Lecture 5: hyperfine structure

Readings: Foot Chapter 6.1-6.2

So far: - gross structure from electrostatic interactions
- fine structure, in large part from the electron spin

Today: - hyperfine structure
- we’ll mostly be interested in new level structure coming from the coupling of the spin of the nucleus and the total angular momentum of the electron
- also, energy corrections due to various nuclear properties
$|F, m_F\rangle$ states are the ones utilized in low-energy (ultracold) atomic physics experiments

$\rightarrow$ Thermal energy scales will be below $\Delta E$, only coherent control of internal D.O.F.

$\Delta E_{\text{gross}} \sim 100 \text{ THz}$

$\Delta E_{\text{fs}} \sim <1-10 \text{ THz}$

$\Delta E_{\text{hfs}} \sim \text{few GHz}$

$k_B T \sim \text{few kHz}$
pseudo-spin states

$|F, m_F\rangle$ states are the ones utilized in low-energy (ultracold) atomic physics experiments.

→ Thermal energy scales will be below $\Delta E$, only coherent control of internal D.O.F.

Example – lithium 6 hyperfine states
pseudo-spin states

Many experiments are based on spin-polarized gases (all population in one hyperfine state)

Lots of interesting physics based on spin mixtures

Greiner group, AFM ordering & ...

Jin group, BEC-BCS

Zwierlein group, Fermi mixture at unitarity
pseudo-spin states

Unlike electrons, not limited to only two spin states

SU(N) Mott insulator (Folling, Fallani, Takahashi, etc.)
pseudo-spin states

Many bosonic mixture experiments too

Tarruel group, quantum droplets

Stamper-Kurn group, spontaneous spin textures
Ground state:
$L = 0$
$S = 1/2$ for the lone electron

$I = 1/2$ for the lone proton

$\vec{F} = \vec{I} + \vec{S} \ (l = 0)$
hydrogen hyperfine structure

Ground state:
\[ L = 0 \]
\[ S = 1/2 \] for the lone electron

\[ I = 1/2 \] for the lone proton

\[ \vec{F} = \vec{I} + \vec{S} \quad (l = 0) \]

\[ F = 0 \text{ or } 1 \]

\[ \Delta E / h \approx 1.42 \text{ GHz} \]

\[ \lambda \approx 21 \text{ cm} \]

really important to observational astronomy
Importance in radio astronomy

F = 1 states are easily excited in thermal equilibrium at $T_{\text{space}} \sim 3\, \text{K}$

$\Delta E / h \approx 1.42\, \text{GHz}$

$\Delta E / k_B \approx 68\, \text{mK}$

Excited states can undergo spontaneous emission $\rightarrow$ very small decay rate, but there’s a lot of “stuff” out there (hydrogen makes up most of baryonic matter)

$\Gamma / 2\pi \sim 10^{-15}\, \text{Hz}$
This was even before the hydrogen maser was invented!
the hydrogen maser

Not the best clock / freq. standard, inaccuracy due to collisions with walls

$\Delta E/h = 1,420,405,751.7667(9) \text{ Hz}$

(measured by comparison to Cs clock)
radio astronomy at 21 cm

Milky Way

NGC 3198
radio astronomy at 21 cm

Detect galaxy rotation through Doppler shifts

NGC 3198

https://ned.ipac.caltech.edu/level5/March05/Bosma/Bosma4_4.html
evidence for dark matter

Stefania.deluca of Wikimedia Commons

NGC 3198

https://ned.ipac.caltech.edu/level5/March05/Bosma/Bosma4_4.html
Hyperfine constants $A_{\text{hfs}}, B_{\text{hfs}}$

Note the range of nuclear spins values and the large variations in $A_{\text{hfs}}$ values

<table>
<thead>
<tr>
<th>Element</th>
<th>Abundance</th>
<th>$I$</th>
<th>$F_g$ ($J_g=\frac{1}{2}$)</th>
<th>$A$ (MHz)</th>
<th>$\Delta E_{\text{hfs}}$ (GHz)</th>
<th>$F_e$ ($J_e=\frac{1}{2}$)</th>
<th>$A$ (MHz)</th>
<th>$F_e$ ($J_e=\frac{3}{2}$)</th>
<th>$A$ (MHz)</th>
<th>$B$ (MHz)</th>
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<tbody>
<tr>
<td>$^1\text{H}$</td>
<td>99.985</td>
<td>$\frac{1}{2}$</td>
<td>0,1</td>
<td>1420.405</td>
<td>10.968</td>
<td>0,1</td>
<td>59.18</td>
<td>1,2</td>
<td>23.67</td>
<td>-</td>
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<tr>
<td>$^6\text{Li}$</td>
<td>7.5</td>
<td>1</td>
<td>$\frac{1}{2}, \frac{3}{2}$</td>
<td>152.137</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$^7\text{Li}$</td>
<td>92.5</td>
<td>$\frac{3}{2}$</td>
<td>1,2</td>
<td>401.752</td>
<td>10.091</td>
<td>1,2</td>
<td>45.914</td>
<td>0,1,2,3</td>
<td>-3.055</td>
<td>-0.221</td>
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<tr>
<td>$^{23}\text{Na}$</td>
<td>100</td>
<td>$\frac{3}{2}$</td>
<td>1,2</td>
<td>885.813</td>
<td>515.53</td>
<td>1,2</td>
<td>94.3</td>
<td>0,1,2,3</td>
<td>18.69</td>
<td>2.90</td>
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<tr>
<td>$^{39}\text{K}$</td>
<td>93.26</td>
<td>$\frac{3}{2}$</td>
<td>1,2</td>
<td>230.859</td>
<td>1730.4</td>
<td>1,2</td>
<td>28.85</td>
<td>0,1,2,3</td>
<td>6.06</td>
<td>2.83</td>
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<tr>
<td>$^{40}\text{K}$</td>
<td>0.0117</td>
<td>4</td>
<td>$\frac{7}{2}, \frac{9}{2}$</td>
<td>-285.731</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5/2, $\frac{7}{2}, \frac{9}{2}, \frac{11}{2}$</td>
<td>-7.59</td>
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<tr>
<td>$^{41}\text{K}$</td>
<td>6.73</td>
<td>$\frac{3}{2}$</td>
<td>1,2</td>
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<td>$^{85}\text{Rb}$</td>
<td>72.17</td>
<td>$\frac{5}{2}$</td>
<td>2,3</td>
<td>1011.910</td>
<td>7123.0</td>
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<td>$^{87}\text{Rb}$</td>
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<td>406.2</td>
<td>1,2</td>
<td>406.2</td>
<td>0,1,2,3</td>
<td>84.845</td>
<td>12.52</td>
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<tr>
<td>$^{133}\text{Cs}$</td>
<td>100</td>
<td>$\frac{7}{2}$</td>
<td>3,4</td>
<td>2298.157</td>
<td>16611.8</td>
<td>3,4</td>
<td>291.90</td>
<td>2,3,4,5</td>
<td>50.34</td>
<td>-0.38</td>
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</tbody>
</table>

TABLE C.4. Fine- and hyperfine structure constants for the various alkali-metal atoms. The parameters $A$ and $B$ can be used in Eqs. 4.2 and 4.3 to calculate the shift and the splitting from the hyperfine interaction. The values for $A$ and $B$ are from Ref. 28.

Metcalf & van der Straten, Laser Cooling and Trapping
the cesium fountain clock

$^{133}\text{Cs}$

$6^2S_{1/2}$

$\Delta E$ determines the SI second (and meter)
the cesium fountain clock

$\Delta E$ determines the SI second (and meter)