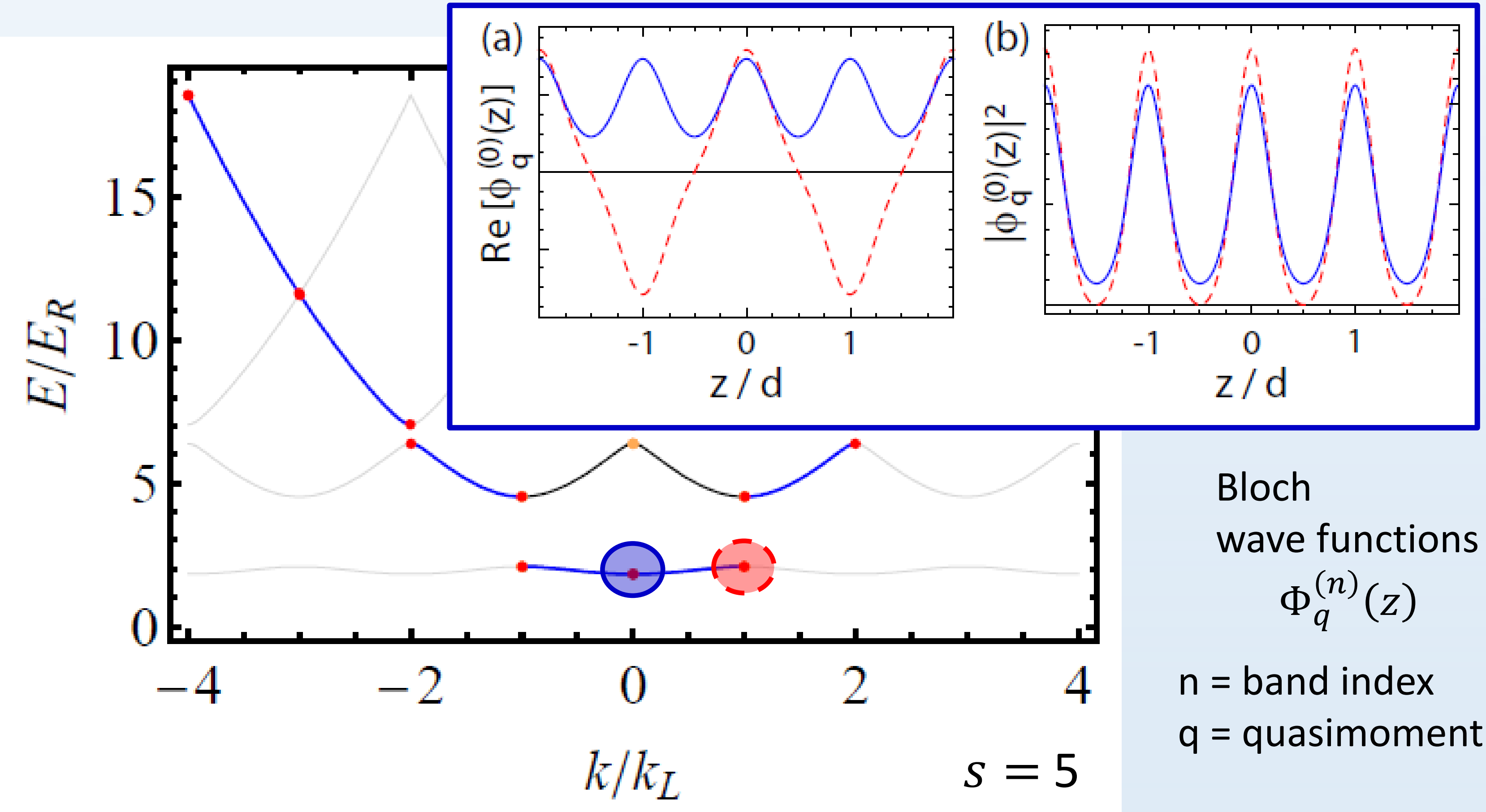
The energy band structure



$$s = \frac{V_0}{E_R} = 5$$

Blue: extended zone scheme energy bands vs. momentum w.r.t. crystal
Black: folded zone scheme band structure

The energy band structure



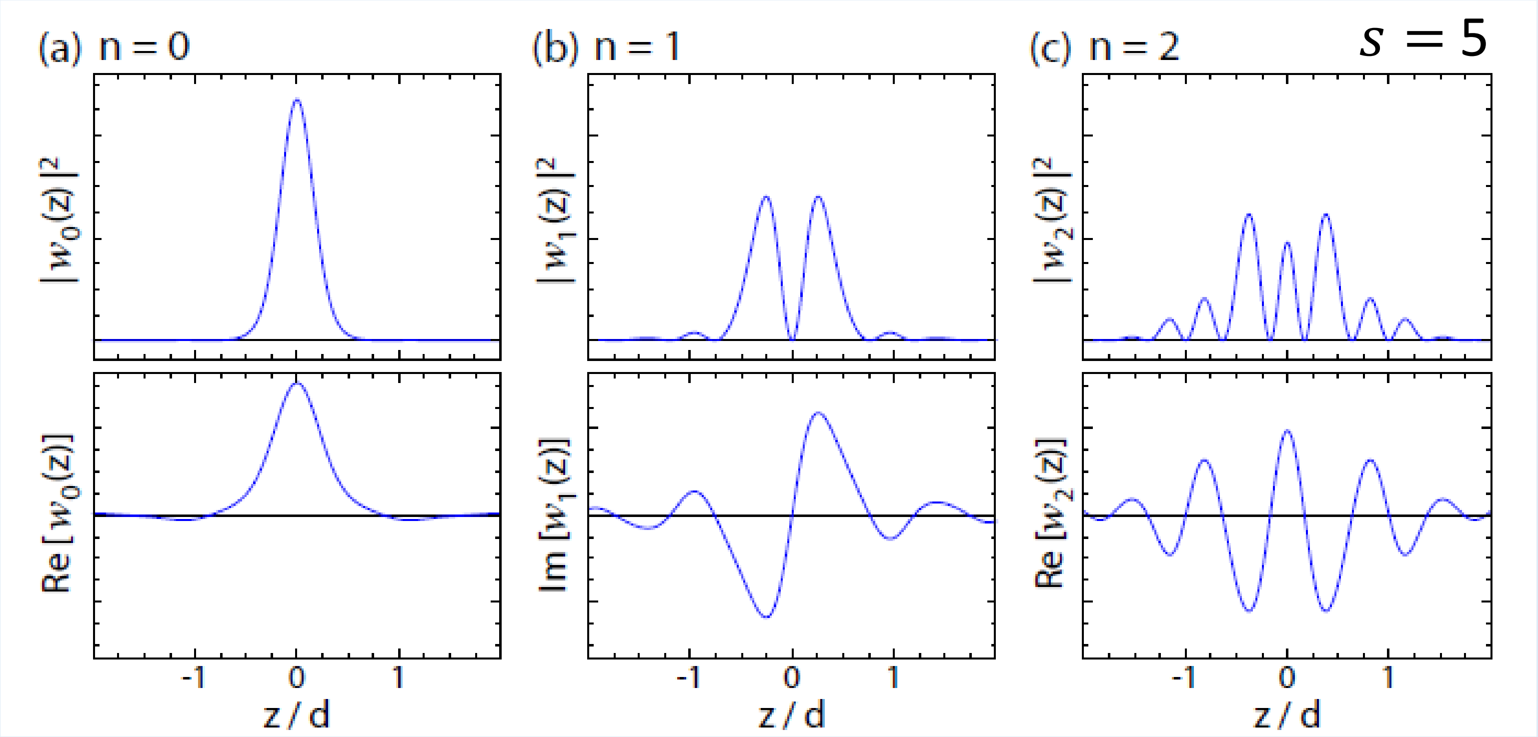
Bloch wave functions $\Phi_q^{(n)}(z)$

n = band index
q = quasimomentum

Blue: extended zone scheme energy bands vs. momentum w.r.t. crystal
Black: folded zone scheme band structure

Note: for sinusoidal potentials, there is an exact, analytical solution for the Bloch wavefunctions / energies in terms of the Mathieu equation (characteristic Mathieu functions)

Localized Wannier orbitals

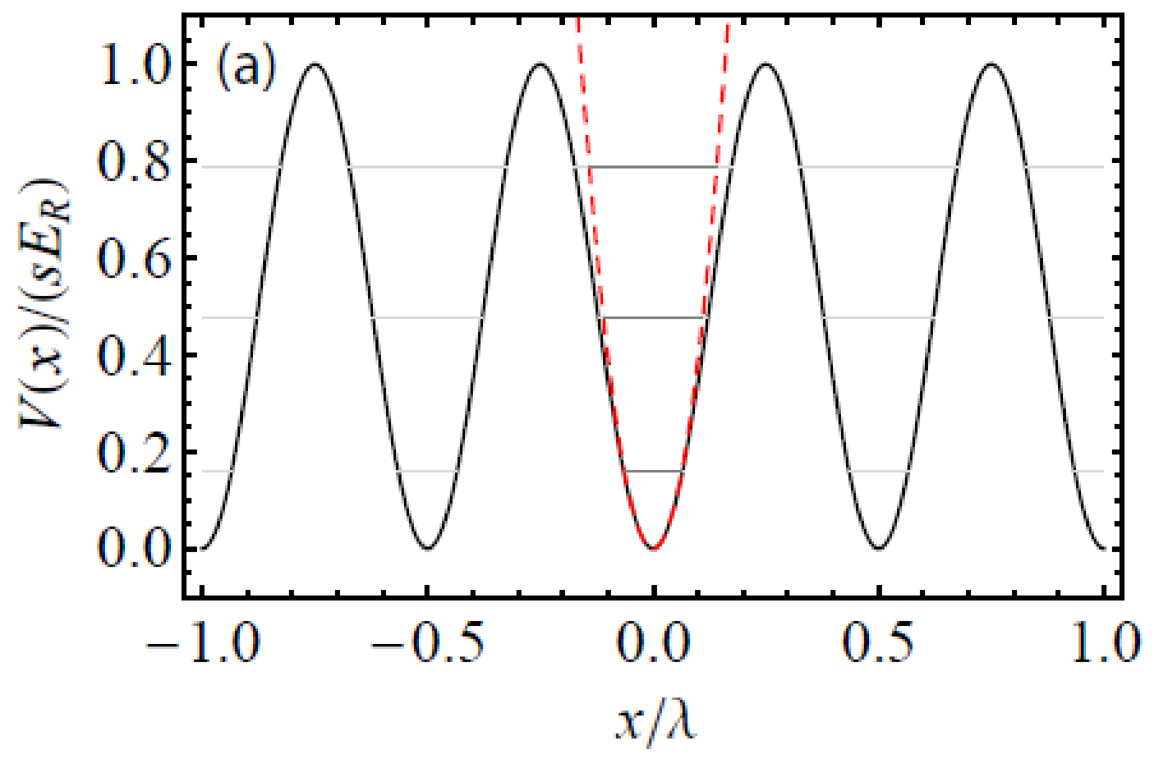


$$w_n(z - z_j) = \frac{1}{\sqrt{\mathcal{N}}} \sum_q e^{-iqz/\hbar} \, \Phi_q^{(n)}(z)$$

Deep lattice (s >> 1), harmonic approx.

$$-\frac{s}{2}E_R\cos(2kz) \approx C + \frac{sE_R}{4}(2kz)^2 \approx C + \frac{1}{2}m\omega^2 z^2$$

$$\hbar\omega \approx 2\sqrt{s}E_R$$

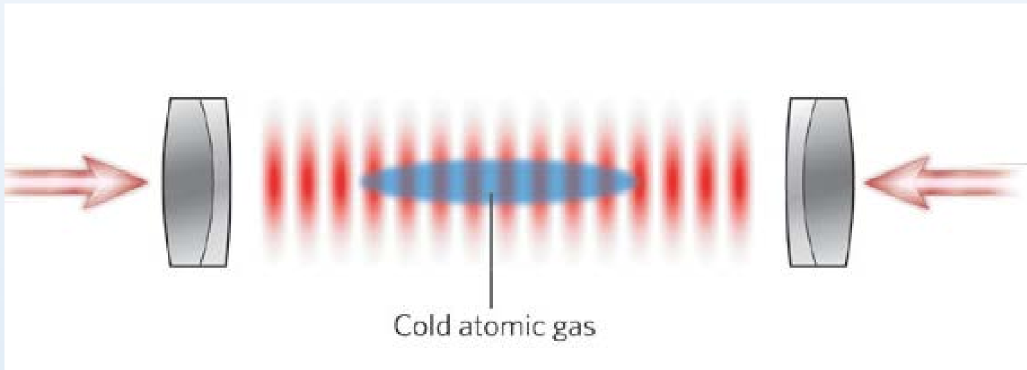


Can approximate the Wannier states as HO orbitals (Gaussian wfs) with $\pi\sigma/d \approx s^{-1/4}$

This approximation is valid, for some HO level n, for $E_n \ll sE_R$

Moving optical lattices

ω_1

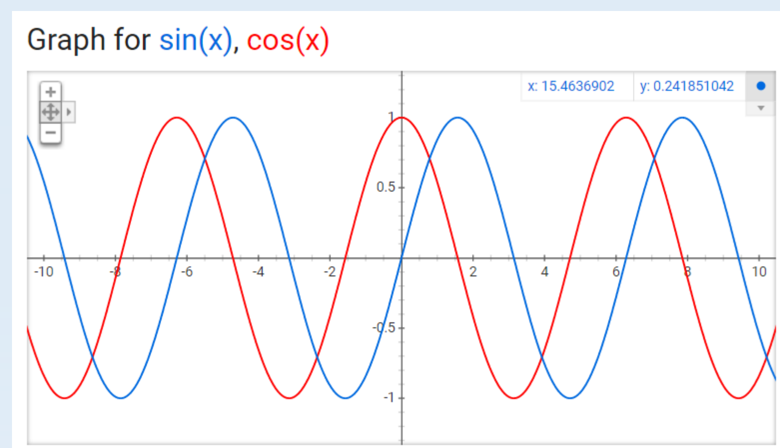


ω_2

Immanuel Bloch
Nature (2008)

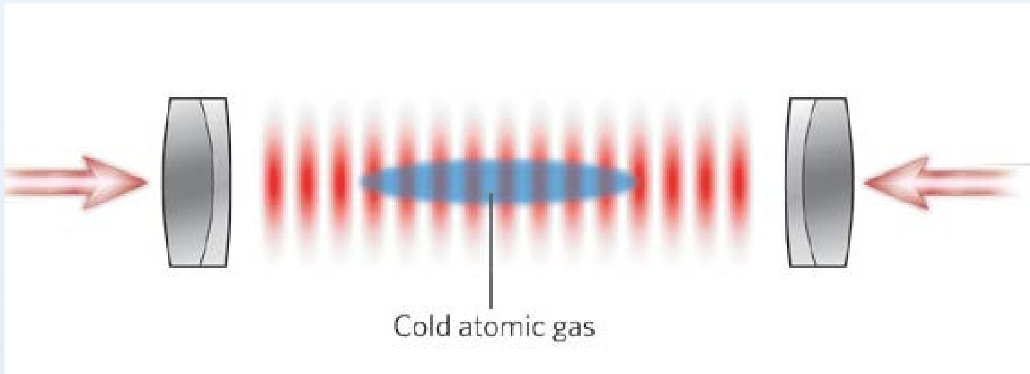
$$V(\vec{r},t) \approx sE_R \cos^2\left(\frac{\pi z}{d} + \frac{\varphi_2 - \varphi_1}{2}\right)$$

A sudden phase jump by $\pi/2$ between the two fields will shift the lattice by ¼ of a wavelength



A continuous linear phase shift will make the lattice move with some fixed velocity --- this is equivalent to a fixed frequency detuning between the interfering beams

Moving optical lattices

ω_1  ω_2

Immanuel Bloch
Nature (2008)

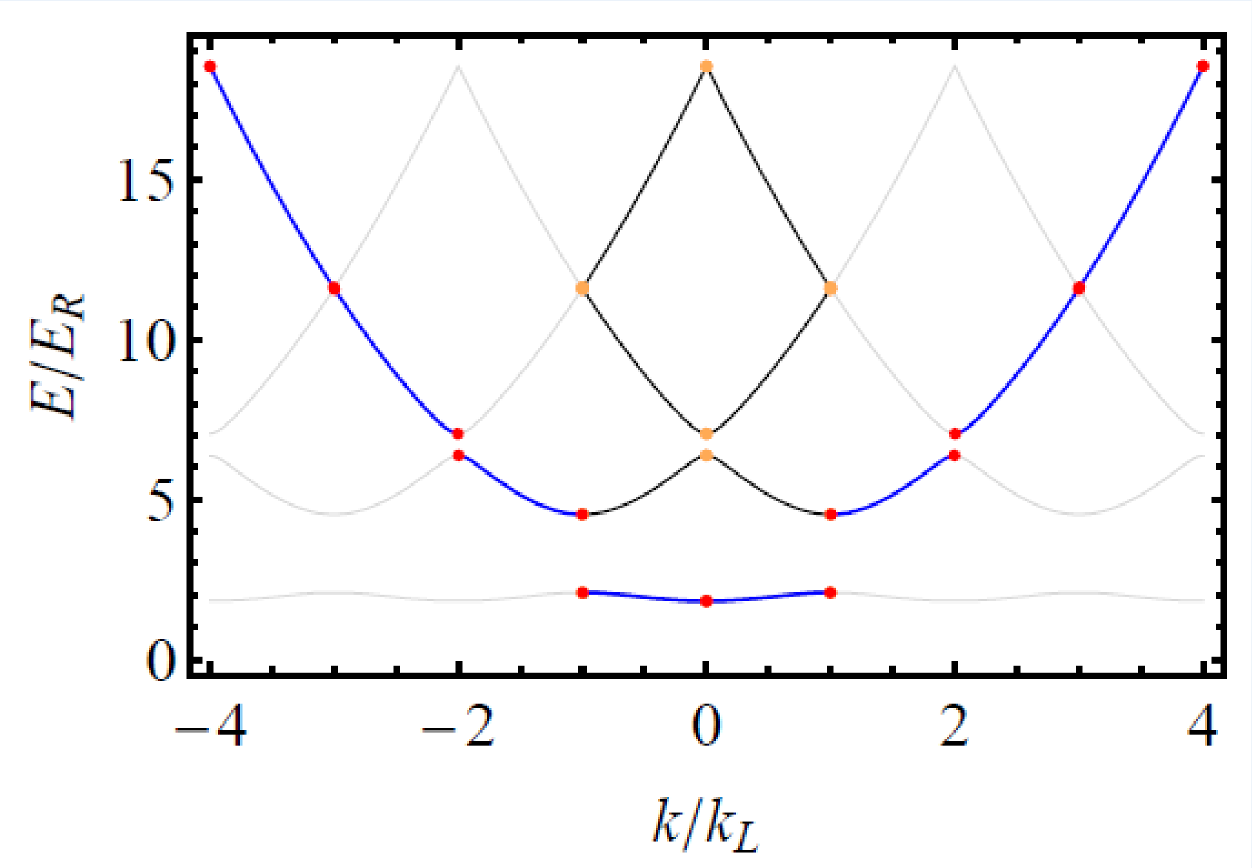
$$V(\vec{r},t) \approx sE_R \cos^2\left(\frac{\pi z}{d} + \frac{\varphi_2 - \varphi_1}{2}\right)$$

A sudden phase jump by $\pi/2$ between the two fields will shift the lattice by ¼ of a wavelength

A continuous linear phase shift will make the lattice move with some fixed velocity --- this is equivalent to a fixed frequency detuning between the interfering beams

$$v_{latt} = \Delta f \times \lambda/2$$

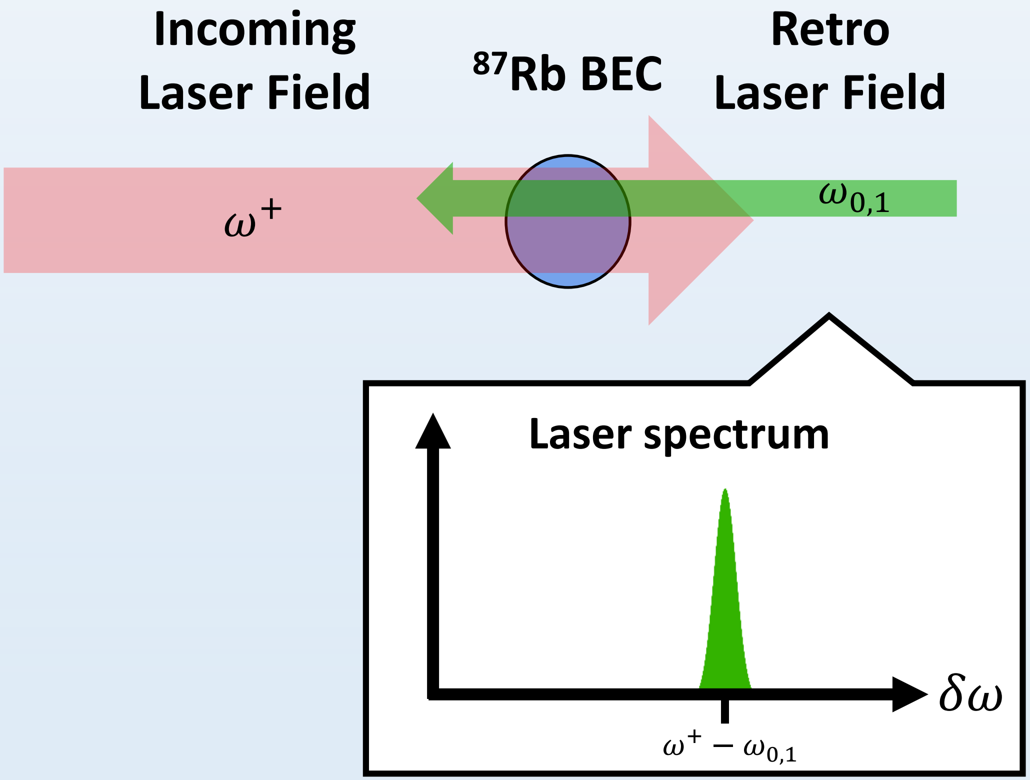
The energy band structure

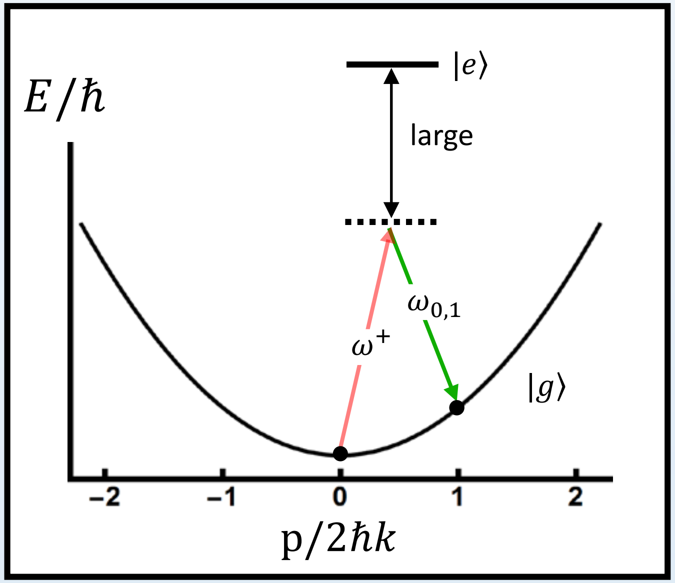


$$s = \frac{V_0}{E_R} = 5$$

What would happen to atoms if you suddenly turned on a moving lattices, such that $v_{latt} = v_R$, or $k = k_L$?

Diffraction from moving lattices



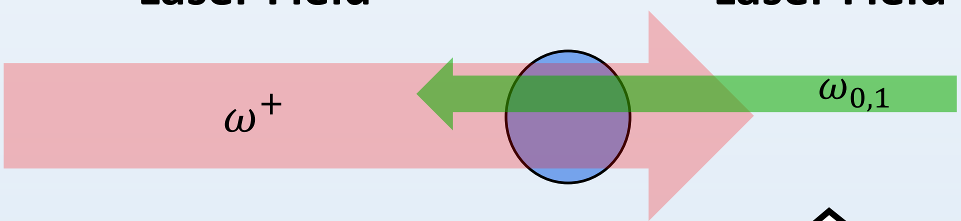


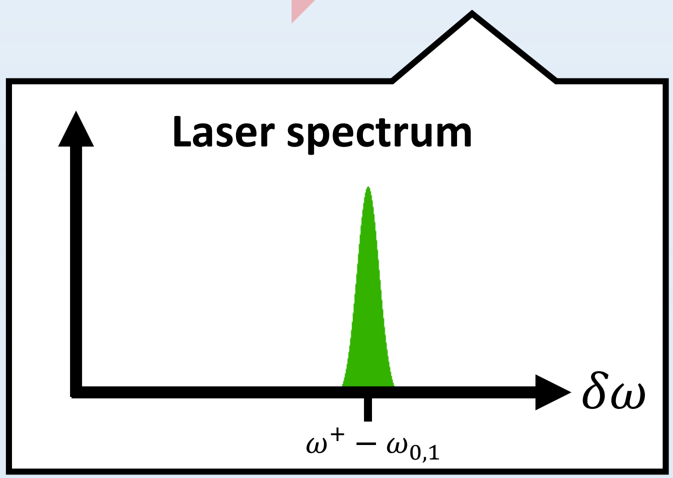
Diffraction from moving lattices

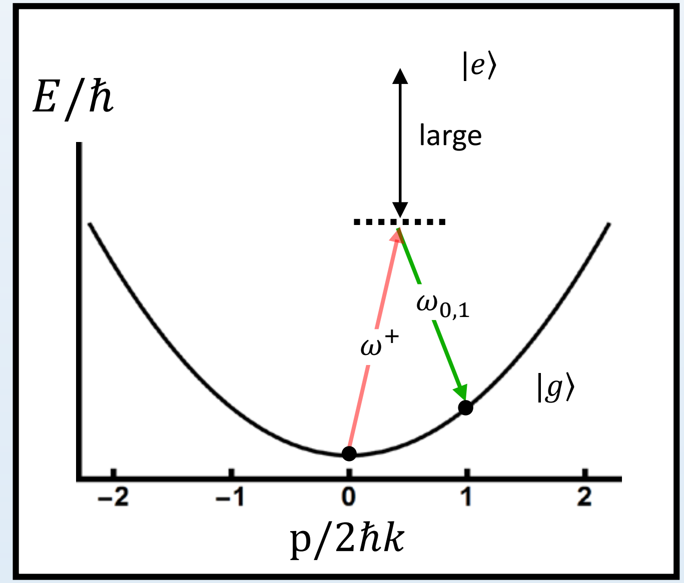
Incoming Laser Field

^{87}Rb BEC

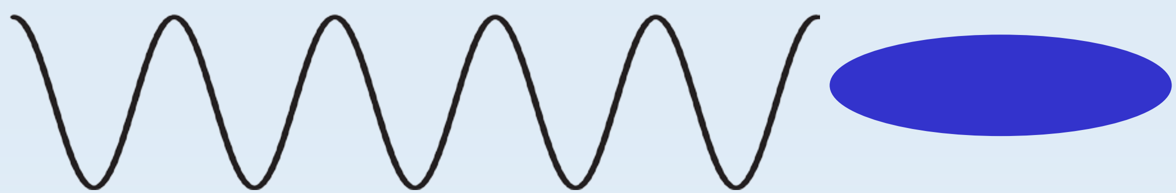
Retro Laser Field



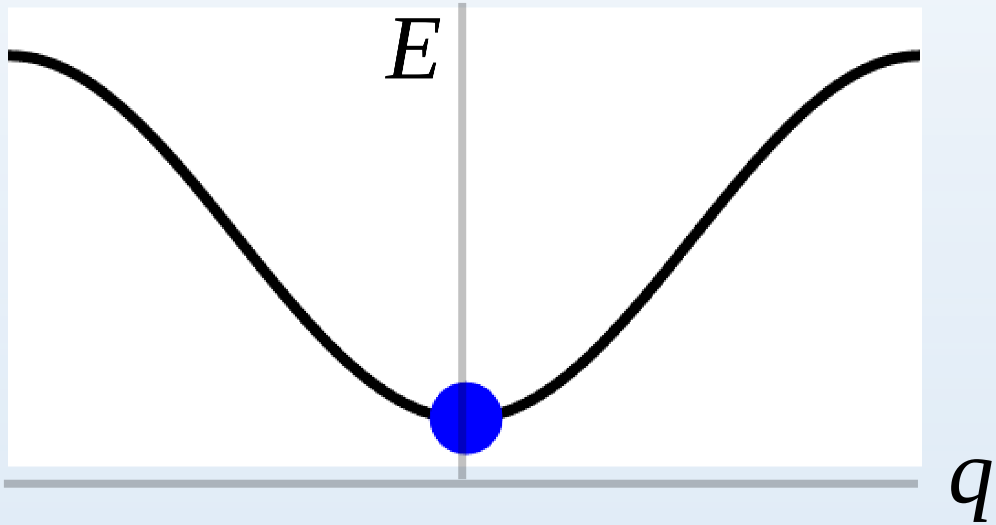


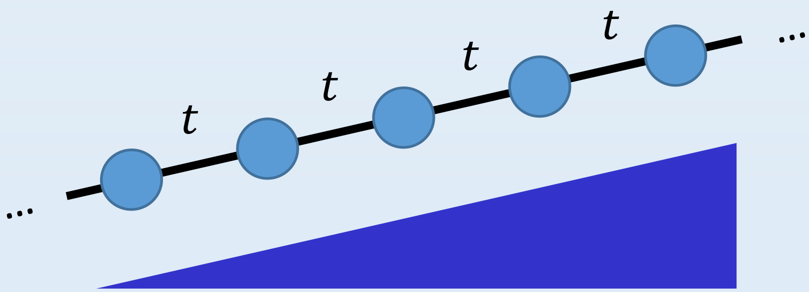


Two-photon Bragg transition



What about applying a force?

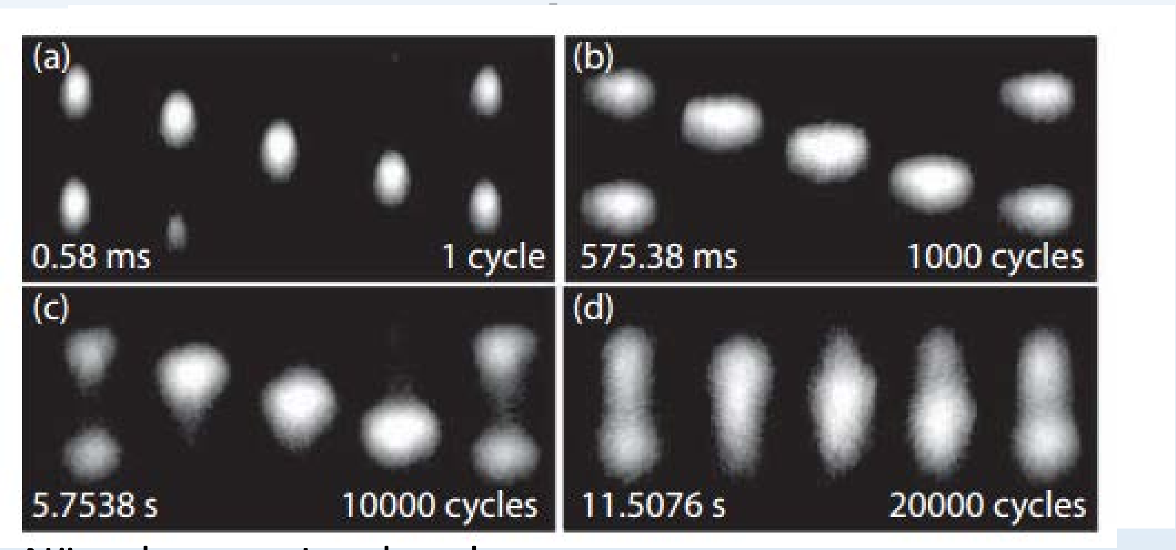




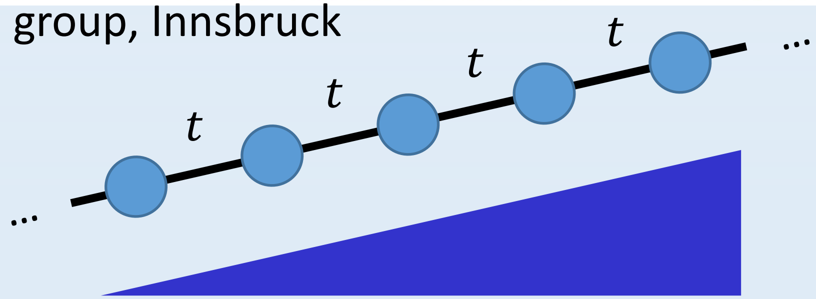
$$V(x) = V_{latt} + Fx$$

$$\Delta\varphi(t) \propto t^2$$

What about applying a force?



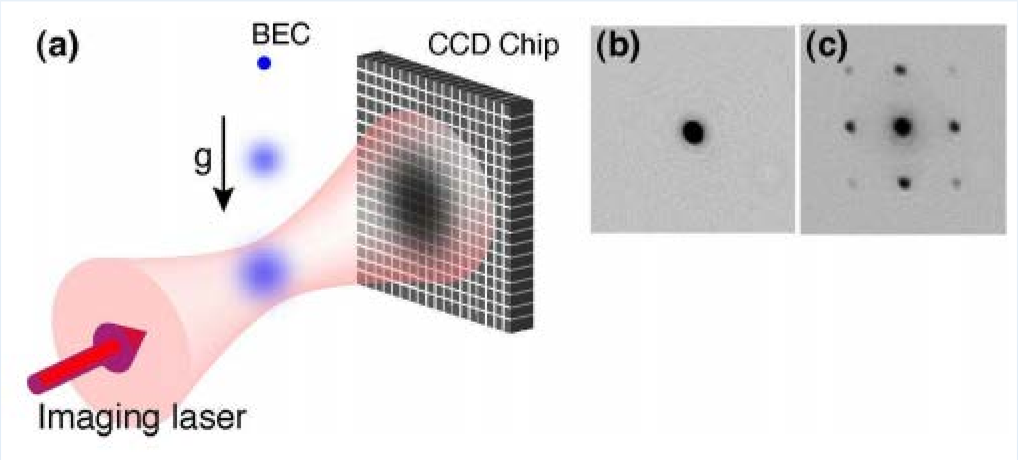
Nägerl group, Innsbruck

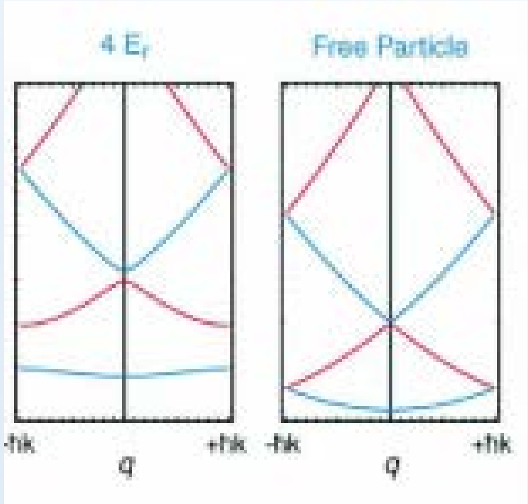


$$V(x) = V_{latt} + Fx$$

$$\Delta\varphi(t) \propto t^2$$

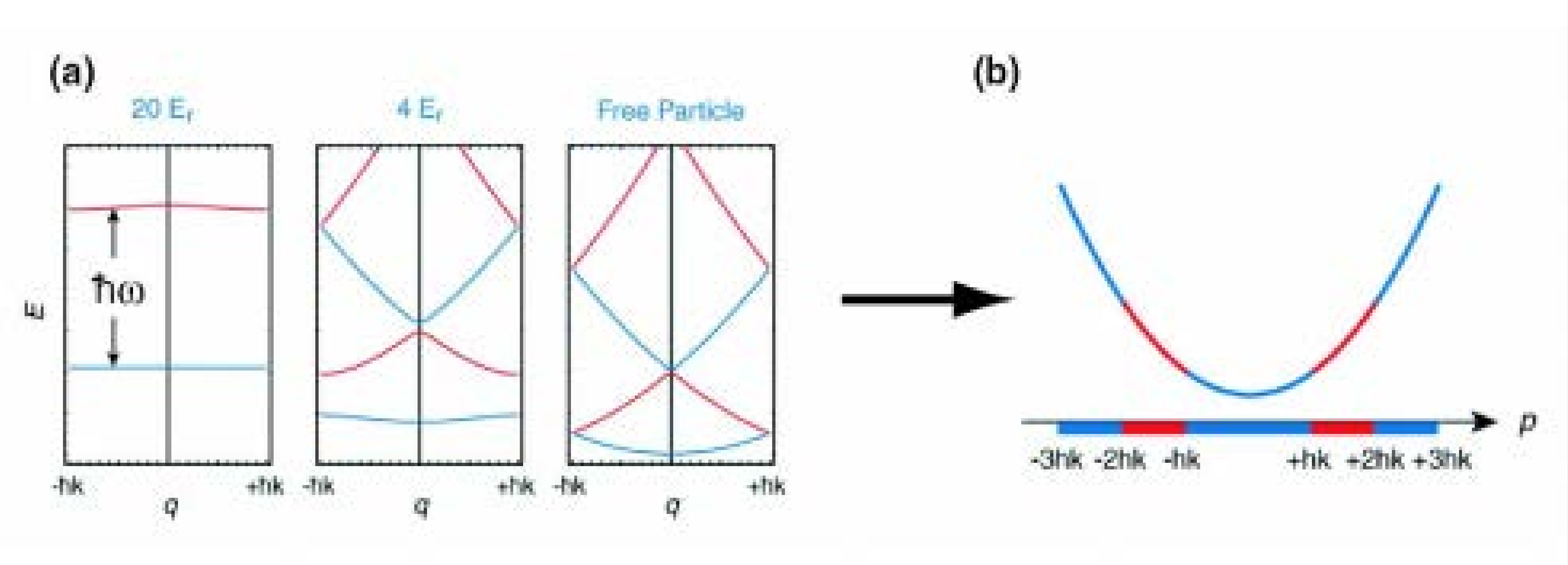
Release of atoms from a lattice



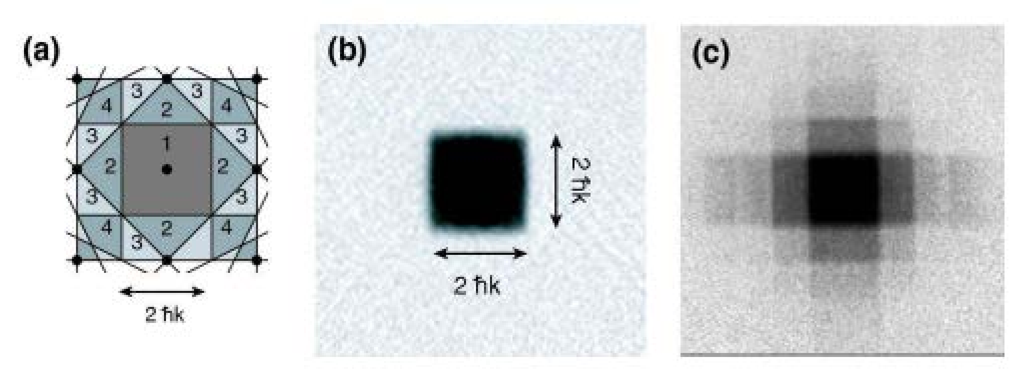


One typically sees a diffraction pattern – the weights of the different momentum orders relates to the projection of the Bloch states onto free particles states

Band-mapping

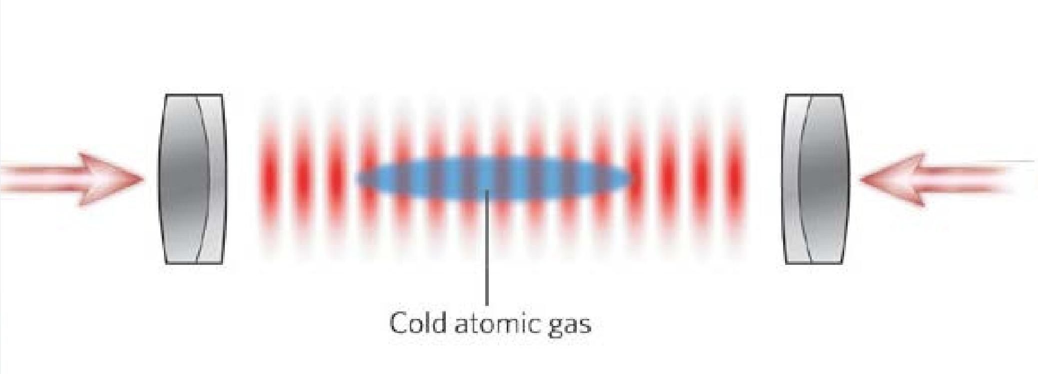


Map population in different lattice bands to unique momentum states. Slow (adiabatic) with respect to band gap, but fast with respect to bandwidth

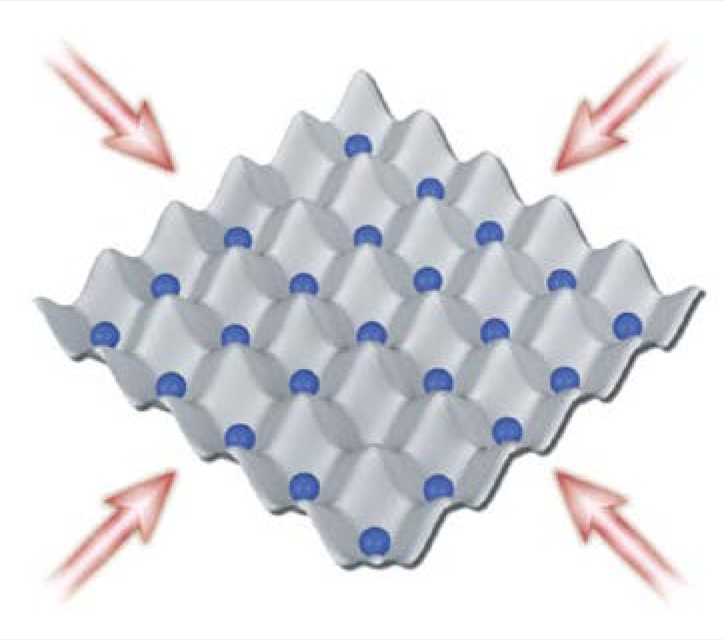


Greiner, et al.

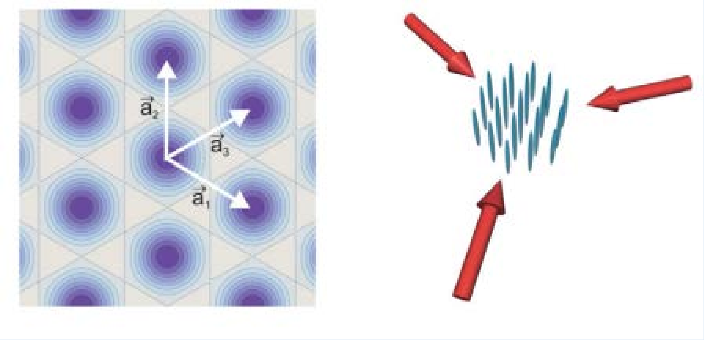
Higher-dimensional lattices



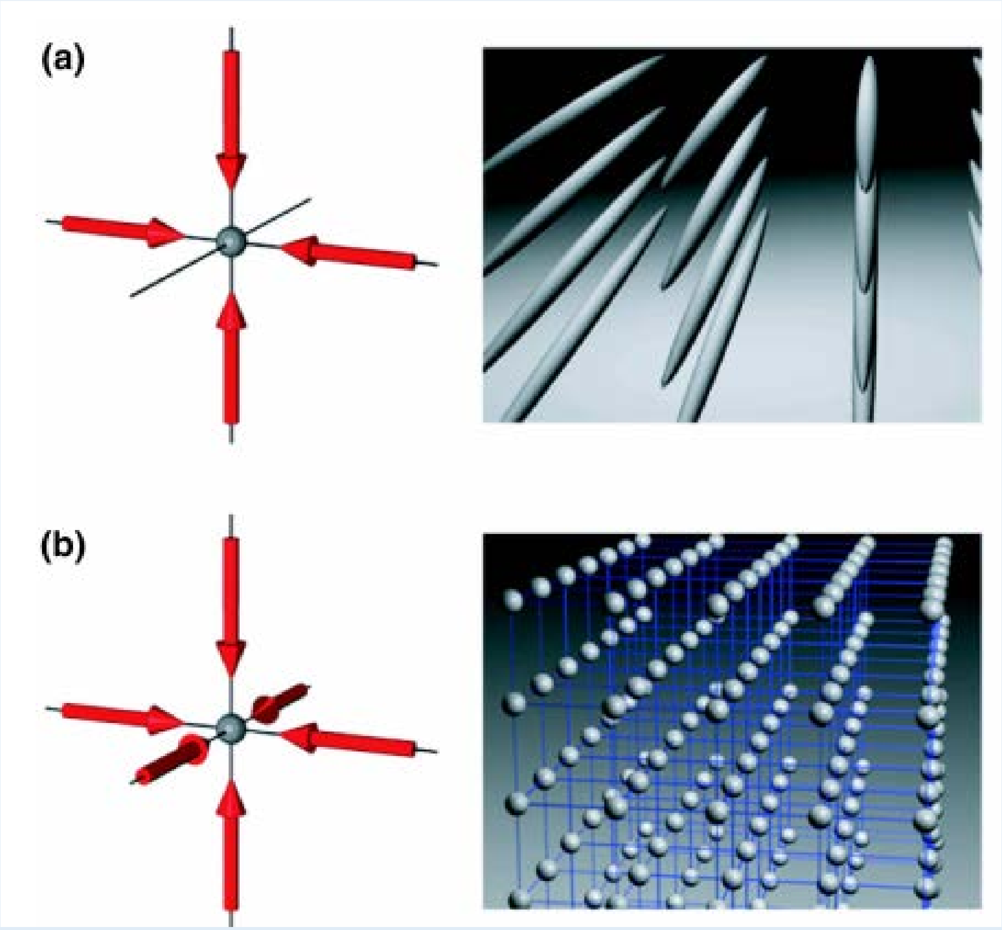
Immanuel Bloch
Nature (2008)



Immanuel Bloch
Nature (2008)

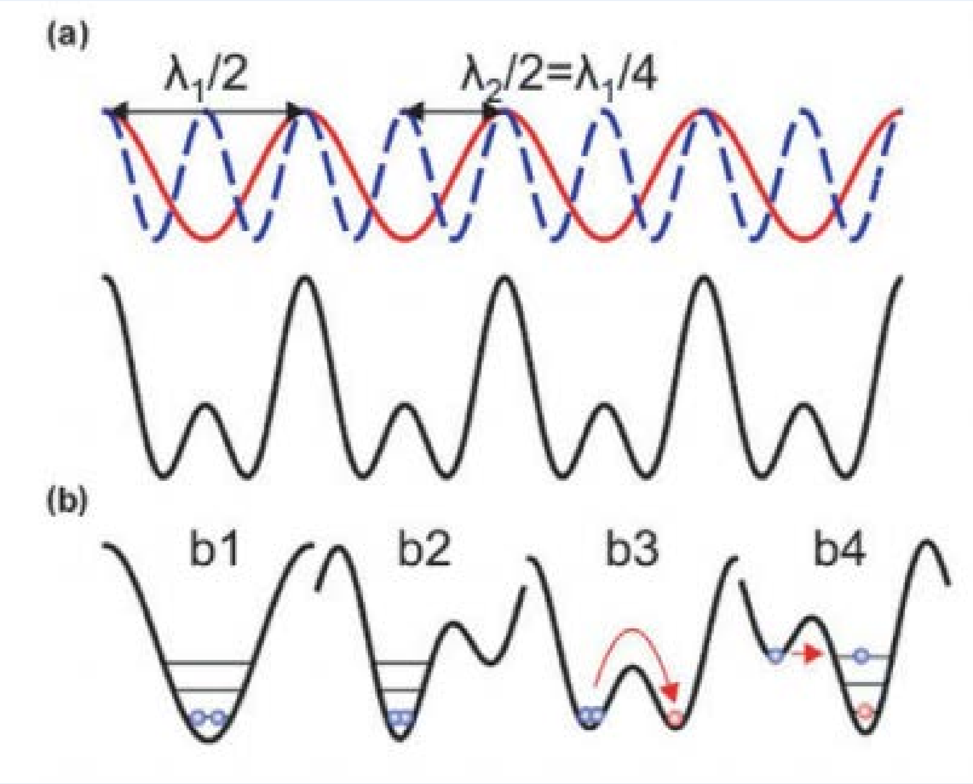


Higher-dimensional lattices (lower-dimensional systems…)



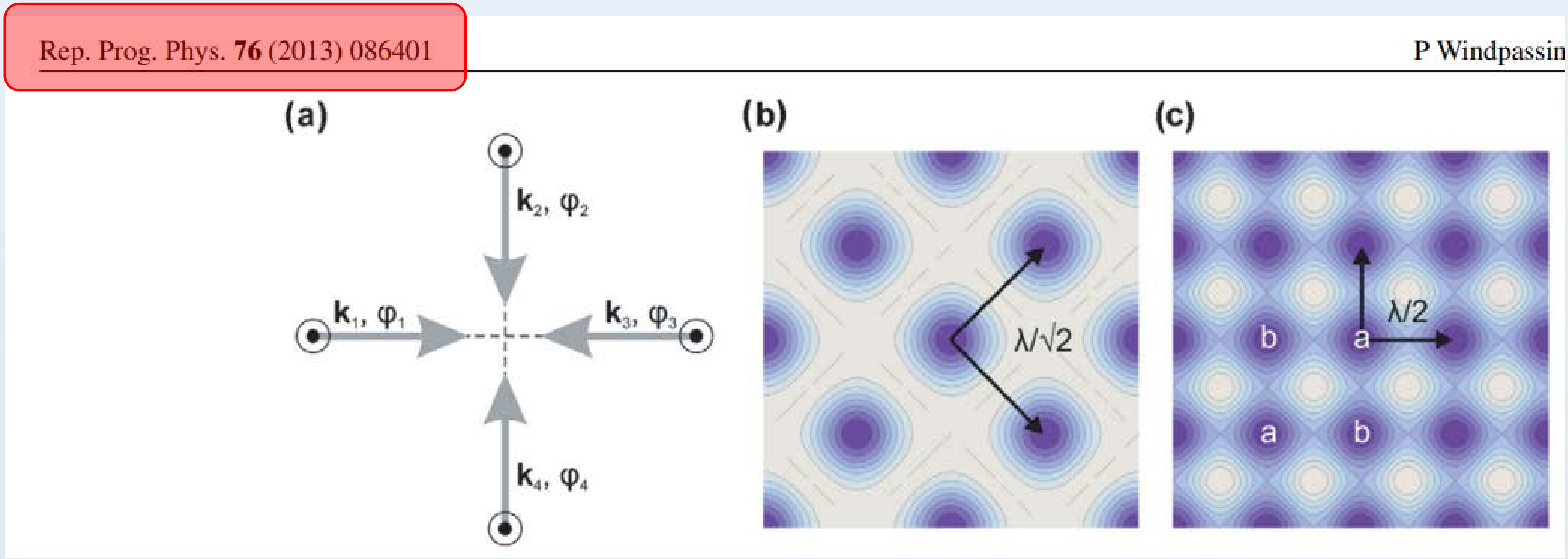
Bloch, Dalibard, Zwerger (2008)

Optical superlattices

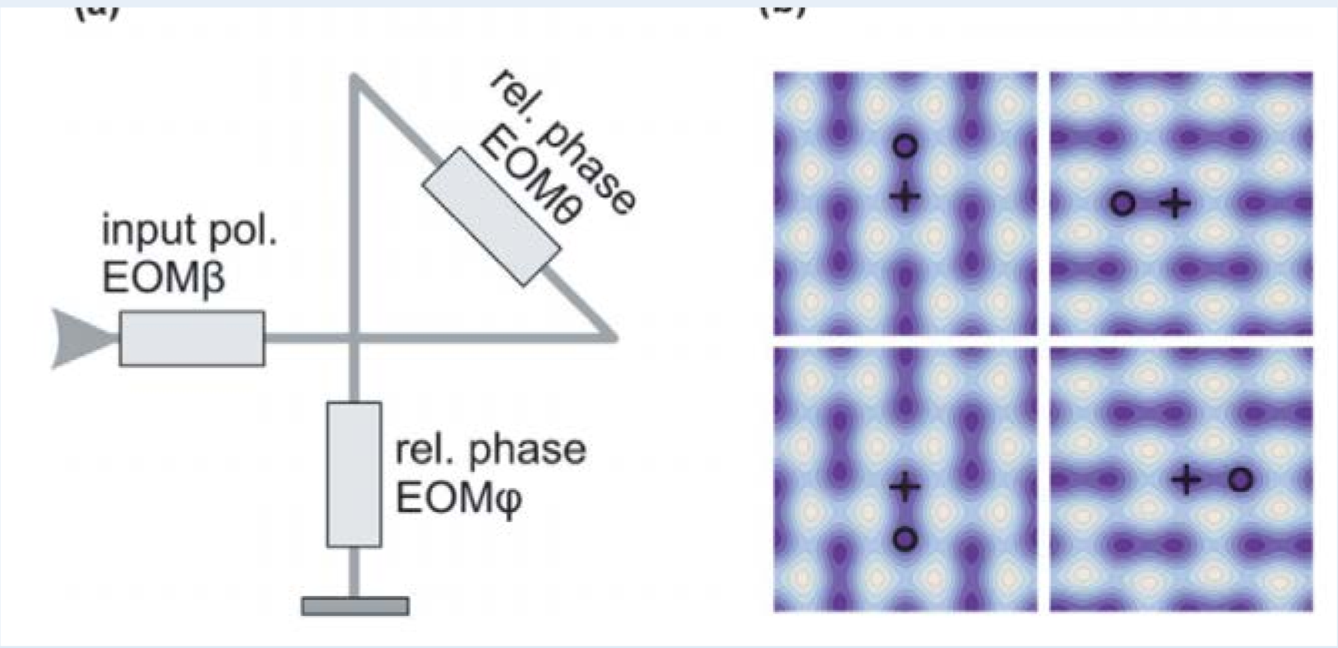


Bloch group , Porto group, others…

Multi-beam lattice structures

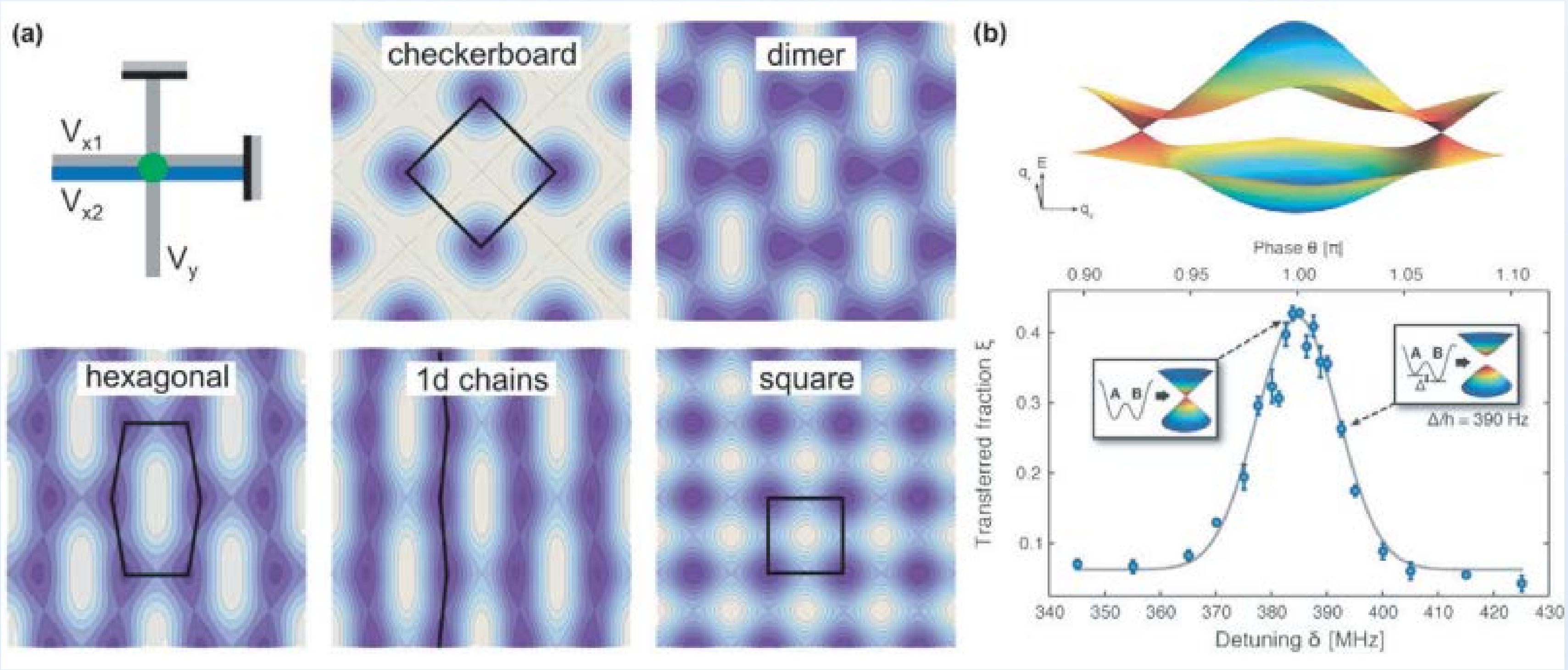


Multi-beam lattice structures



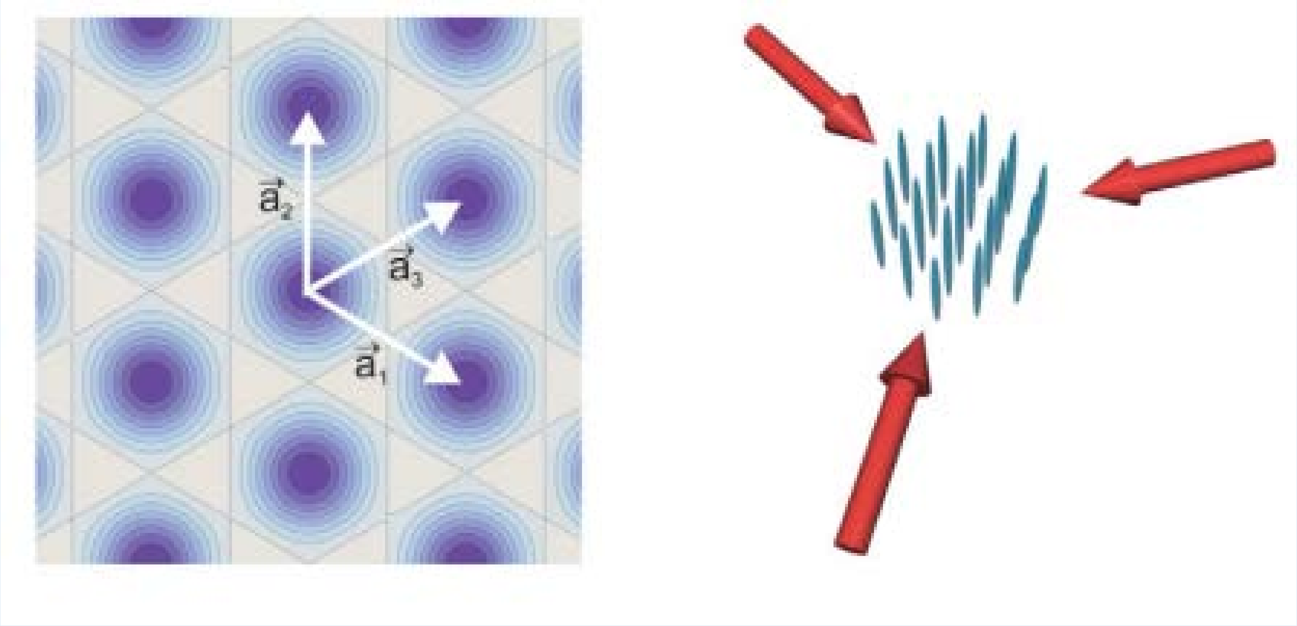
Tunable 4-beam lattice (Porto group)

Multi-beam lattice structures



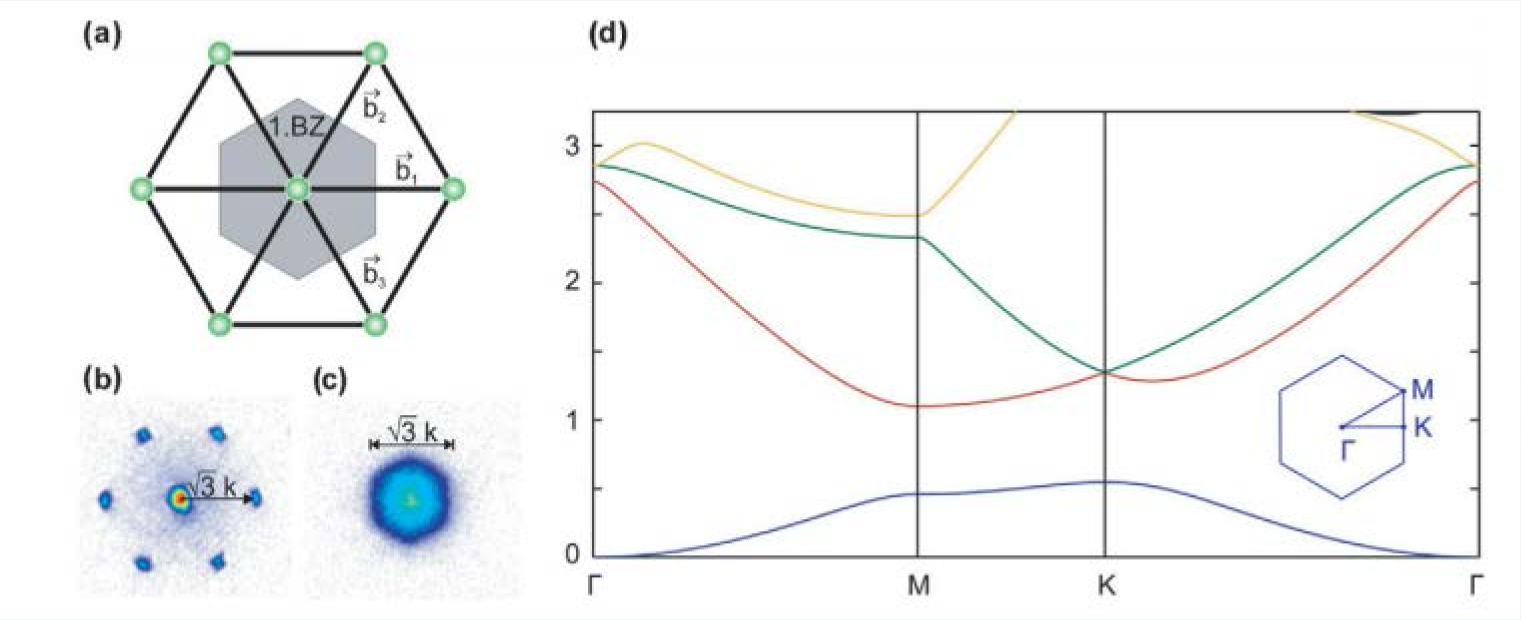
Tunable 6-beam lattice (Esslinger group)

Triangular lattices



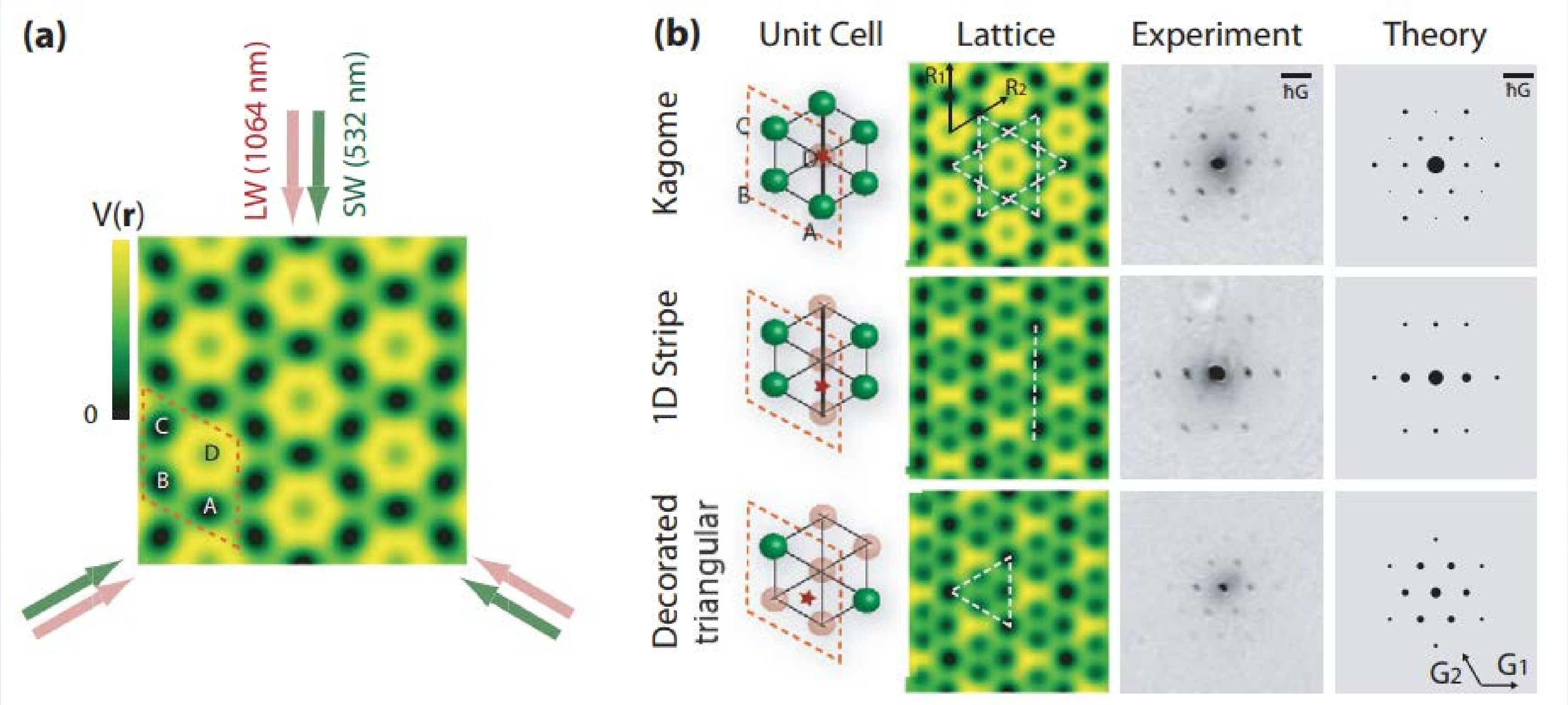
Sengstock group

Triangular lattices



Sengstock group

Kagome lattices



Stamper-Kurn group