Artificial gauge fields for neutral atoms

Lecture 25
PHYS 598-AQ
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Cold neutral atoms are charge neutral, and thus do not couple to electromagnetic gauge potentials in the same way that electrons do. We review some experimental techniques developed to engineer effective, artificial gauge fields for neutral atoms.

Rotation

by having atoms in a trap that rotates (i.e. a time-dependent potential), physics in the rotating ~~frame~~ displays a Coriolis force, which mimics the Lorentz force for charged particles in a magnetic field

Consider the Hamiltonian

$$H(t) = \frac{\hat{p}^2}{2m} + V(\vec{r}\,')$$

describing motion in a trap that rotates about the $\hat{z}$ axis at a rate Ω_{rot}.

(rotation about $\vec{\Omega} = \Omega_{rot}\hat{z}$)

such that

(inertial)

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ in the lab frame,

and also $\vec{p} = p_x\hat{x} + p_y\hat{y} + p_z\hat{z}$,

~~and~~

while $\vec{r}\,' = \underbrace{x'\hat{x} + y'\hat{y}} + z\hat{z}$ describes the position vector in the rotating frame, i.e. with

$$x' = x\cos(\Omega_{rot}t) + y\sin(\Omega_{rot}t)$$
$$y' = -x\sin(\Omega_{rot}t) + y\cos(\Omega_{rot}t)$$

the time-dependent, lab-frame Hamiltonian is related to a stationary rotating-frame Hamiltonian by the transformation

$$H(t) = R_z(t)\left[\frac{\hat{p}^2}{2m} + V(\vec{r})\right]R_z^{+}(t) \quad ,$$

w/ $R_z(t) = e^{-i\,t(\vec{\Omega}_{rot}\cdot\vec{L})/\hbar}$, w/ $\vec{L} = \vec{r}\times\vec{p}$

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the effective, time-independent Hamiltonian for the rotating frame is found by considering the ~~a~~ time-dependent Schrödinger equation for the states ~~$|\Psi_{rf}\rangle$~~ :

$|\Psi\rangle = R_z(t)\,|\Psi'\rangle$, inserting this into the form

$i\hbar\partial_t|\Psi\rangle = H(t)\,|\Psi\rangle$ yields $\boxed{i\hbar\partial_t|\Psi'\rangle = H'|\Psi'\rangle}$

w/ H' time-independent one of the form

$$H' = \frac{\vec{p}^{\,2}}{2m} + V(\vec{r}) - \vec{\Omega}_{rot} \cdot \vec{L}$$

w/ $\vec{\Omega}_{rot} \cdot \vec{L} = (\vec{\Omega}_{rot} \times \vec{r}) \cdot \vec{p}$

where $\vec{\Omega}_{rot} \cdot \vec{L} = i\hbar\, R_z^{+}(t)\, \partial_t\, R_z(t)$

(i.e., it comes from the time-dependence of the rotating frame)

this can be recast as

$$H' = \frac{(\vec{p} - \vec{A})^2}{2m} + V(\vec{r}) + W_{rot}(\vec{r}),$$

i.e. in terms of an effective vector potential

$$\vec{A} = m\vec{\Omega}_{rot} \times \vec{r} = \underbrace{m\Omega_{rot}(x\hat{y} - y\hat{x})}_{\text{symmetric gauge, relating to } B_z \hat{z}}$$

what we're looking for

an an "extra" part $W_{rot}(\vec{r}) = -\frac{1}{2} m\Omega_{rot}^2 (x^2 + y^2)$

upshot: this rotation of the system at a frequency Ω_{rot} gives us an effective magnetic field $\vec{B} = \vec{\nabla} \times \vec{A} = 2m\Omega_{rot}\hat{z}$,

[relating to the Coriolis force on the atoms]

as well as an anti-confining harmonic potential $W_{rot}(\vec{r})$ [stemming from the centrifugal potential]

If the atoms were simply held in a trap

$V(r) = \frac{1}{2} m \omega^2 (x^2 + y^2)$, then the system

is unstable for $\underbrace{\Omega_{rot} > \omega}$

[atoms fly away from the trap].

In experiment, one can equivalently either rotate the trap with respect to the atoms, or alternatively rotate the atoms (apply some orbital angular momentum) relative to the trap.

The latter is generally what has been done in experiment.

Geometric gauge potentials

An effective gauge potential shows up quite naturally in a cold atom system whenever some internal d.o.f. of the atom is coupled to the atom's center of mass motion [e.g., this shows up naturally in Sisyphus cooling]

So, we have ~~two~~ contributions to the Hamiltonian acting on some atom $\Rightarrow$ $\hat{H}_{\text{external}}$ and $\hat{H}_{\text{internal}}$

let's assume that $\hat{H}_{ext}$ is explicitly independent of the atom's internal d.o.f., and is of the form

$$\hat{H}_{ext} = \left[\frac{\vec{p}^{\,2}}{2m} + V(\vec{r},t)\right] \otimes \mathbb{1}_{int}$$

To get a geometric gauge potential, we'll assume that $\hat{H}_{\text{internal}}$ has some dependence on position or momentum, i.e. that the internal structure of the atom depends on position, e.g., which could be accomplished w/ a "coupling" field that has a spatially-varying amplitude, frequency, or phase.

we'll just write $\hat{H}_{int} = \hat{H}_{int}(\vec{r},t)$

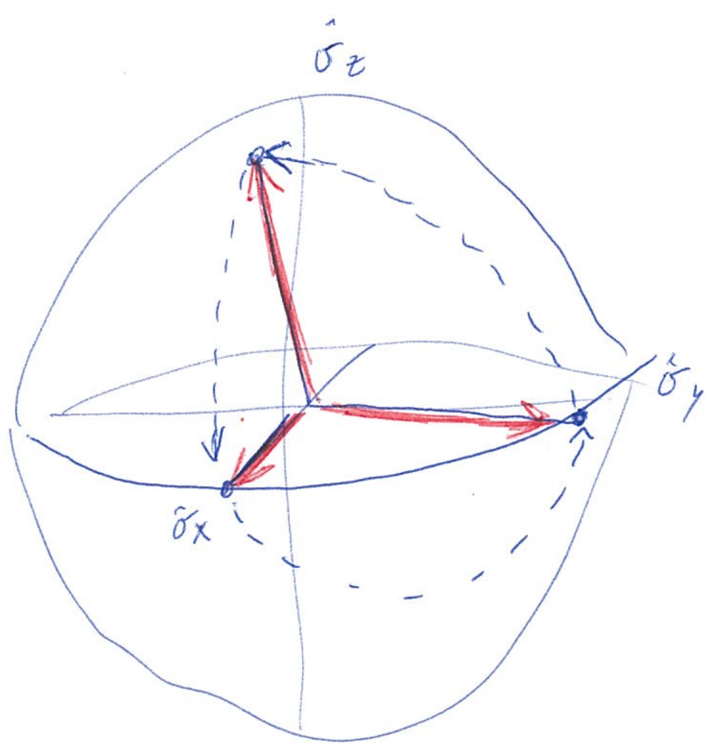
One specific (simplified) picture of how this can give rise to a geometric gauge potential comes from considering the fact that the eigenstates of $\hat{H}_{int}$ will vary as a function of $\underline{\vec{r}}$ and $\underline{t}$, in general.

Let's ignore t-dependence, and assume only that $\hat{H}_{int}$ varies with position. Let's imagine that we prepare an atom that is localized in space at some specific $\vec{r}$, in the ground state of $\hat{H}_{int}(\vec{r})$.

As we drag the atom around in space, <u>slowly</u>, it will remain in the ground state of $\hat{H}_{int}(\vec{r})$, but it's internal state composition will change, reflecting the <u>local</u> ground state properties.

By making some closed path in real space, we will return the state of the atom to it's original spin configuration. However, it has the possibility of picking up a global, geometric (Berry's) phase if the path has some enclosed area in space. That is, if the path in real-space encloses some area, then this can be represented as a path on the Bloch sphere [here we assume just two internal state

with some enclosed geometric phase.



This geometric phase acquired upon traveling along closed paths in space can be directly analogous to the Aharanov-Bohm phase acquired by charged particles in an electromagnetic vector potential.

More detail (for N internal states)

assume $$\hat{H}_{int}(\vec{r},t) = \sum_{n,m=1}^{N} |n\rangle M_{nm}(\vec{r},t) \langle m|$$

w/ $|m\rangle, |n\rangle$ the "bare" internal states and M_{nm} some matrix elements relating to coupling fields or state-dependent potentials.

The dressed states (eigenstates) can be expressed in terms of the bare states $|m\rangle$ by some unitary transformation

$$|\eta_m(\vec{r},t)\rangle = \hat{R}|m\rangle, \quad w/ \quad \hat{R}^{+} \hat{H}_{int}(\vec{r},t) \hat{R} = \hat{\varepsilon}$$

one can eventually show that an effective Hamiltonian of the form

$$\hat{H} = \frac{(\vec{p} - \hat{A})^2}{2m} + \vec{V}(\vec{r},t) + \hat{\varepsilon} + \hat{\Phi} \quad \text{emerges,}$$

where $$\hat{A} = i\hbar \hat{R}^{+} \underbrace{\vec{\nabla} \hat{R}}_{} \quad \text{and} \quad \hat{\Phi} = -i\hbar \hat{R}^{+} \underbrace{\partial_t \hat{R}}_{}$$

needs spatial variations of $\hat{R}$

needs temporal dependence

this can give effective $\vec{B}$-fields, e.g.

this can give effective $\vec{E}$-fields, e.g.

Laser-assisted tunneling

The motion of cold atoms in static optical lattices is generally derived from quantum tunneling between lattice orbitals w/ similar energies.

- Some consequences:

- The tunneling matrix elements decay w/ spatial separation in an exponential fashion (usually only nearest-neighbor is dominant),

- Usually, i.e. in the absence of a strong gradient, the tunneling is orbital / bond preserving

- the quantum mechanical tunneling element is necessarily real-valued $\quad t_{ij} = t_{ji} = t_{ij}^*$

thus, one does not naturally get terms reminiscent of effective magnetic fields or a vector potential $\hat{A}$, where by the Peierls substitution.

$E_0(\hbar k \to \vec{p} - \hat{A})$

$$t^x_{mn} = t e^{i\varphi_x(m,n)}, \quad t^y_{mn} = t e^{i\varphi_y(m,n)}$$

$$\boxed{U_{jk} = e^{i\varphi_{jk}} = \exp\left[\frac{i}{\hbar}\int_j^k \hat{A}\cdot d\vec{l}\right]}$$

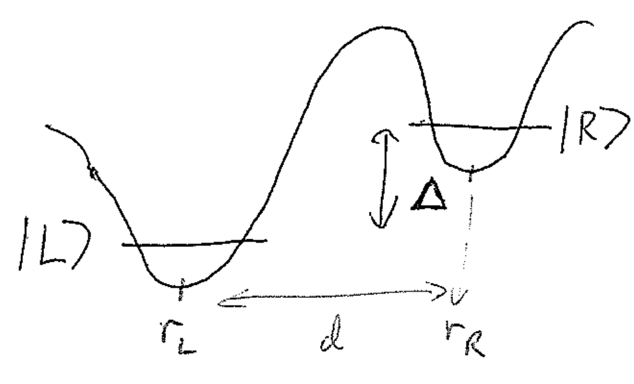
One way to engineer a tunneling phase,

and thus some artificial gauge field (by judicious choice of these phases

is to use light-driven tunneling to transfer population between sites. Similar to 2-state dynamics of internal states, as represented on a Bloch sphere, the phase of the applied oscillating field (parametric coupling) sets the effective torque vector relating to the dynamics, or equivalently the coupling phase.

To accomplish this, one has to first suppress normal quantum tunneling. One can apply a linear potential offset [which for neighboring sites mimics a B_z field] to accomplish this.

$\Delta >> |t|$



tunneling is suppressed.

Laser-induced transitions will occur if

① the modulation frequency of some perturbation matches $\hbar\omega_{mod} = \Delta$, and if

② the applied perturbation can physically couple (has non-zero matrix element) the states $|L\rangle$ and $|R\rangle$.

by applying the perturbation

$$V_{couple}(\vec{r},t) = \hbar\Omega\cos(\vec{q}\cdot\vec{r} - \Delta\omega t + \Delta\varphi)$$

[representing a moving lattice potential formed by interfering lasers]

one can get an effective coupling term between Wannier functions at the L + R wells of

$$J^{eff}_{L\to R} = \frac{\hbar\Omega}{2}\int w_R^*(\vec{r}-\vec{r}_R)\, w_L(\vec{r}-\vec{r}_L)\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r}\; e^{i\Delta\varphi}$$

if $\underbrace{\Delta\omega = \Delta/\hbar}$ (resonant coupling)

by setting $\vec{q}$ in a 2D geometry, one can mimic 2D artificial vector potentials