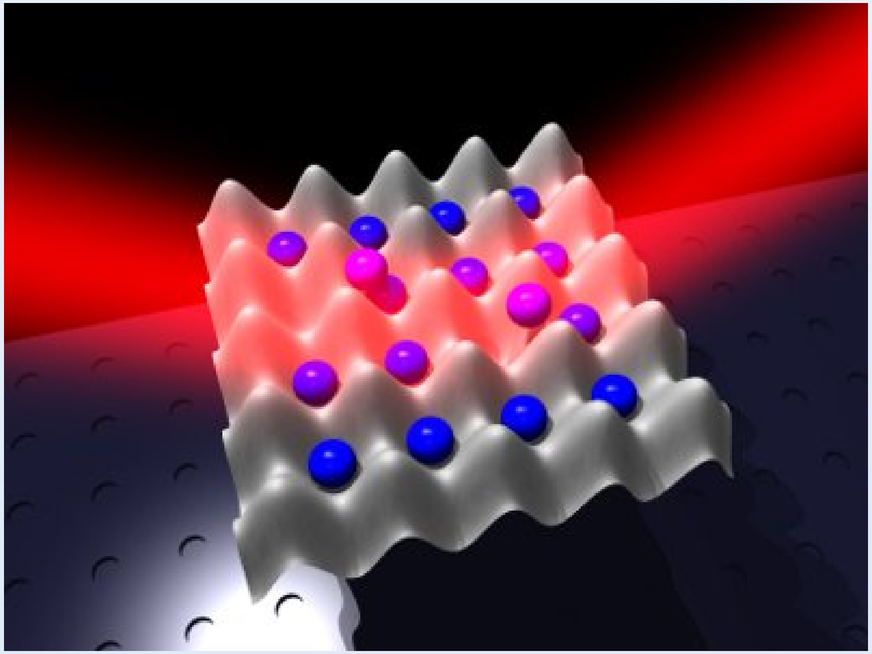
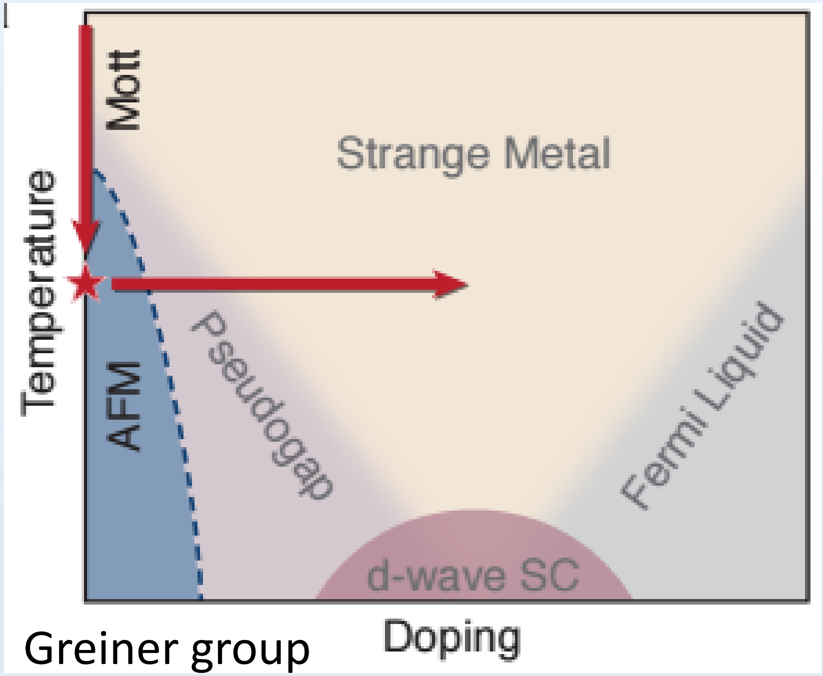
Cold-atom Hubbard models



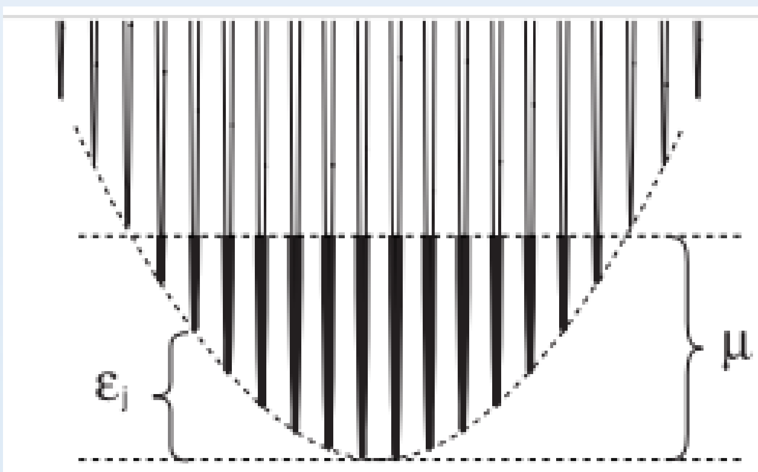
www.lens.unifi.it

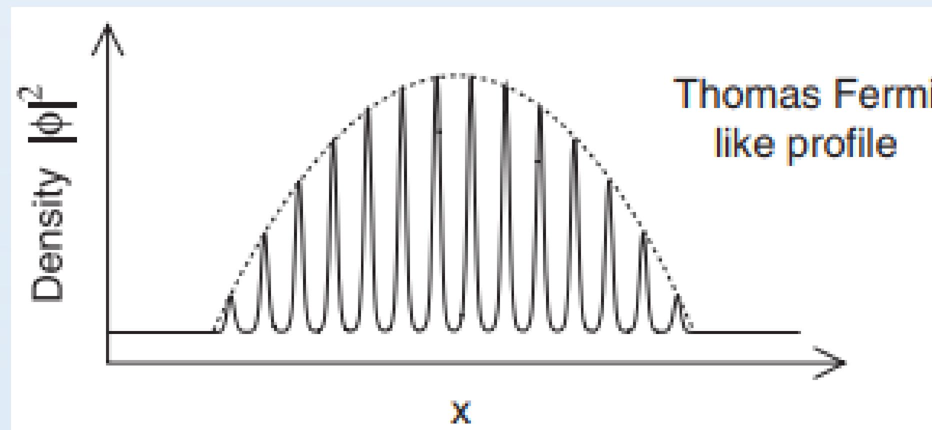


Greiner group

Interacting gases in optical lattices

For neutral atoms in a 1D lattice, forming tunnel-coupled "pancakes," many properties are qualitatively similar to what you have in a simple 3D trap

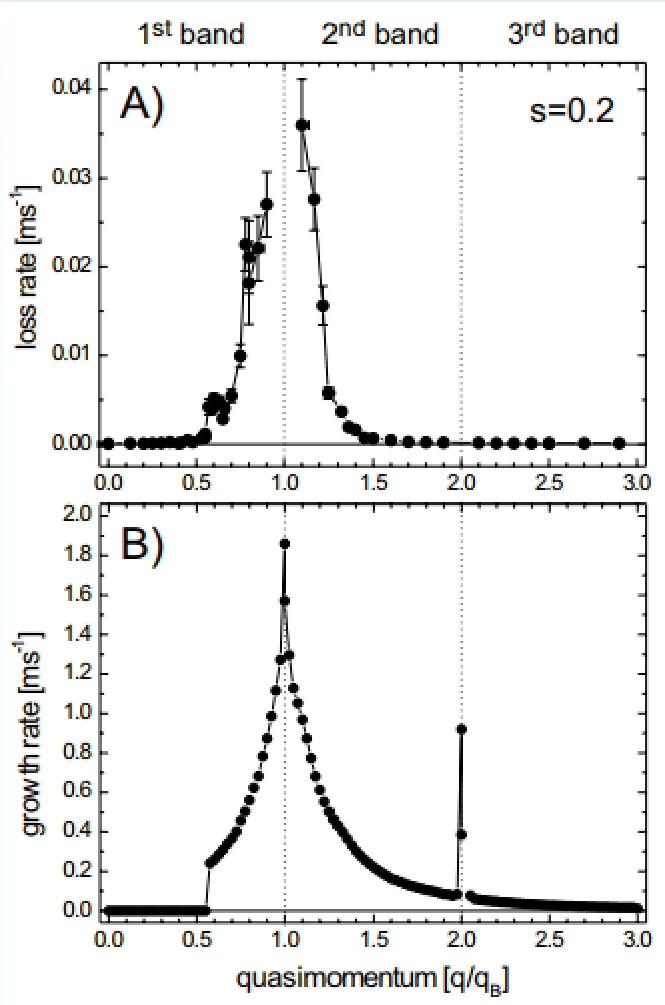




Greiner thesis

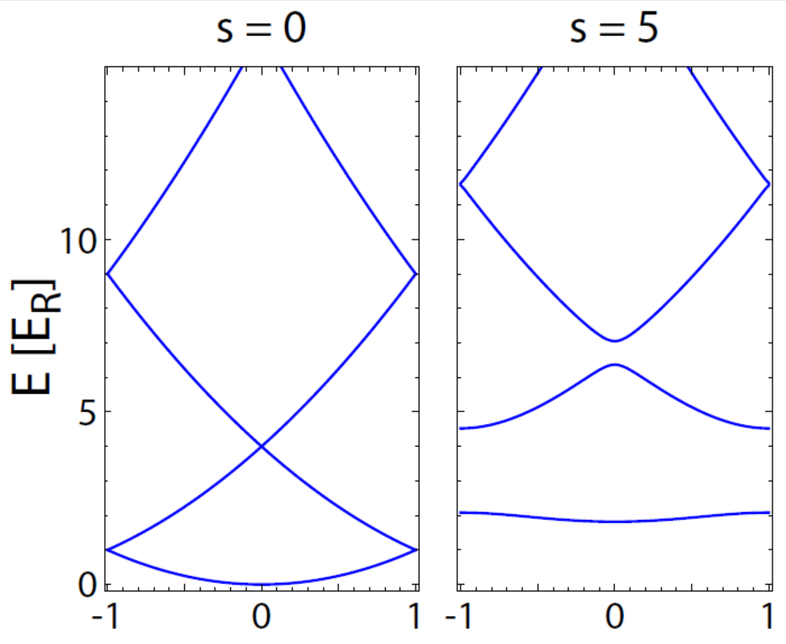
Interacting gases in optical lattices

Some qualitatively new effects can show up, however:



Dynamical & energetic instabilities due to nonlinear interactions / band dispersion effects

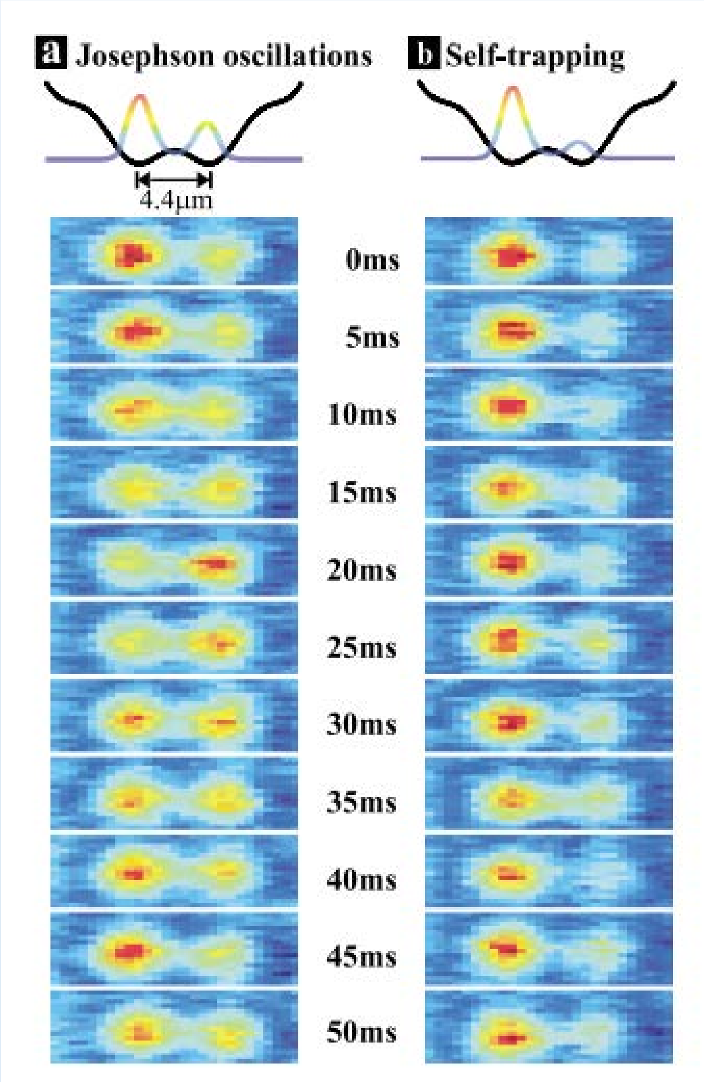
$$1/m_{eff} = d^2E(q)/dq^2$$



Fallani, et al. 2004 PRL

Interacting gases in optical lattices

Some qualitatively new effects can show up, however:



The dynamics of a superfluid (bosonic or fermionic) in a coupled multi-well system has some parallels to the dynamics in junctions / arrays of superconductors

→ correlated many-body physics

In a 3D lattice, where the on-site density of individual atomic wavefunctions is much larger, particles can get strongly correlated at the two- and few-body level

Albiez, et al. 2005 PRL
also, Steinhauer group

Mapping to discrete lattice model

For N-interacting particles (all in the same internal state), our full description of the system would look like:

$$H = \sum_n \int dx \hat{\psi}_n{}^\dagger(\boldsymbol{x}) \left[\hat{H}_{sp}\right] \hat{\psi}_n(\boldsymbol{x}) + \sum_{n,n'} \frac{1}{2} \iint d\boldsymbol{x} d\boldsymbol{x'} \hat{\psi}_n{}^\dagger(\boldsymbol{x}) \hat{\psi}_{n'}{}^\dagger(\boldsymbol{x'}) \boldsymbol{V_{int}}(\boldsymbol{x}, \boldsymbol{x'}) \hat{\psi}_n(\boldsymbol{x}) \hat{\psi}_{n'0}(\boldsymbol{x})$$

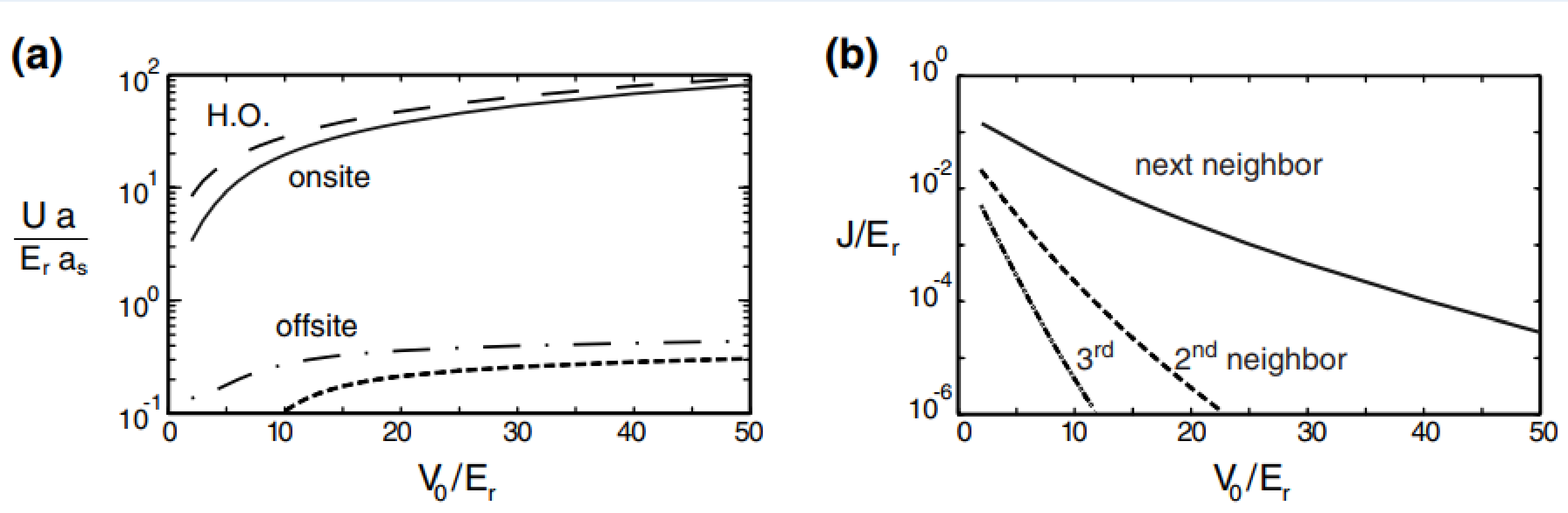
We can get a much simpler form if we expand in terms of local Wannier orbitals, and only keep those for bands that are relevant

$$H = -J \sum_n \left(\hat{c}_{n+1}^\dagger \hat{c}_n + \hat{c}_n^\dagger \hat{c}_{n+1}\right) + \frac{U}{2} \sum_n \hat{c}_n^\dagger \hat{c}_n^\dagger \hat{c}_n \hat{c}_n + \sum_n \varepsilon_n \hat{c}_n^\dagger \hat{c}_n$$

tunneling term — interaction term — site energy term

Hubbard models - approximations

Lots of terms ignored (long-range tunneling, off-site interactions, inter-band transitions, etc.)



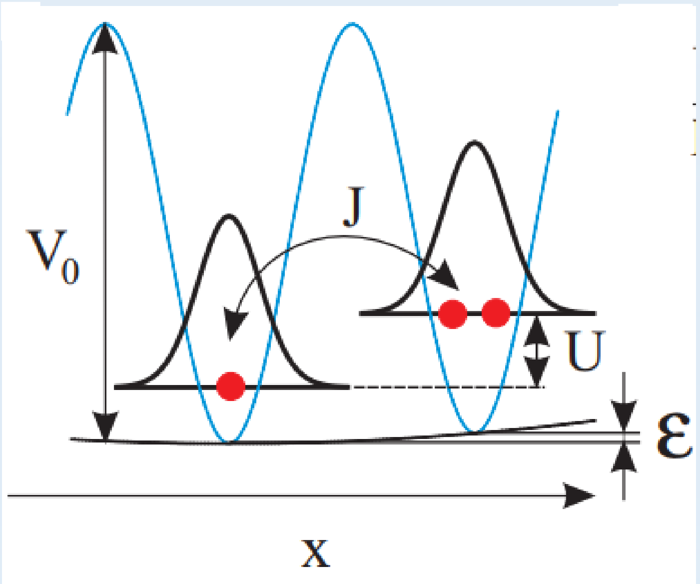
Jaksch

Hubbard models

Minimal model describing the influence of interactions on transport properties [metal-insulator transitions] of particles in lattice systems

$$H = -J\sum_n \left(\hat{c}^\dagger_{n+1}\hat{c}_n + \hat{c}^\dagger_n\hat{c}_{n+1}\right) + \frac{U}{2}\sum_n \hat{c}^\dagger_n\hat{c}^\dagger_n\hat{c}_n\hat{c}_n + \sum_n \varepsilon_n\hat{c}^\dagger_n\hat{c}_n$$

[Most basic version, for scalar particles in single band w/ NN tunneling & on-site interactions]



Jaksch, et al. PRL 1998

J → nearest-neighbor tunneling
U → local, on-site interaction energy
ε → local site energy (due to trap, etc.)

Note: developed for describing metal-insulator transitions in electronic systems.

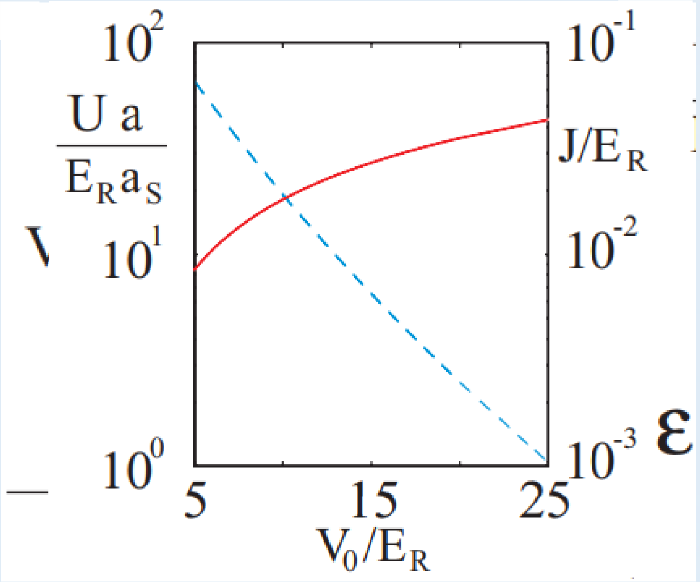
Local U for Coulomb is not ***that*** crazy, due to screening

Hubbard models

Minimal model describing the influence of interactions on transport properties [metal-insulator transitions] of particles in lattice systems

$$H = -J\sum_n \left(\hat{c}^\dagger_{n+1}\hat{c}_n + \hat{c}^\dagger_n\hat{c}_{n+1}\right) + \frac{U}{2}\sum_n \hat{c}^\dagger_n\hat{c}^\dagger_n\hat{c}_n\hat{c}_n + \sum_n \varepsilon_n\hat{c}^\dagger_n\hat{c}_n$$

[Most basic version, for scalar particles in single band w/ NN tunneling & on-site interactions]



Jaksch, et al. PRL 1998

$J \rightarrow$ nearest-neighbor tunneling
$U \rightarrow$ local, on-site interaction energy
$\varepsilon \rightarrow$ local site energy (due to trap, etc.)

can *tune* these parameters with lasers & B-fields!

Bose-Hubbard model (aside)

$$H = -J\sum_n \left(\hat{c}_{n+1}^\dagger \hat{c}_n + \hat{c}_n^\dagger \hat{c}_{n+1}\right) + \frac{U}{2}\sum_n \hat{c}_n^\dagger \hat{c}_n^\dagger \hat{c}_n \hat{c}_n$$

Tunneling term – prefers off-diagonal long-range order (coherent delocalization across lattice)

On-site interaction – penalizes higher occupancies, energy reduced when density is uniform

Two competing terms
→ simple model for understanding quantum phase transitions,
formal analogy to model for interacting spins

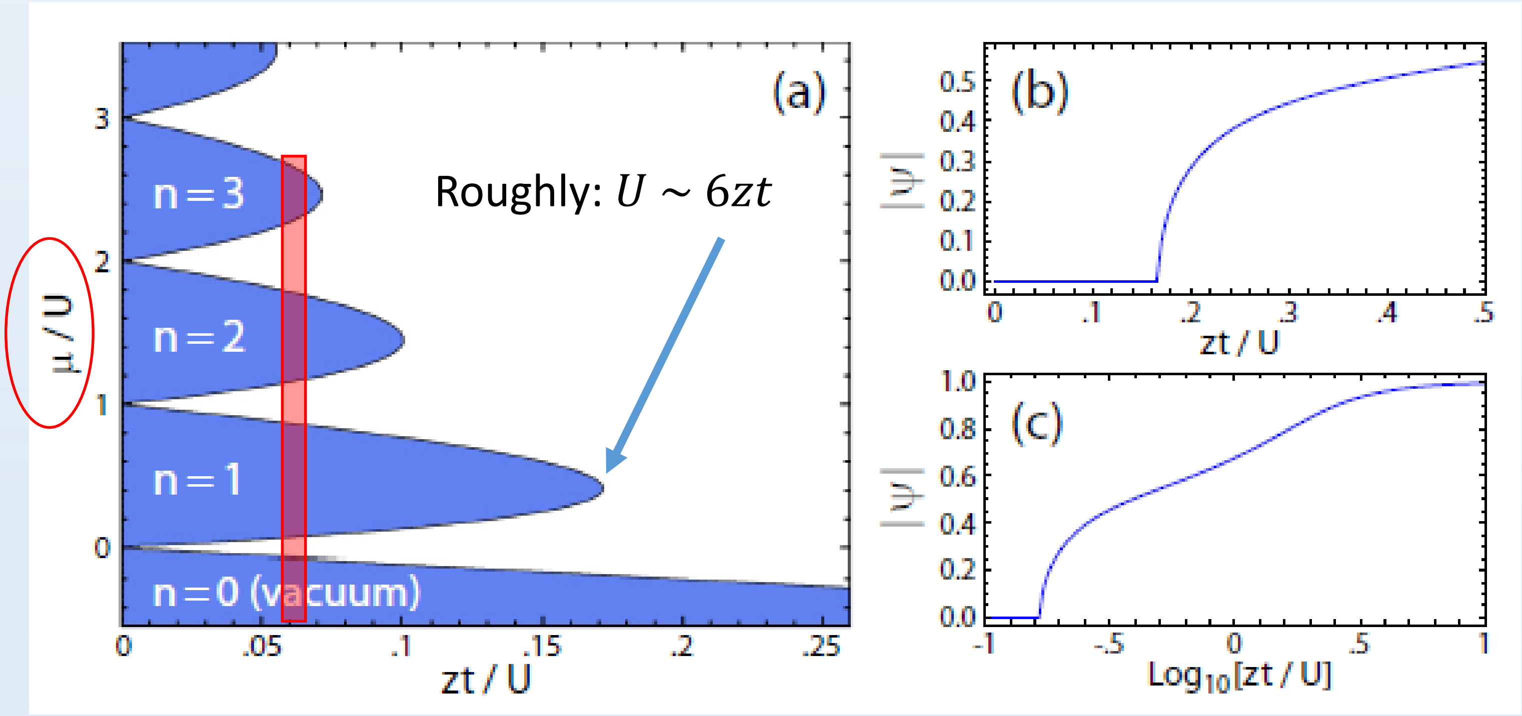
$$H = -B_x \sum_n S_{x,n} + J_z \sum_n S_{z,n} S_{z,n+1}$$

Lots of general interest in using these systems to explore quantum criticality in the vicinity of a quantum phase transitions

Bose-Hubbard model

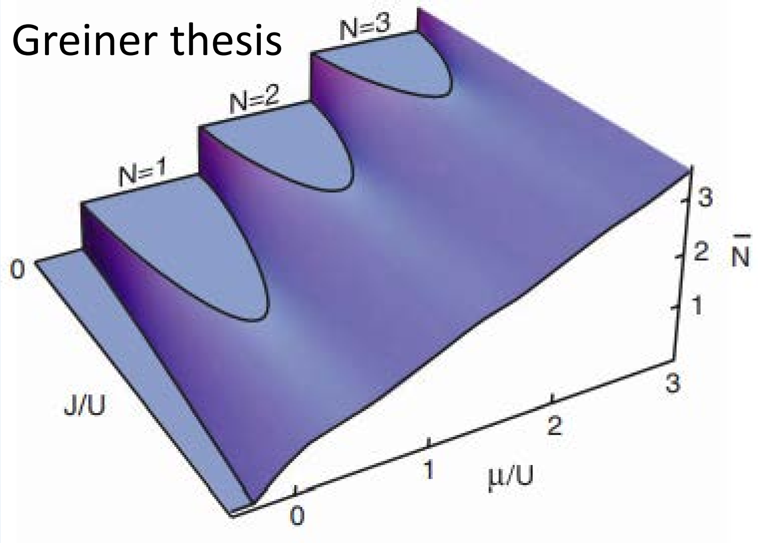
Phase diagram from site-decoupled mean-field theory

(Phys. Rev. A **63**, 053601, 2001)

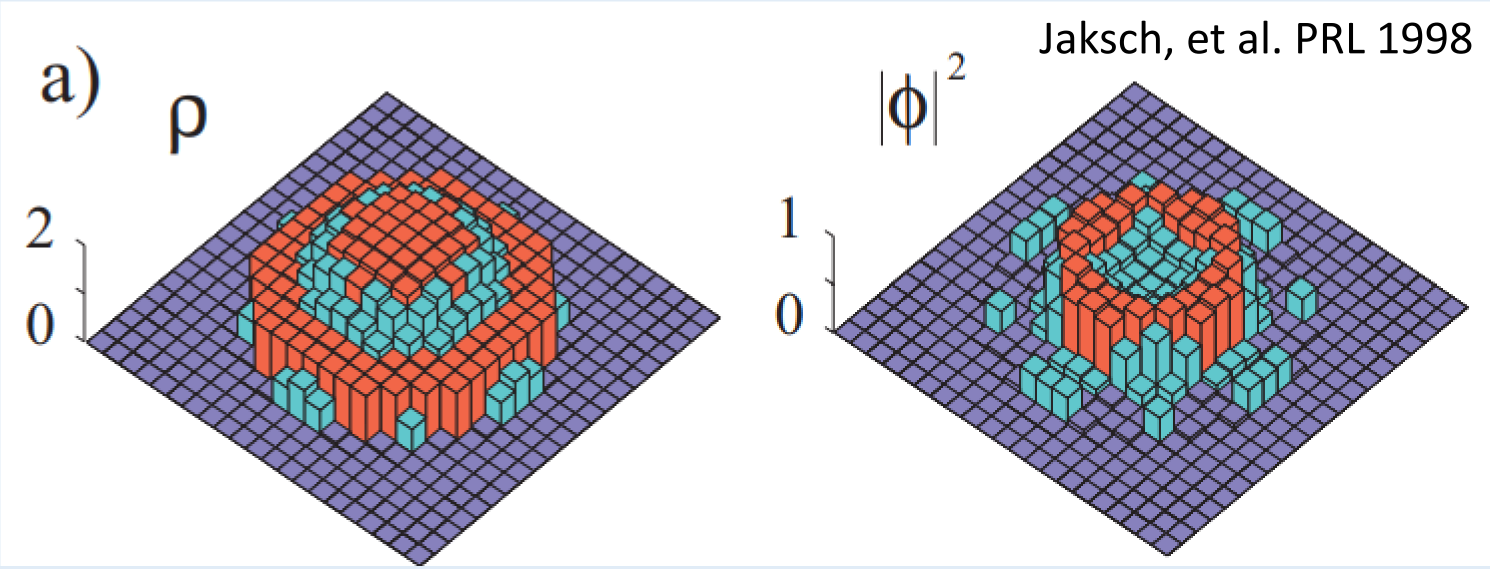


For a trapped system, the chemical potential is going to vary across the system

Bose-Hubbard model



Incompressible Mott lobes, i.e.
Insulating regions, for strong interactions

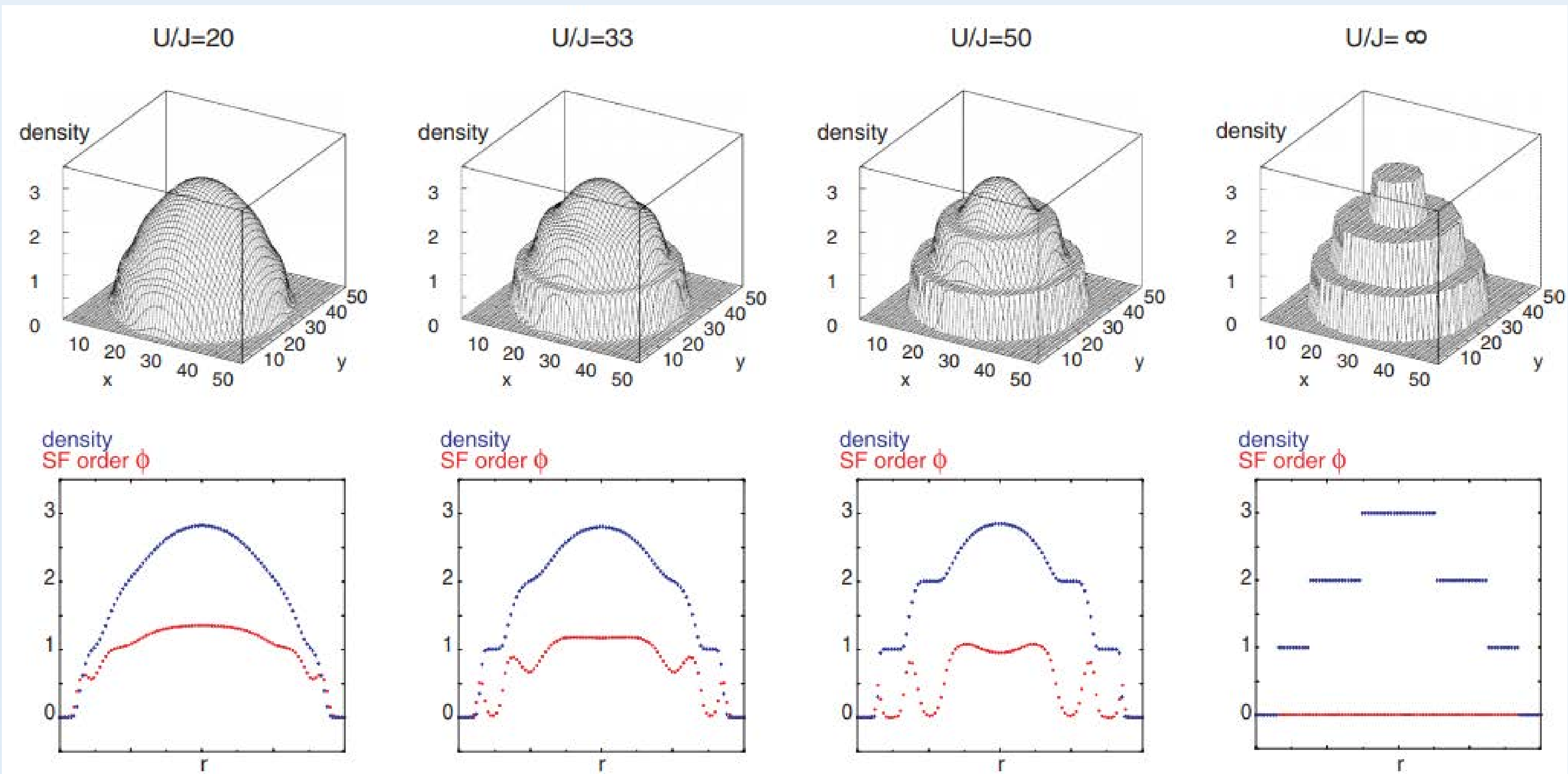


Bose-Hubbard model

Cuts of the density in a trapped system:

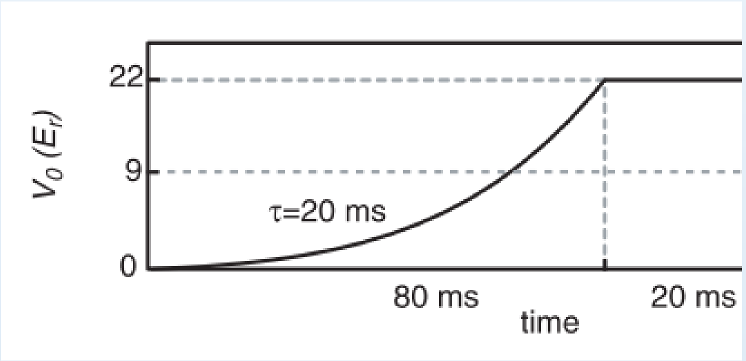
“wedding cake structure” / ziggurat structure

Greiner thesis

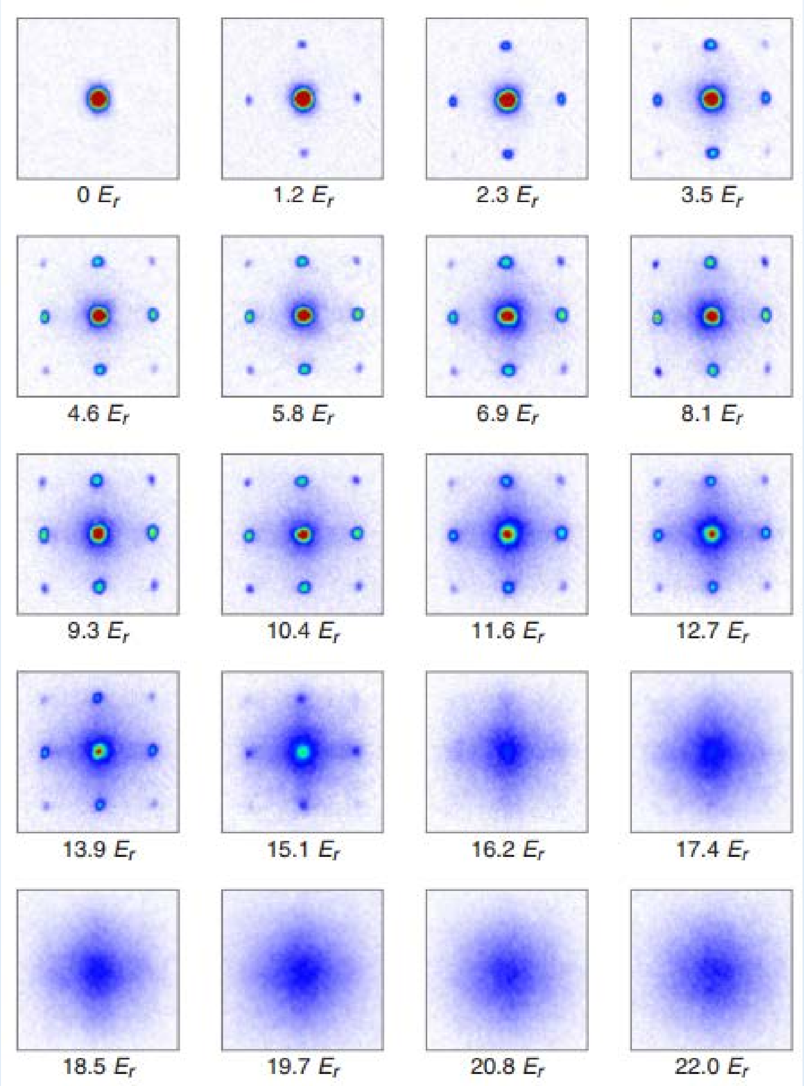


Experimental signatures

Time-of-flight (single-particle) interference: CONTRAST   &   PEAK WIDTH



Experimental ramp protocol in original experiment



Greiner, et al. Nature 2002

Experimental signatures

Time-of-flight (single-particle) interference: CONTRAST & PEAK WIDTH

$$S(k) \propto \sum_{j,j'=1}^{M} e^{i(j-j')kd} \langle \phi_g | \hat{a}_j^\dagger \hat{a}_{j'} | \phi_g \rangle$$

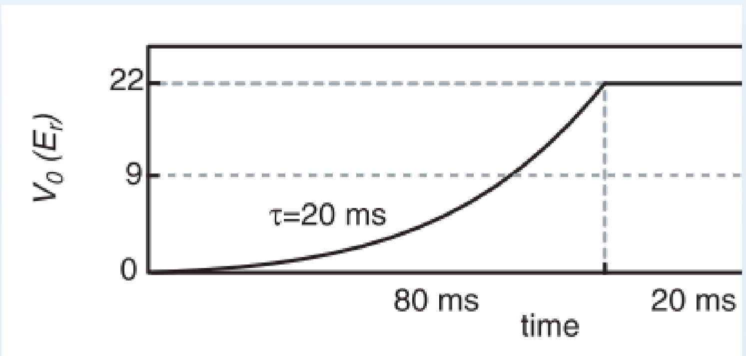
Interference pattern in k-space

$$D_j(r) \propto \langle \phi_g | \hat{a}_j^\dagger \hat{a}_{j+r} | \phi_g \rangle / \sqrt{\hat{n}_j \hat{n}_{j+r}}$$

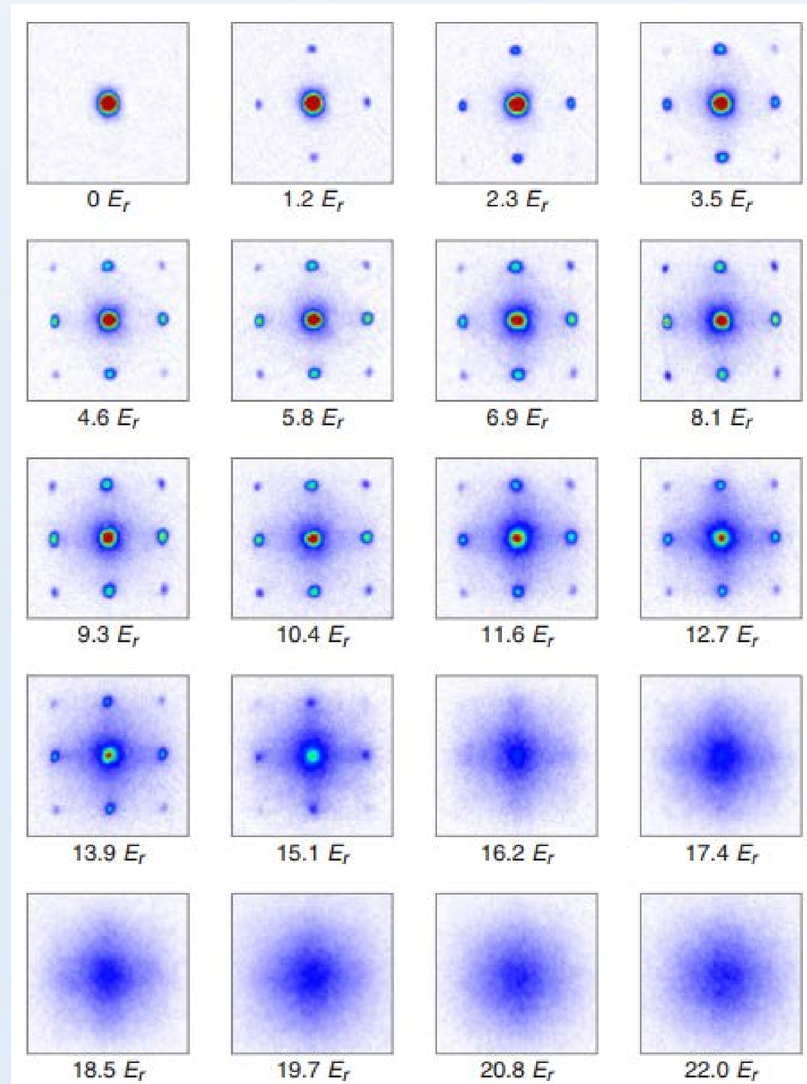
off-site coherence in real-space

Experimental signatures

Time-of-flight (single-particle) interference: CONTRAST   &   PEAK WIDTH



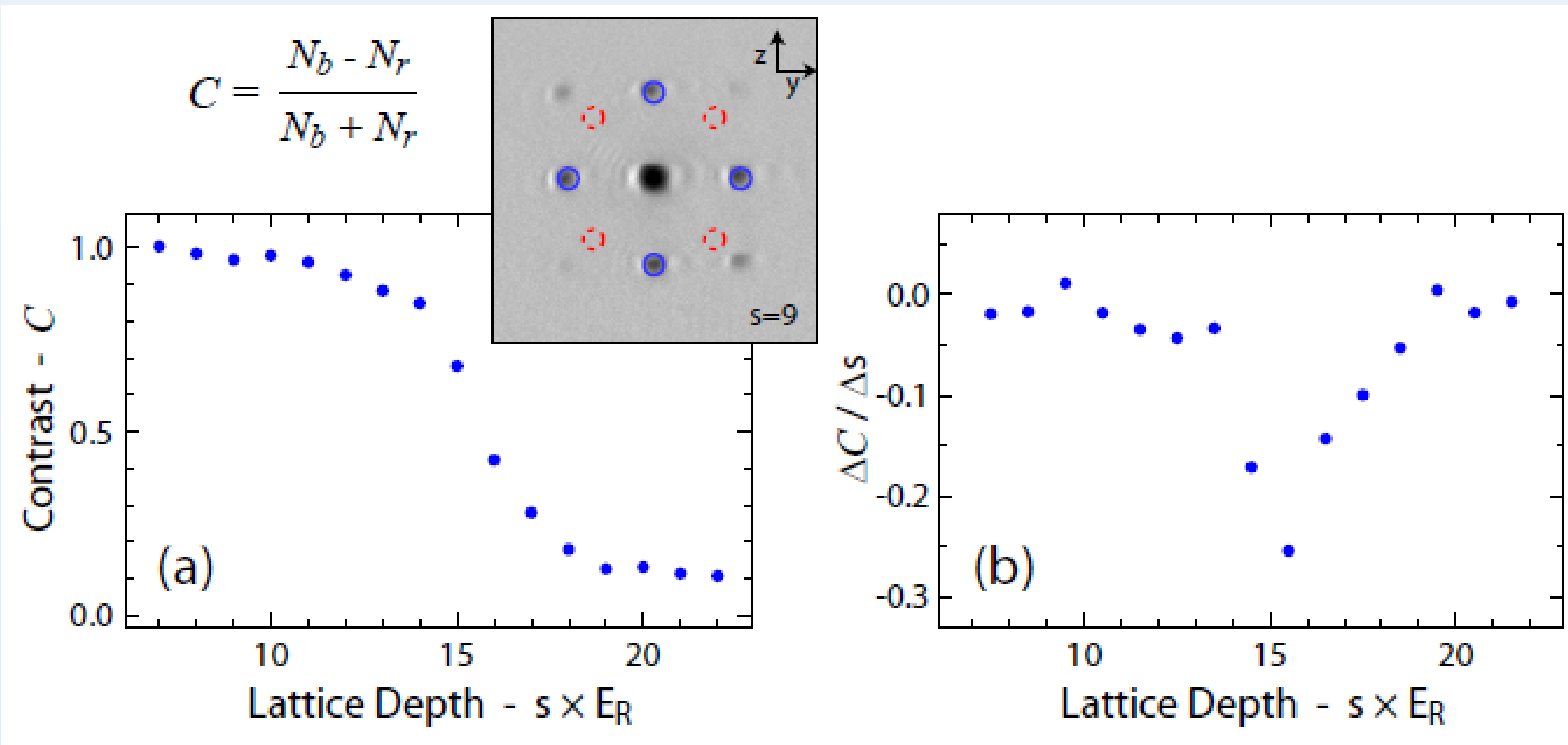
Experimental ramp protocol in original experiment



Greiner, et al. Nature 2002

Experimental signatures

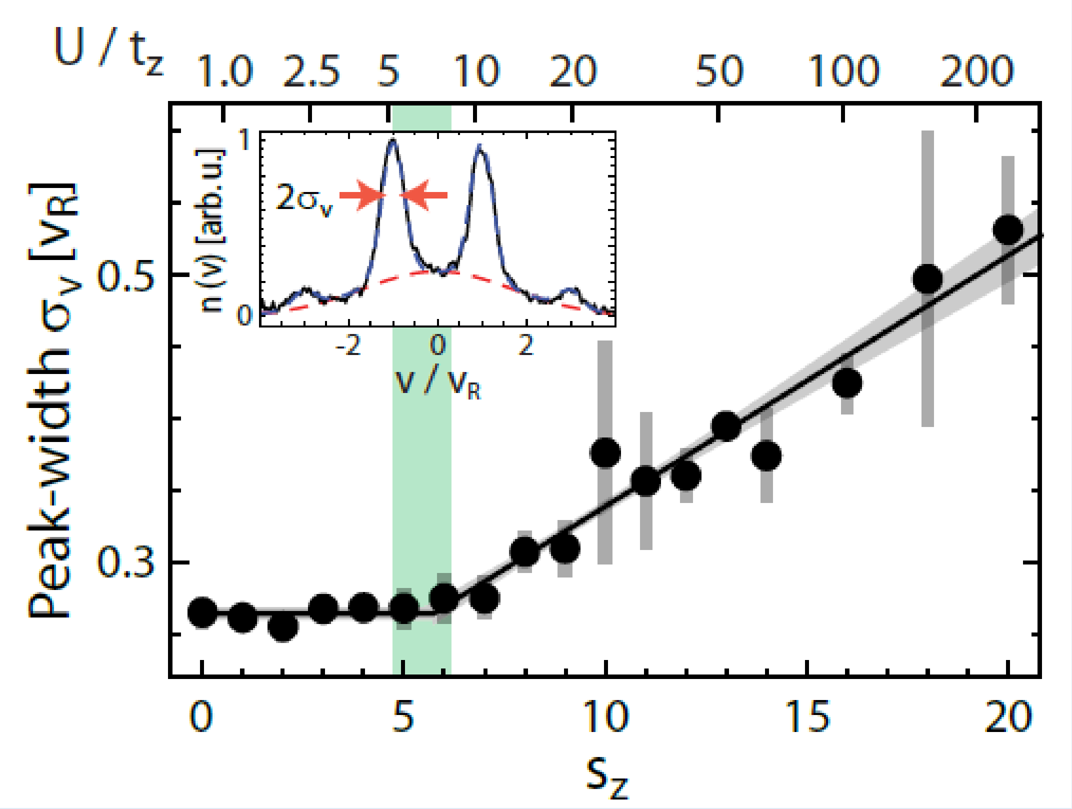
Time-of-flight (single-particle) interference: <u>CONTRAST</u> & PEAK WIDTH



Greiner, et al. Nature 2002

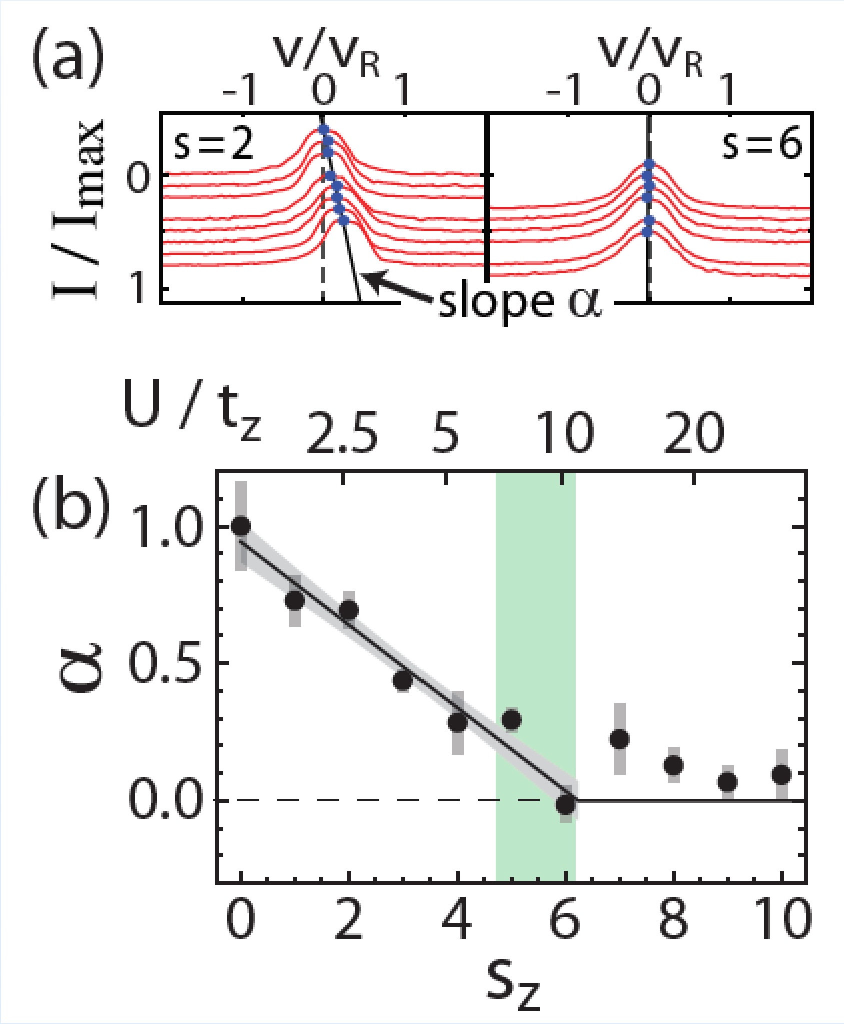
Experimental signatures

Time-of-flight (single-particle) interference: CONTRAST & PEAK WIDTH



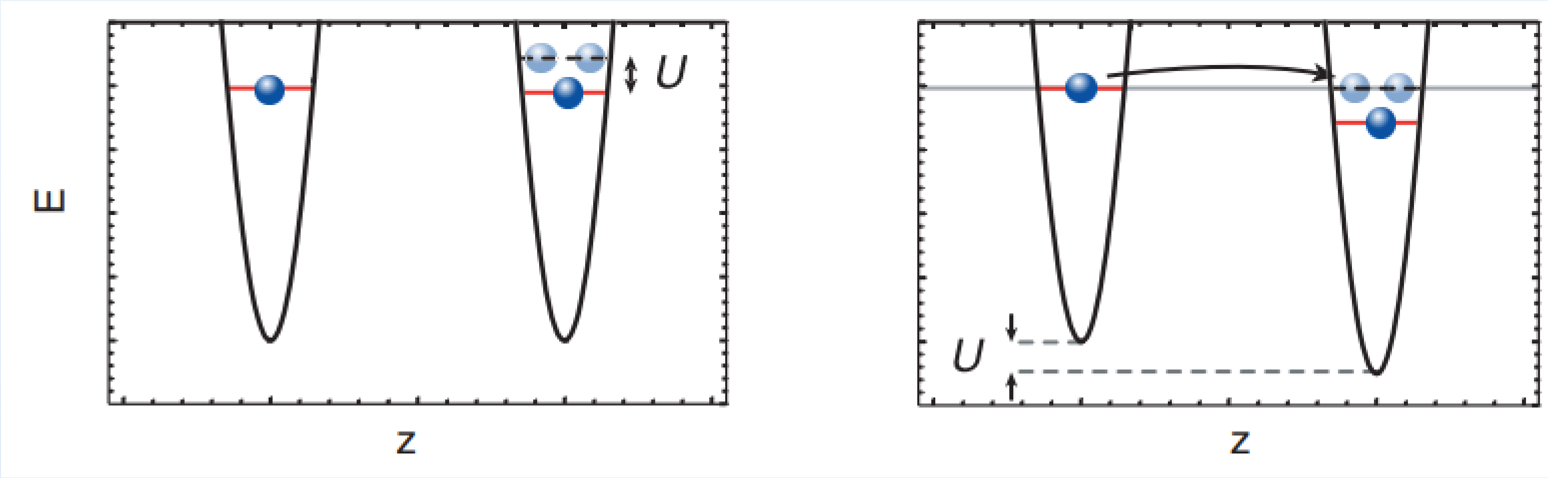
Experimental signatures

Impulse response – give the cloud a kick and see if the density responds



Experimental signatures

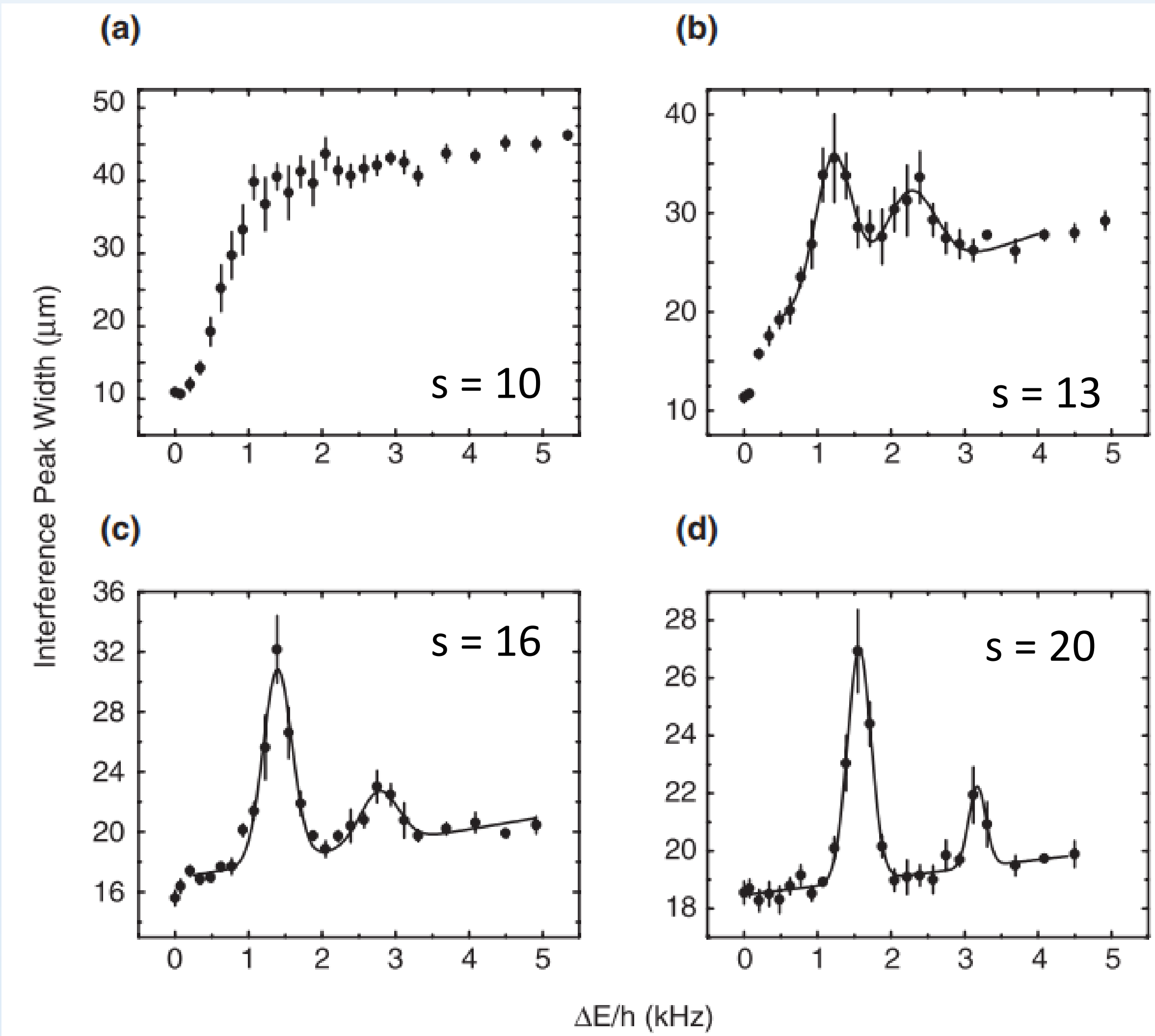
Measuring the charge gap U



Experimental signatures

Measuring the charge gap U

Greiner, et al. Nature 2002

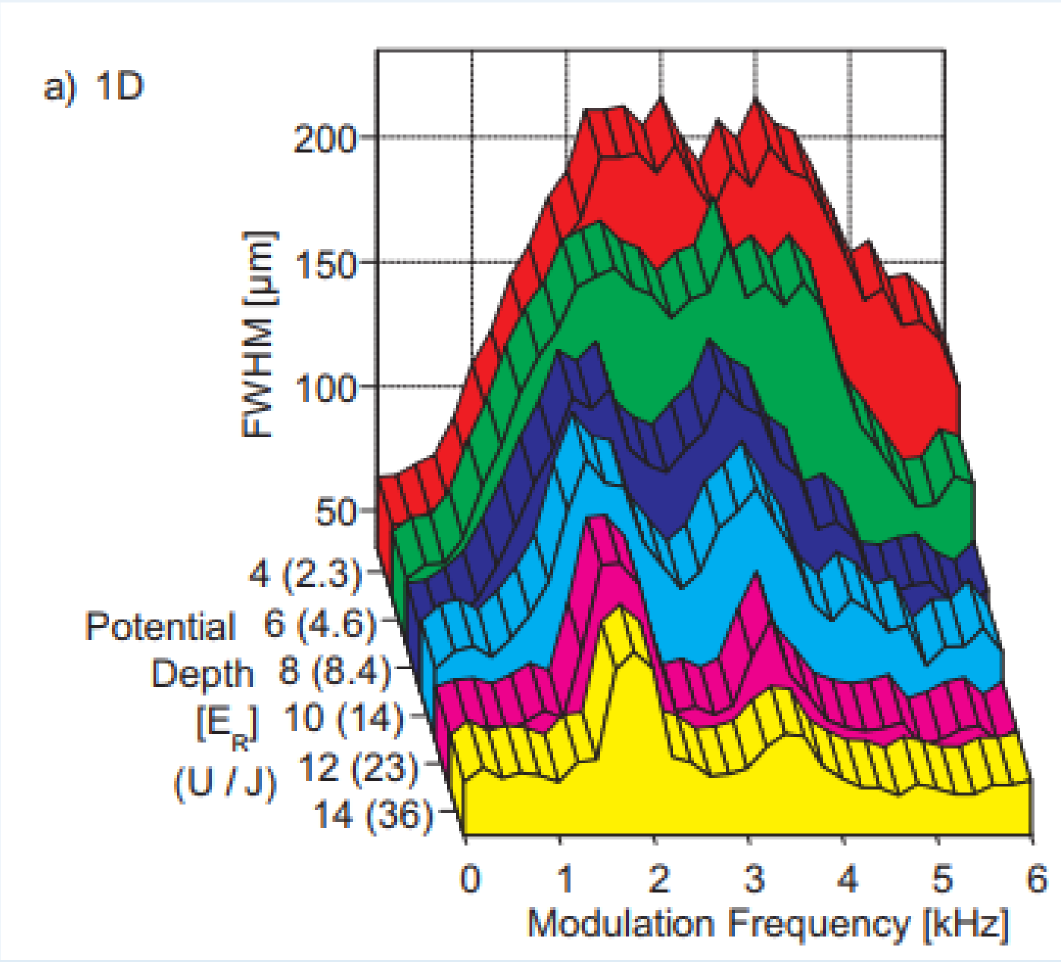


linear gradient

Experimental signatures

Measuring the charge gap U

Esslinger group, 2003

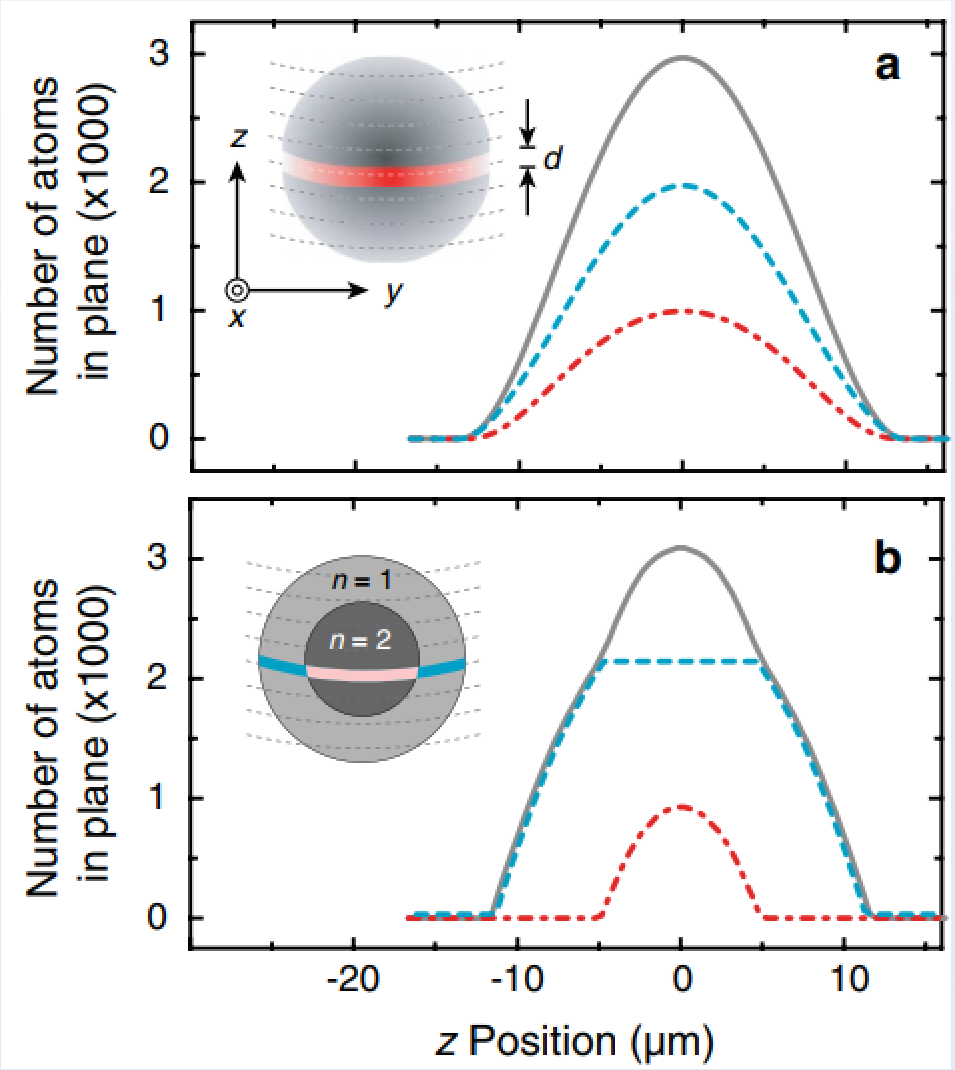


modulated lattice depth:
~Bragg spectroscopy

Experimental signatures

Imaging the Mott shells

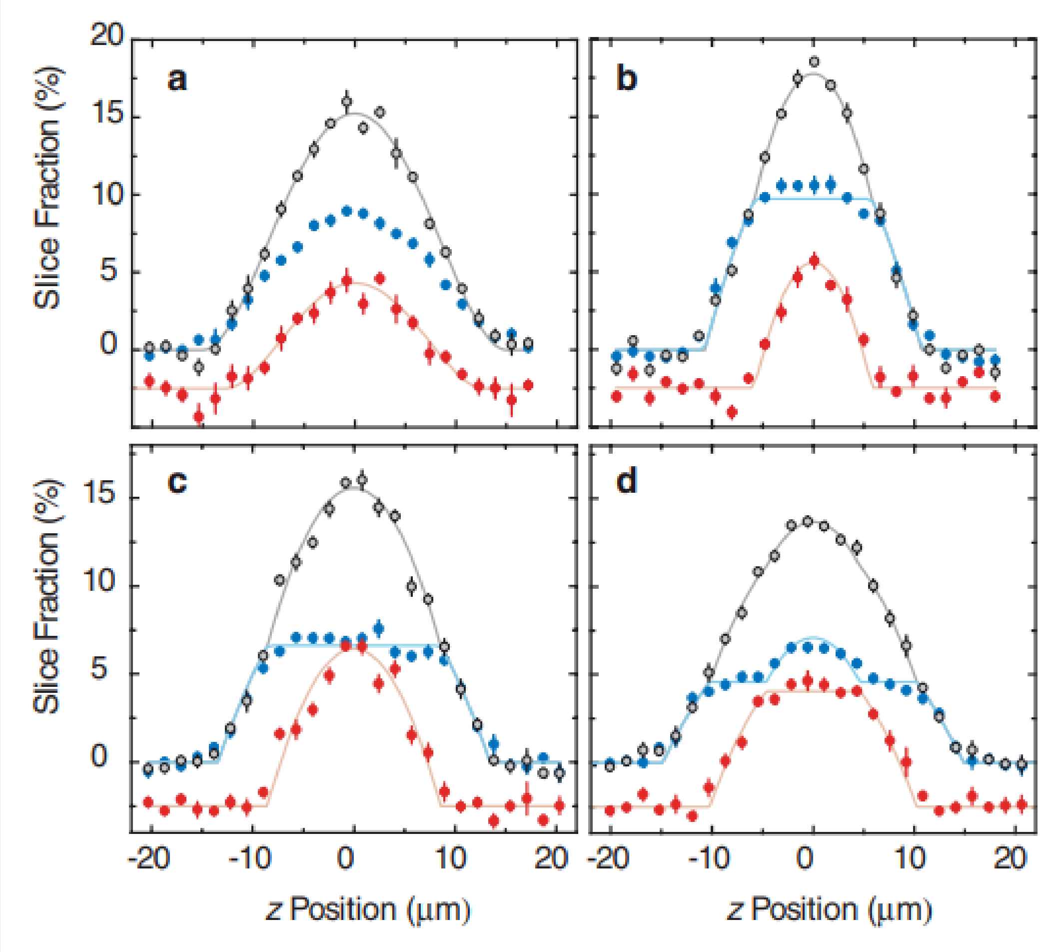
Fölling, et al. 2006



Experimental signatures

Imaging the Mott shells

Fölling, et al. 2006

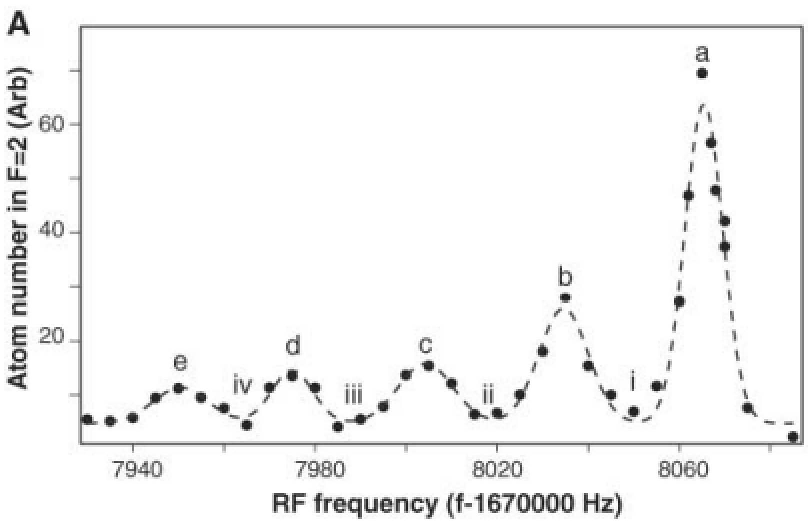


Local addressing + spin-changing collisions

Experimental signatures

Imaging the Mott shells

Campbell, et al. 2006



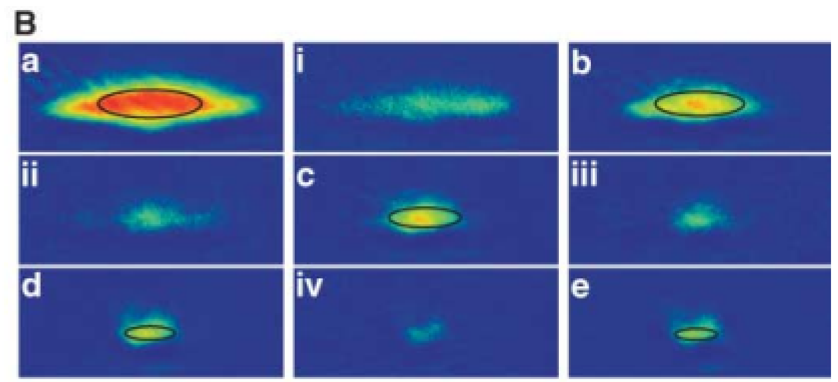
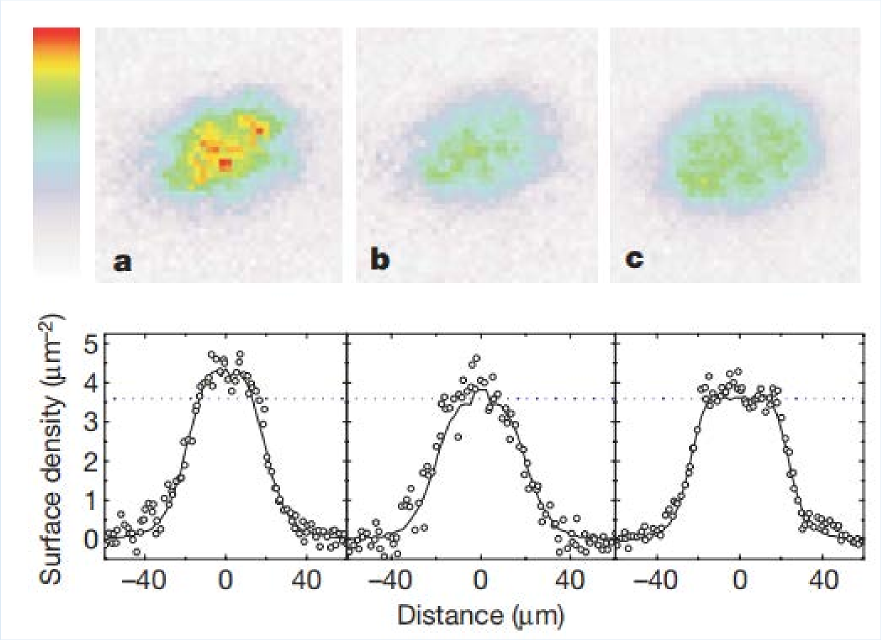


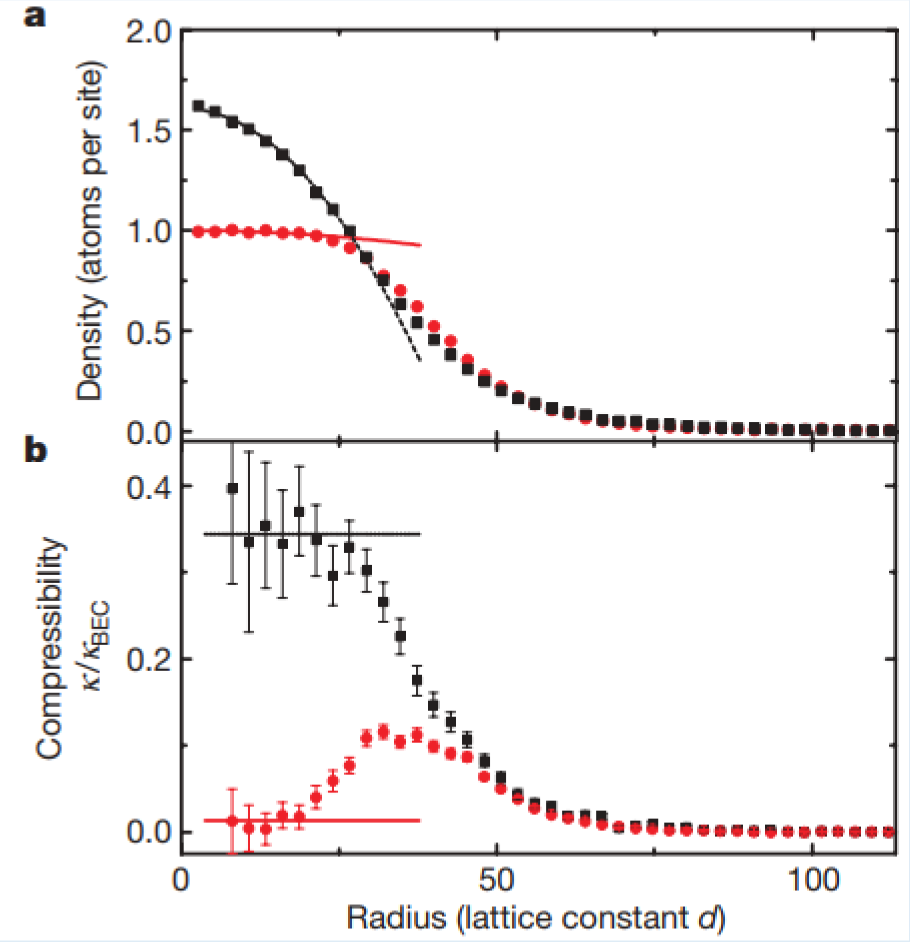
Fig. 3. Imaging the shell structure of the MI. (**A**) Spectrum of the MI at V = $35E_{rec}$. (**B**) Absorption images for for decreasing rf frequencies. Images a to e were taken on resonance with the peaks shown in (A) and display the spatial distribution of the *n* =1 to *n* = 5 shells. The solid lines shows the predicted contours of the shells. Absorption images taken for rf frequencies between the peaks (images i to iv) show a much smaller signal. The field of view was 185 μm by 80 μm.

Experimental signatures

Imaging the Mott shells

Gemelke, et al. 2009

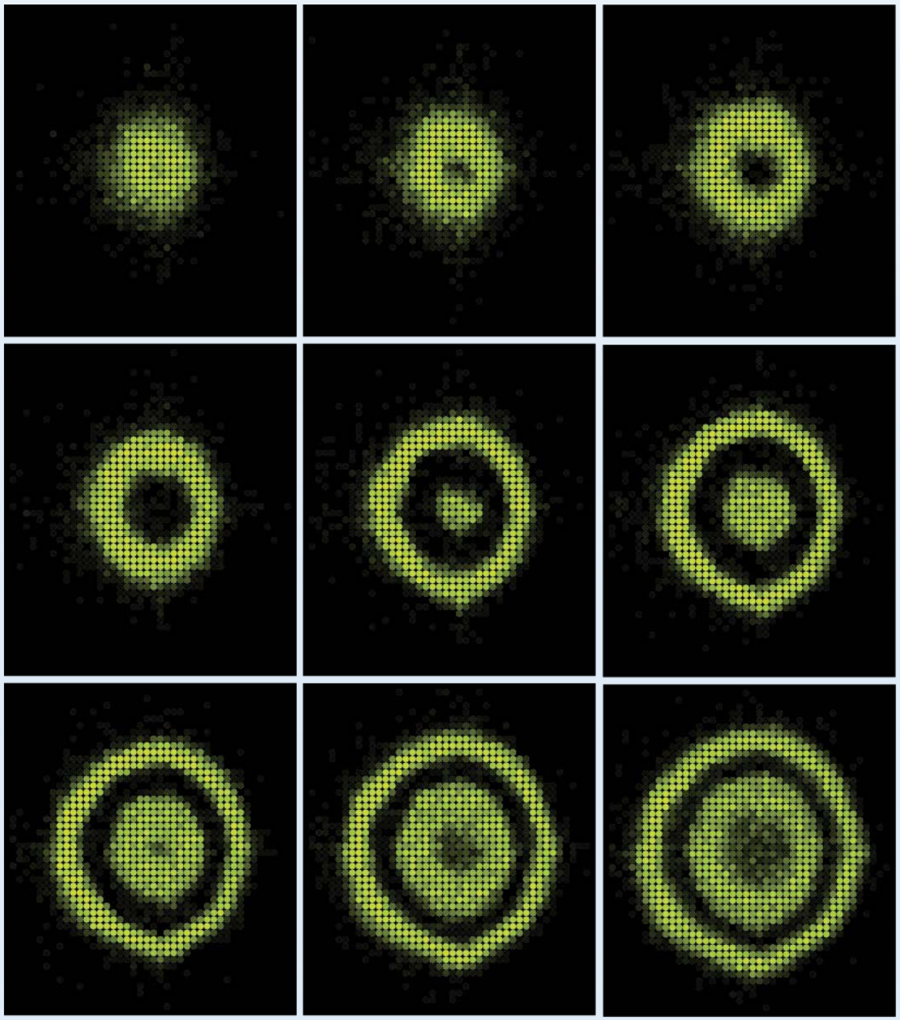




Experimental signatures

Imaging the Mott shells

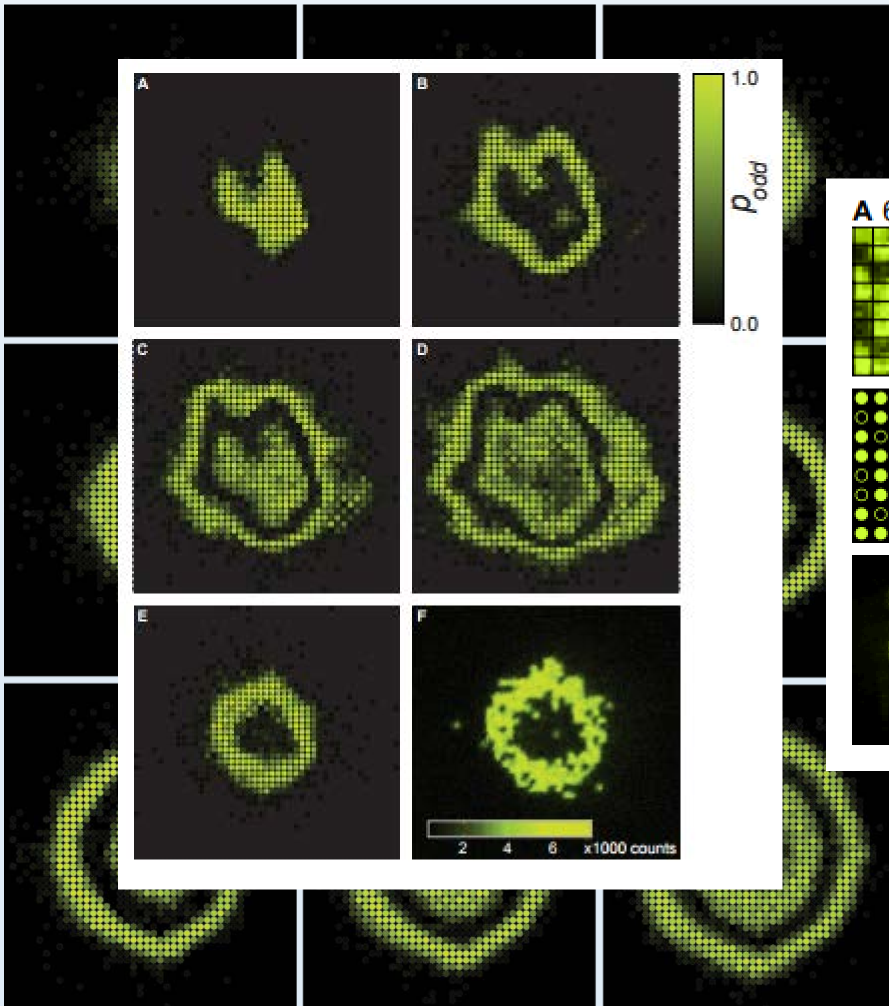
Greiner group & Bloch group, et al. 2010

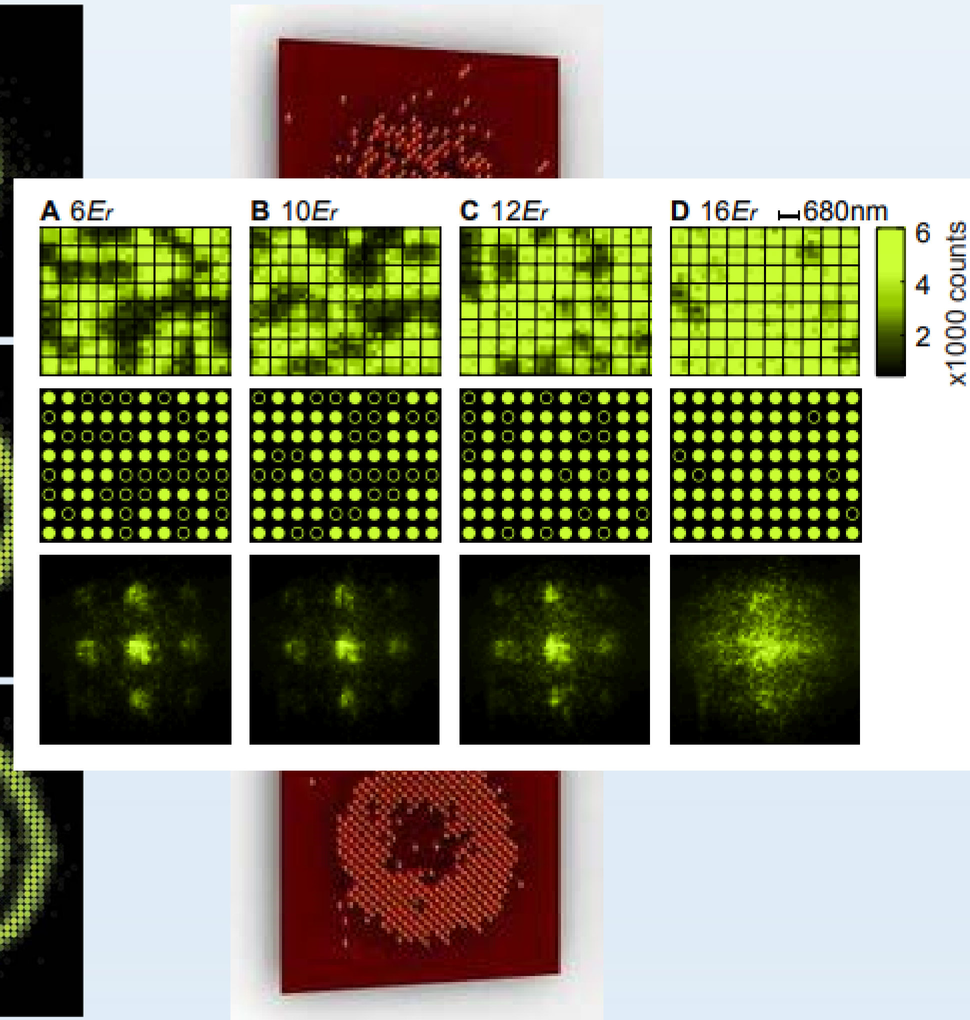


Experimental signatures

Greiner group & Bloch group, et al. 2010

Imaging the Mott shells

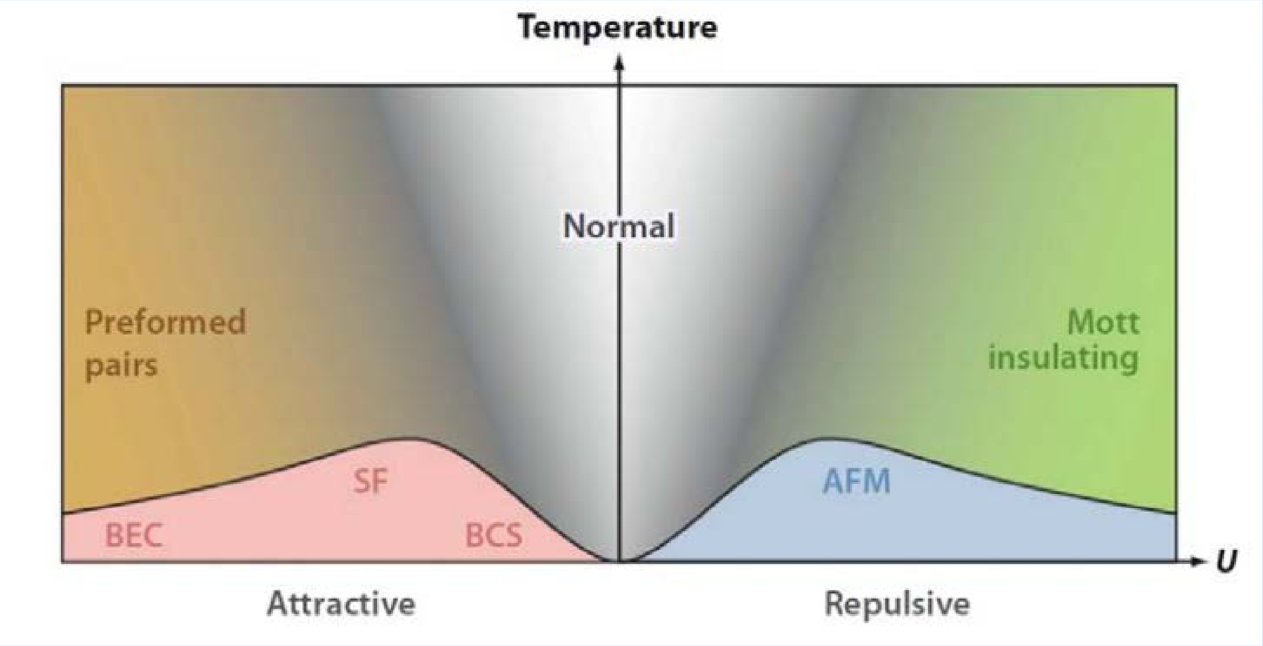




Two-component Fermi Hubbard model

Let's focus on main two terms, in simplest case: spin-independent hopping and on-site interaction

$$H = -t\sum_{n,\sigma}\left(\hat{c}^{\dagger}_{\sigma,n+1}\hat{c}_{\sigma,n} + \hat{c}^{\dagger}_{\sigma,n}\hat{c}_{\sigma,n+1}\right) + U\sum_{j}\hat{n}_j^{\uparrow}\hat{n}_j^{\downarrow}$$

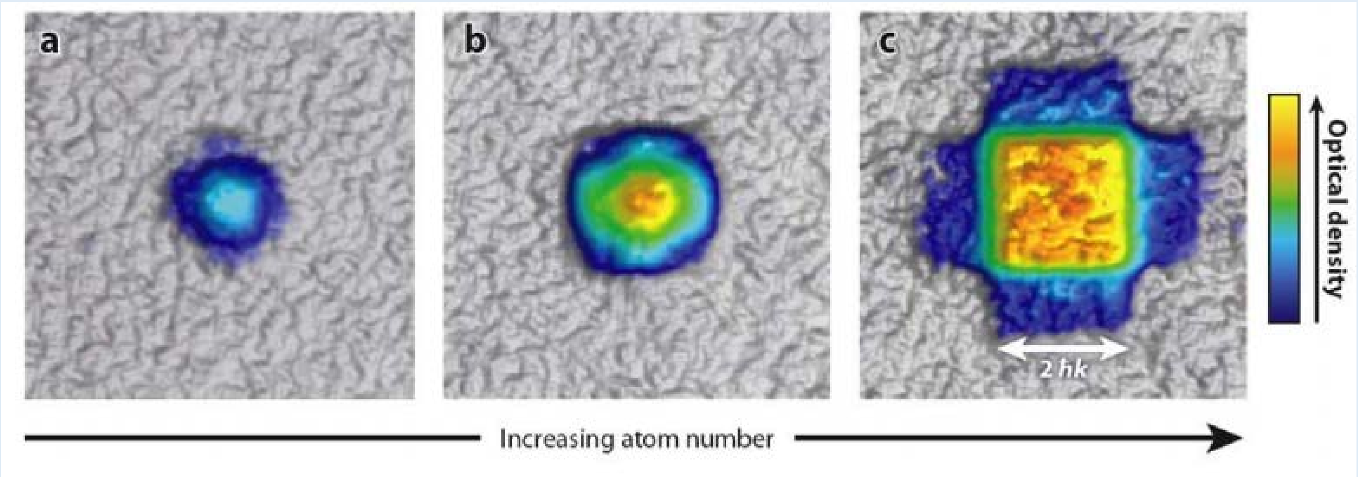


Phase diagram for fixed T at half filling (# of particles = # of sites, but 2 spin states)

Two-component Fermi Hubbard model

Even for weak interactions or a spin-polarized gas, sample becomes incompressible when all the states of the lowest energy band are filled (band insulator at unit occupancy)

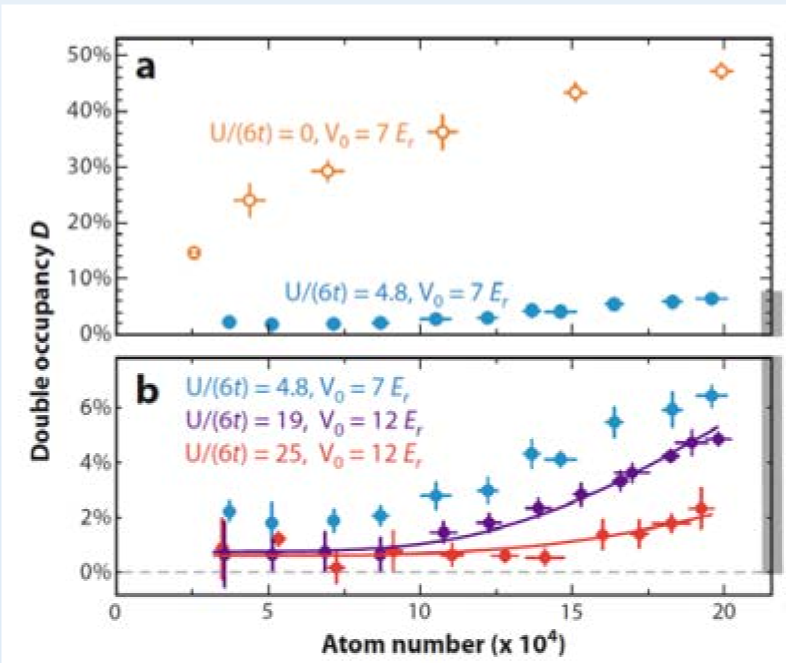
$$H = -t\sum_{n,\sigma}\left(\hat{c}^{\dagger}_{\sigma,n+1}\hat{c}_{\sigma,n} + \hat{c}^{\dagger}_{\sigma,n}\hat{c}_{\sigma,n+1}\right) + U\sum_{j}\hat{n}^{\uparrow}_{j}\hat{n}^{\downarrow}_{j}$$

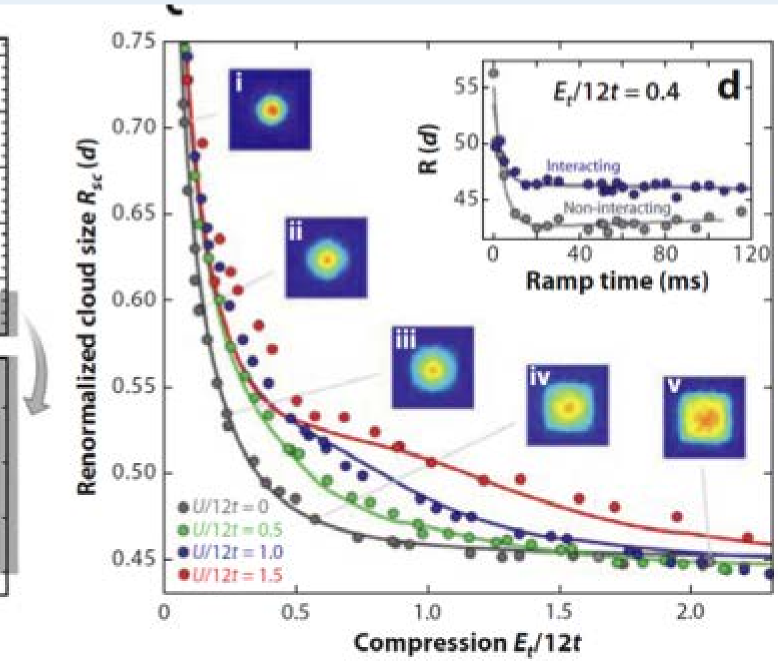


Experimental signatures of Mott insulator

Counting “doublons” for fixed number and increasing interaction strength

Look for modification of compressibility

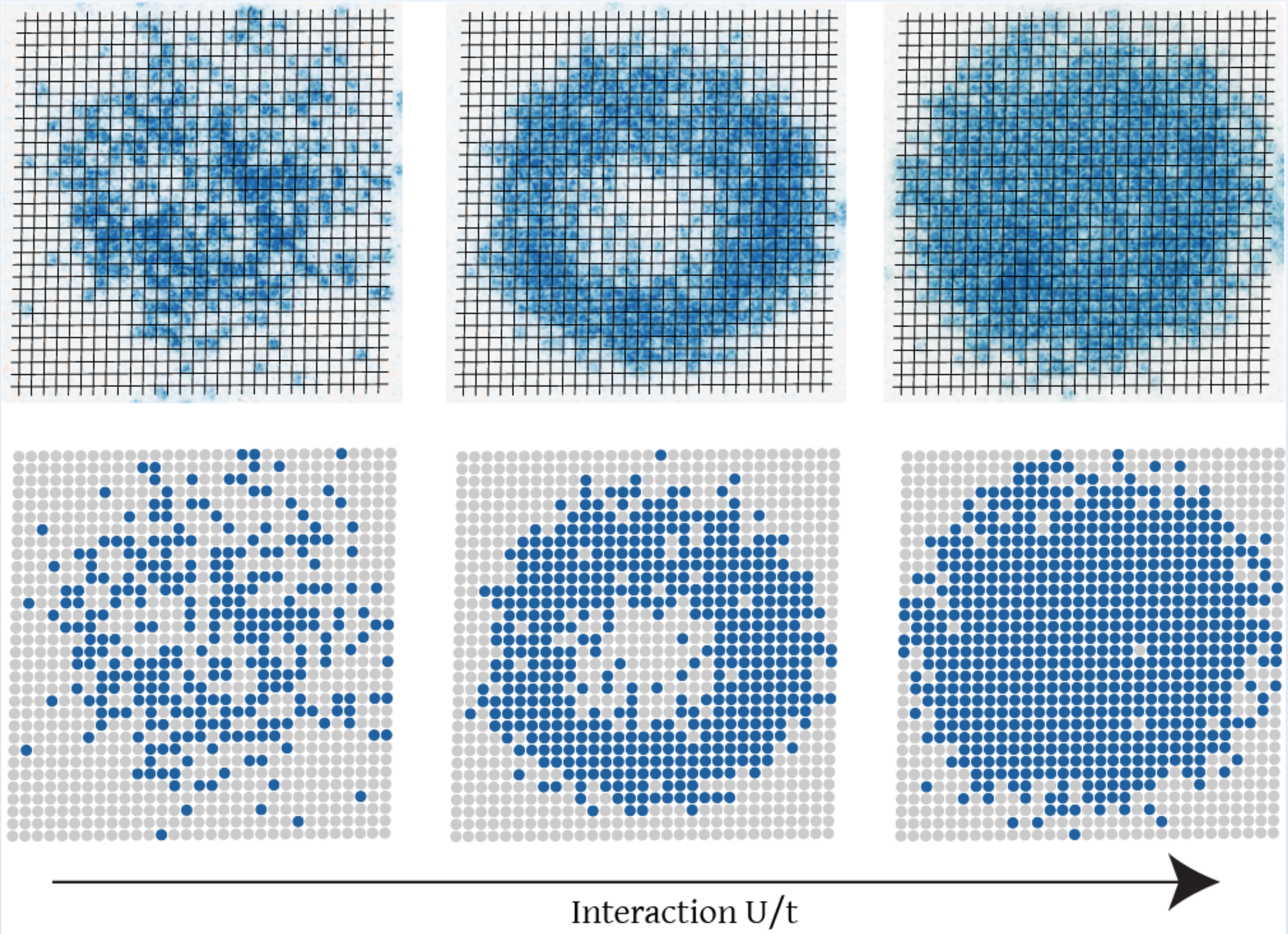




Jördens, et al. 2008

Schneider, et al. 2008

Experimental signatures of Mott insulator



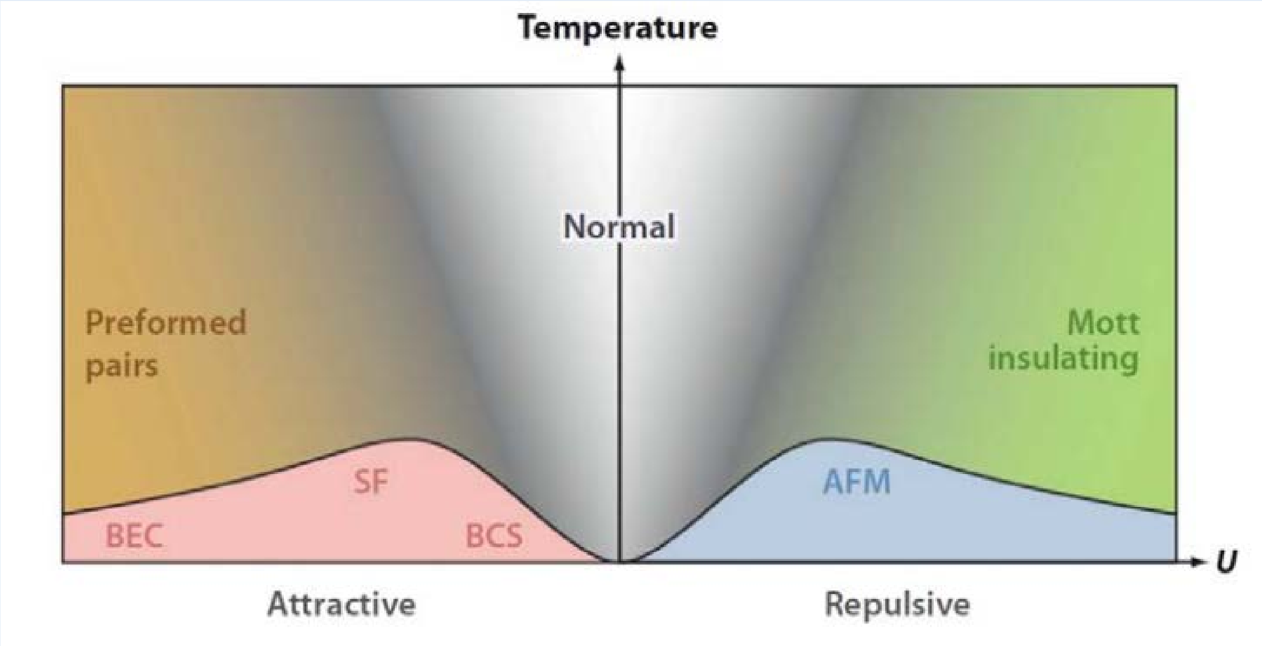
Greiner group (also, Bloch group, Zwierlein group)

Directly look for absence of doublons (holes in the middle)

Density gets "frozen," what then?

With U/t >> 1, density fluctuations are suppressed. But spins can still "hop" through second-order tunneling

$$H = -t \sum_{n,\sigma} \left(\hat{c}^{\dagger}_{\sigma,n+1} \hat{c}_{\sigma,n} + \hat{c}^{\dagger}_{\sigma,n} \hat{c}_{\sigma,n+1} \right) + U \sum_j \hat{n}^{\uparrow}_j \hat{n}^{\downarrow}_j$$



$$H = \pm J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j,$$

$$J = 4t^2/U$$

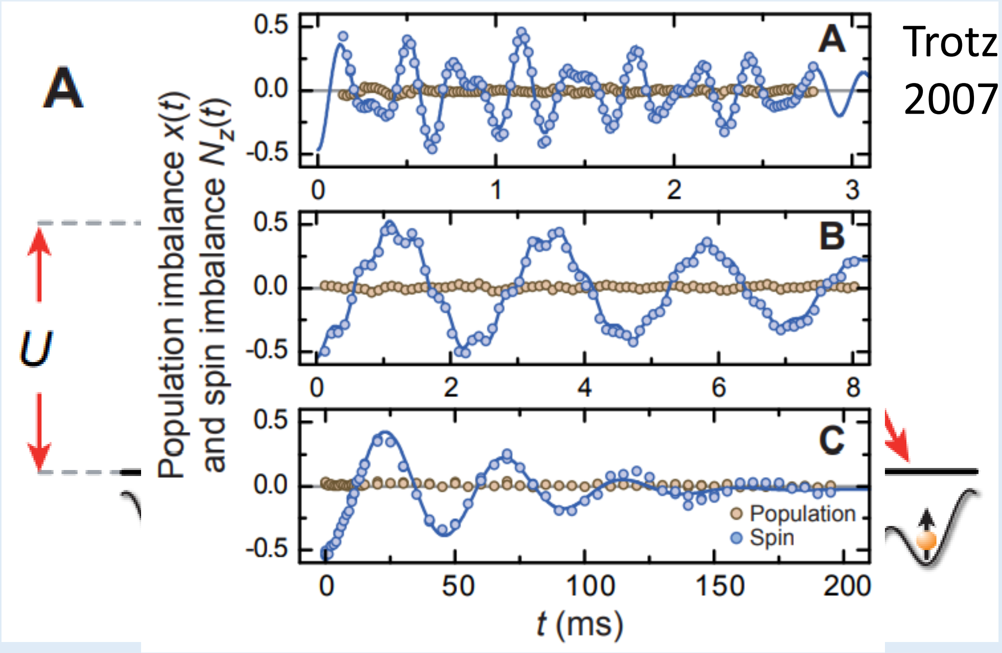
[negative sign for fermions (AFM correlations), positive for bosons (FM correlations)]

$$k_B T_{Neel} \sim J$$

Density gets “frozen,” what then?

With U/t >> 1, density fluctuations are suppressed. But spins can still “hop” through second-order tunneling

$$H = -t\sum_{n,\sigma}\left(\hat{c}^{\dagger}_{\sigma,n+1}\hat{c}_{\sigma,n} + \hat{c}^{\dagger}_{\sigma,n}\hat{c}_{\sigma,n+1}\right) + U\sum_{j}\hat{n}^{\uparrow}_{j}\hat{n}^{\downarrow}_{j}$$



Trotzky, et al. 2007

$$H = \pm J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j,$$

$$J = 4t^2/U$$

[negative sign for fermions (AFM correlations), positive for bosons (FM correlations)]

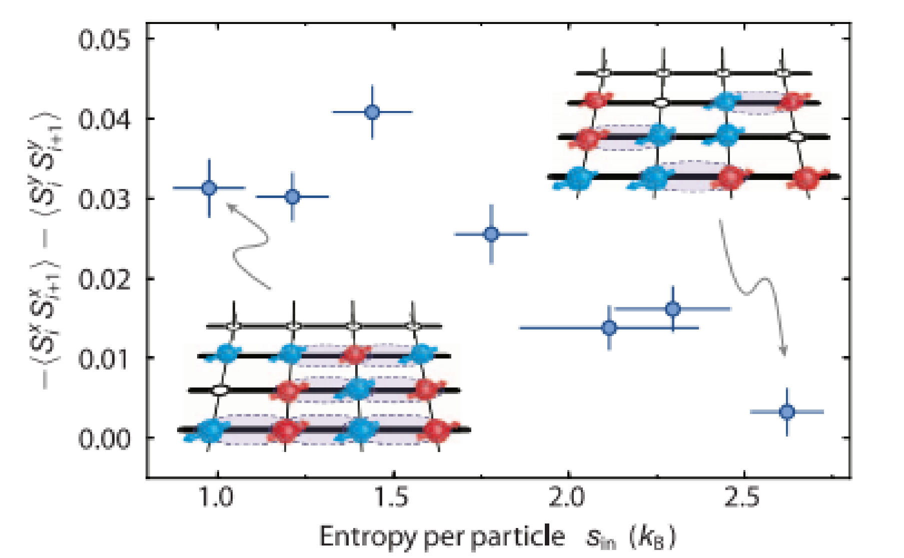
$$k_B T_{Neel} \sim J$$

Note: bosons have spin exchange too, but positive sign

AFM ordering in Fermi-Hubbard gases

Signatures:

Doublon-production rate



g. 4. **Nearest-neighbor antiferromagnetic order. (A)** Transverse spin

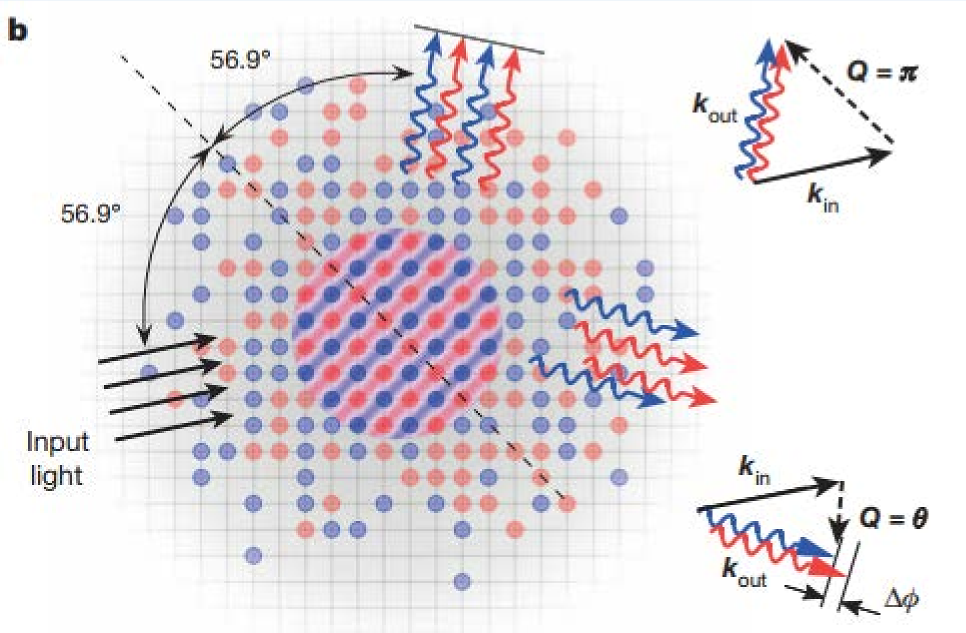
Greif, et al. 2013

AFM ordering in Fermi-Hubbard gases

Signatures:

Doublon-production rate

Bragg scattering



Hart, et al. 2013

AFM ordering in Fermi-Hubbard gases

Signatures:

Doublon-production rate

Bragg scattering

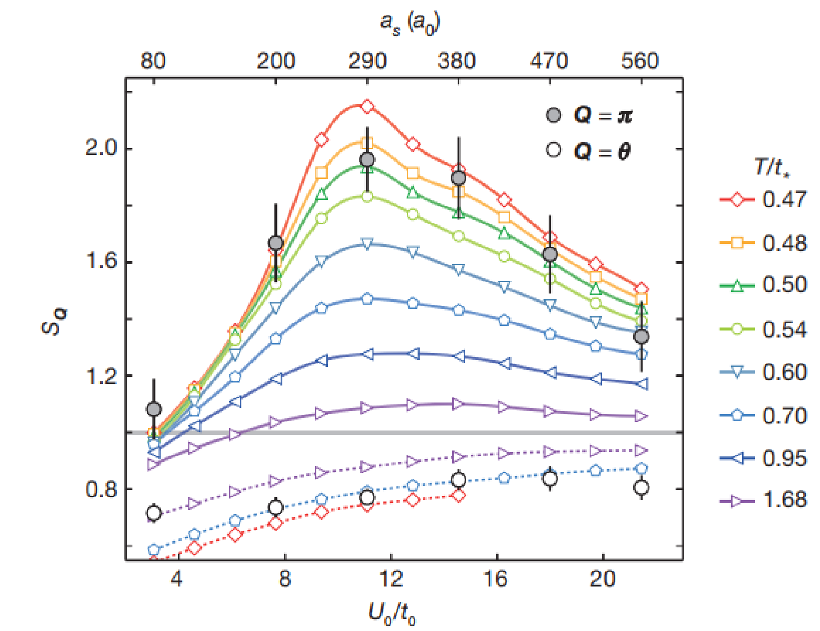
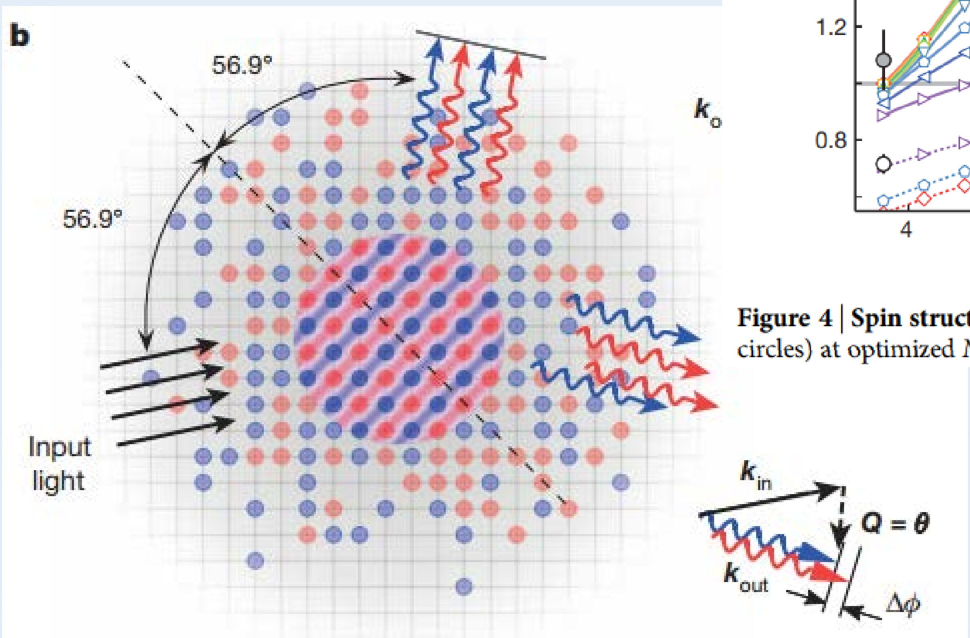


Figure 4 | Spin structure factor. Measured S_π (filled circles) and S_θ (open circles) at optimized N (see text) for various U_0/t_0. The values of the s-wave



Hart, et al. 2013

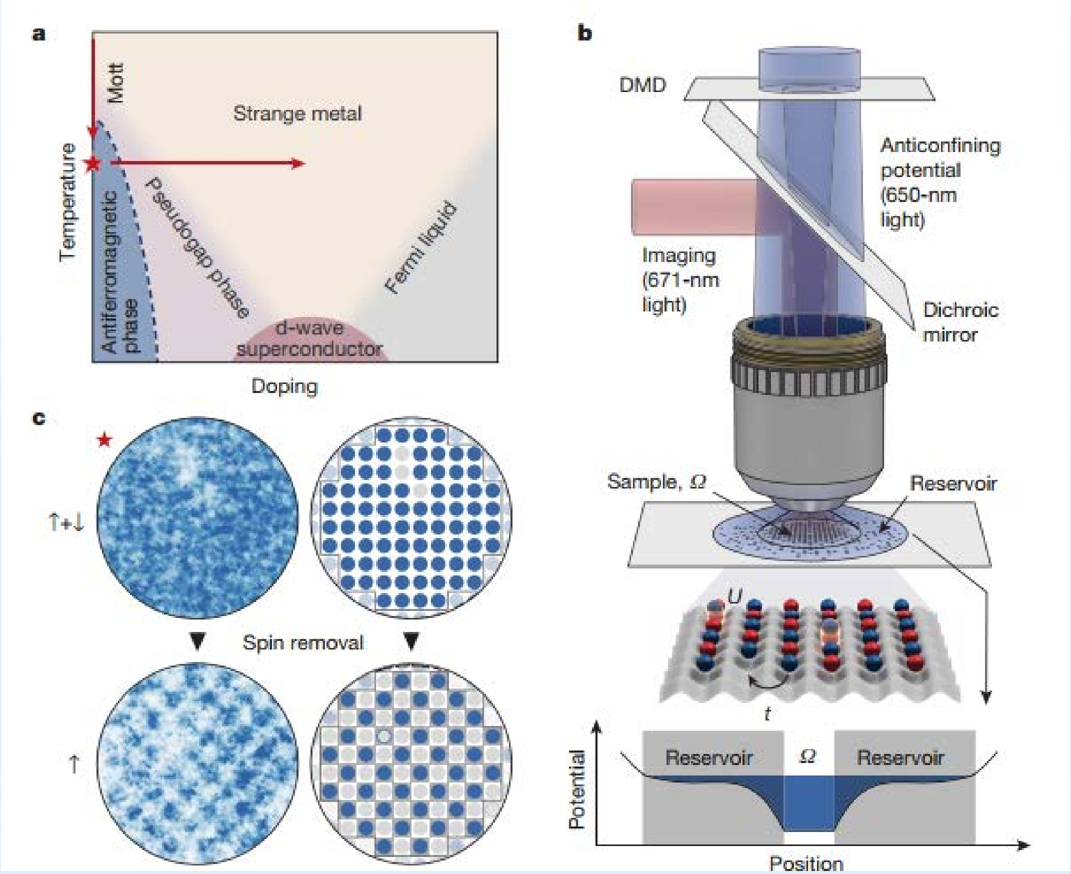
AFM ordering in Fermi-Hubbard gases

Signatures:

Doublon-production rate

Bragg scattering

Build a microscope, and then just look at density of a single spin component



Mazurenko, et al. 2017